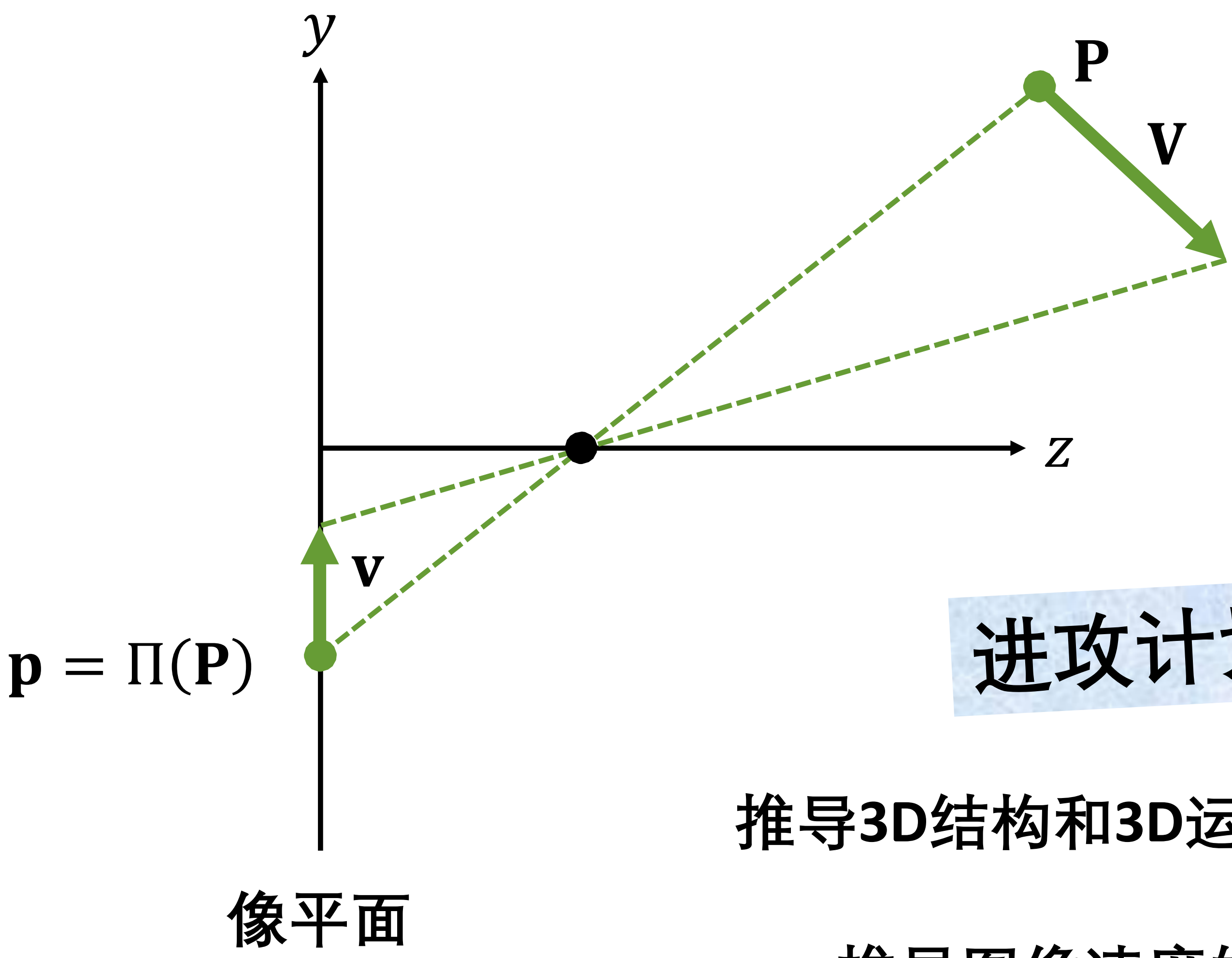
计算机视觉

姿态估计

中国传媒大学
COMMUNICATION UNIVERSITY OF CHINA



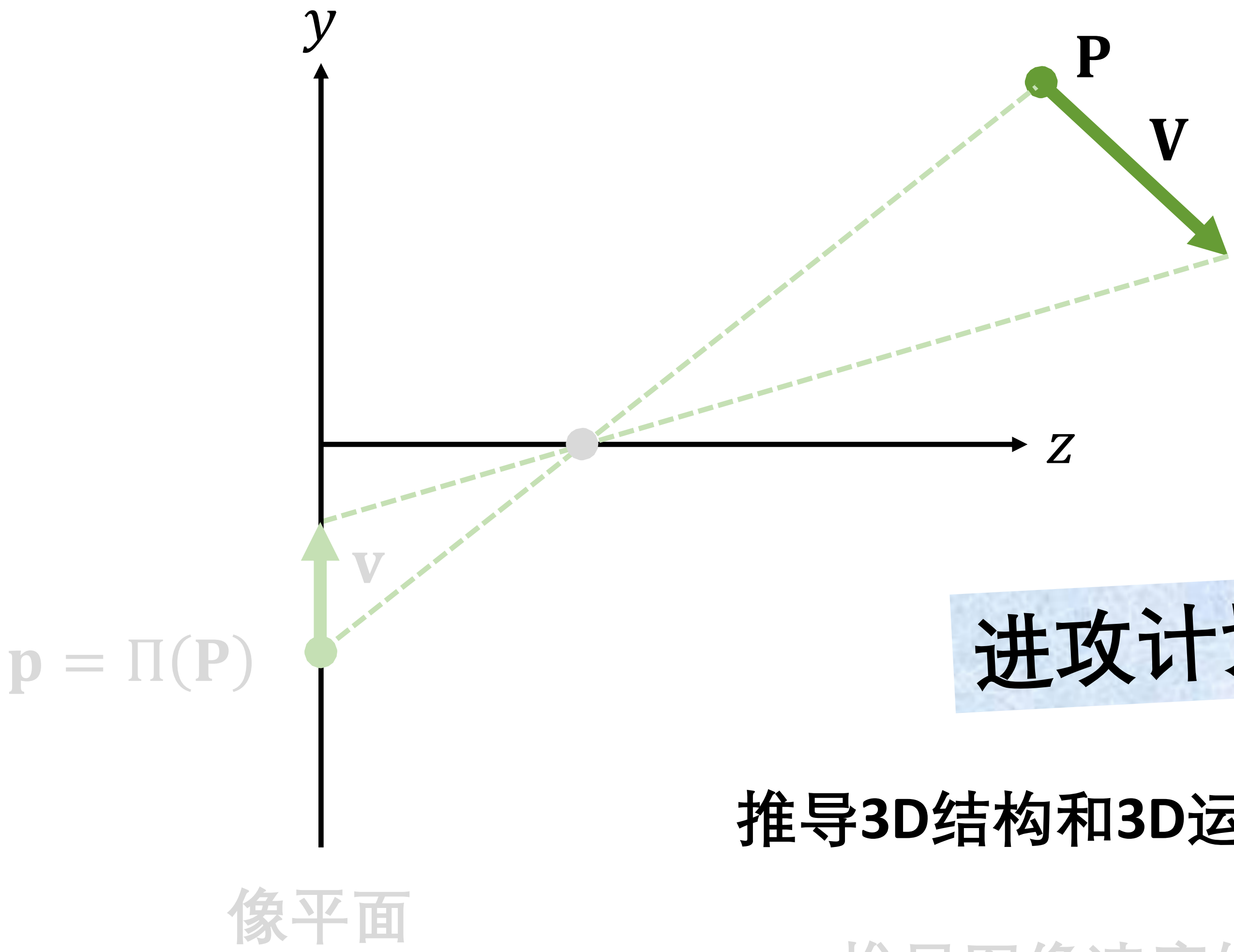
由运动恢复结构

进攻计划

推导3D结构和3D运动的表达式

推导图像速度的表达式

关联3D和2D参数



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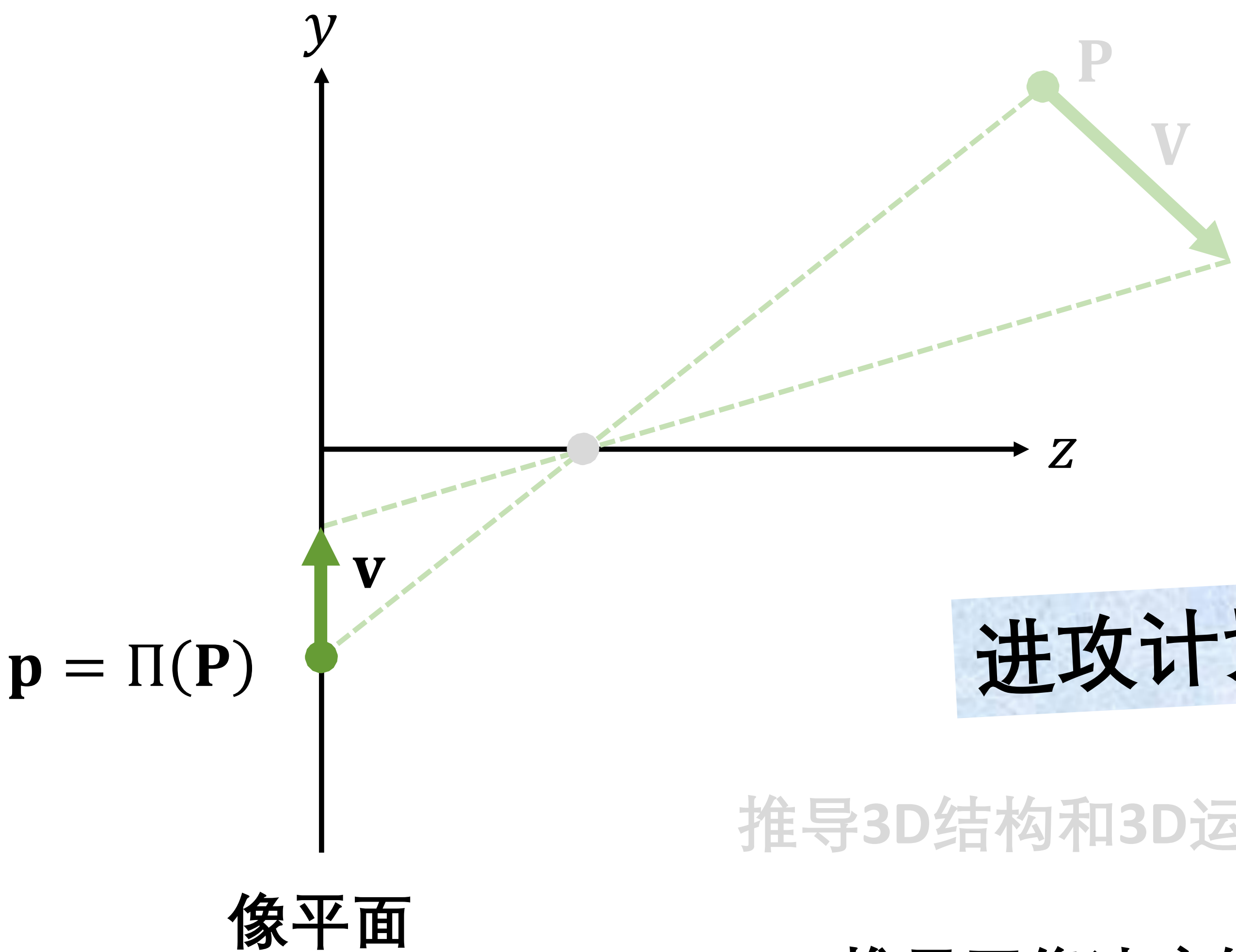

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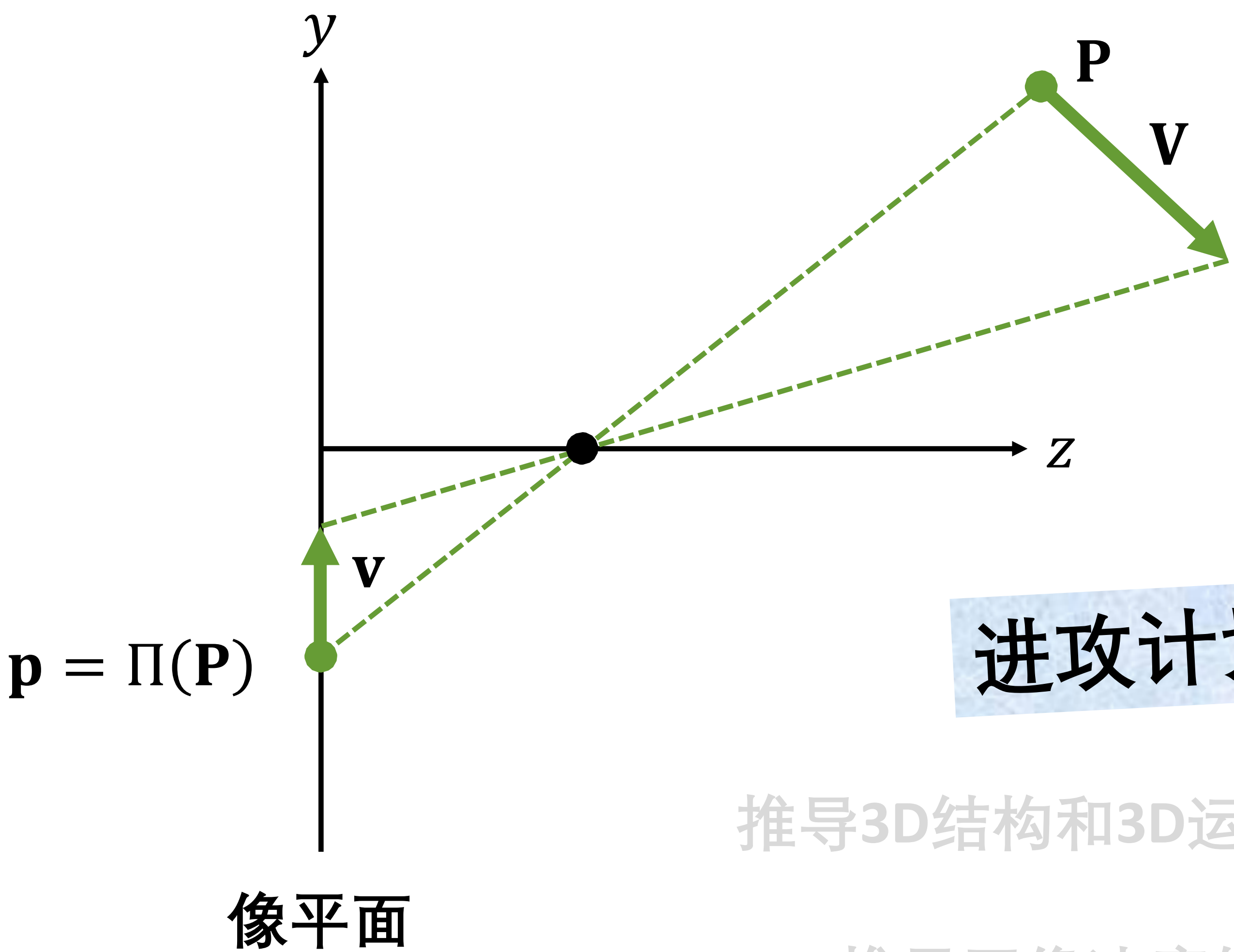

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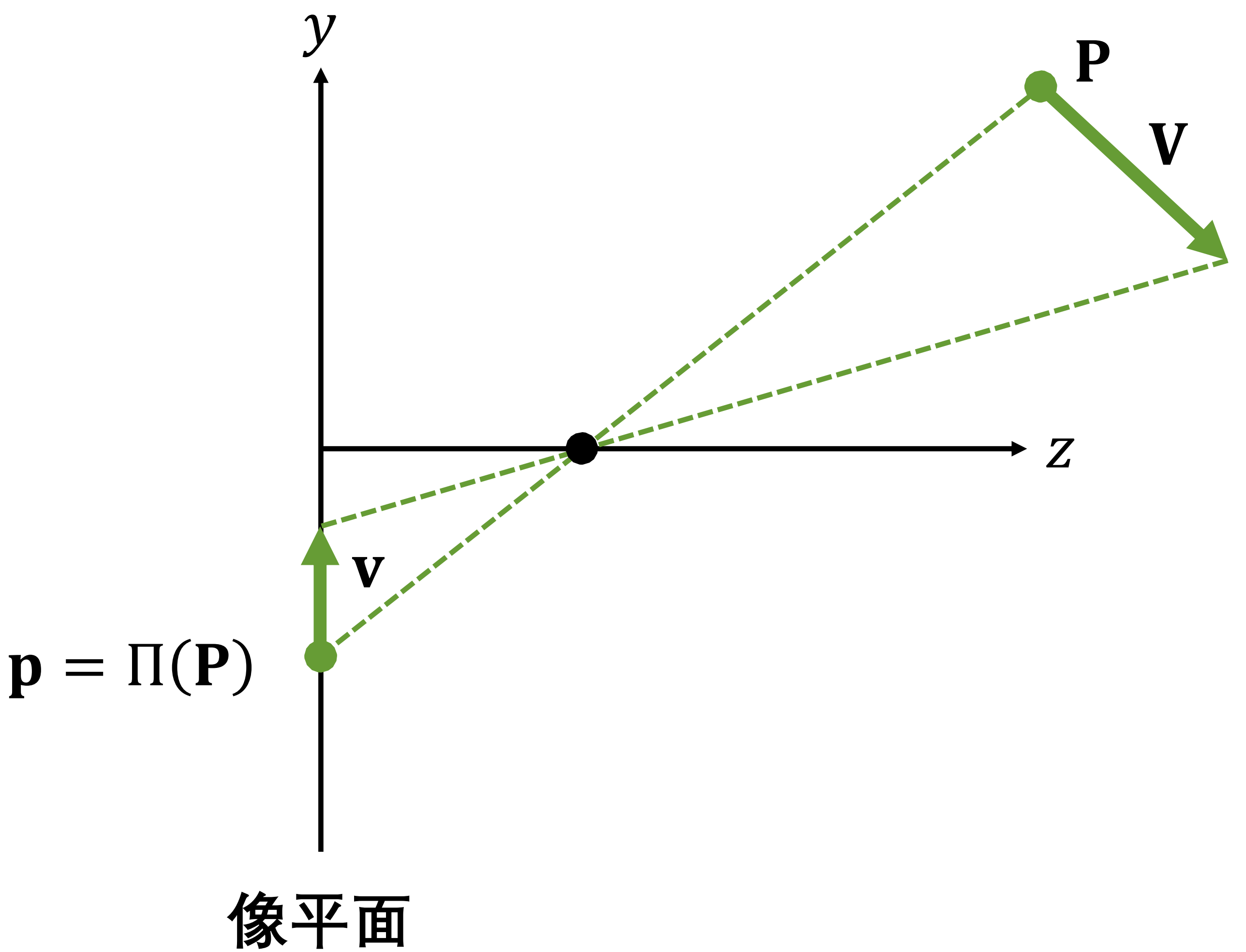


由运动恢复结构

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

如何估计相机的姿态?

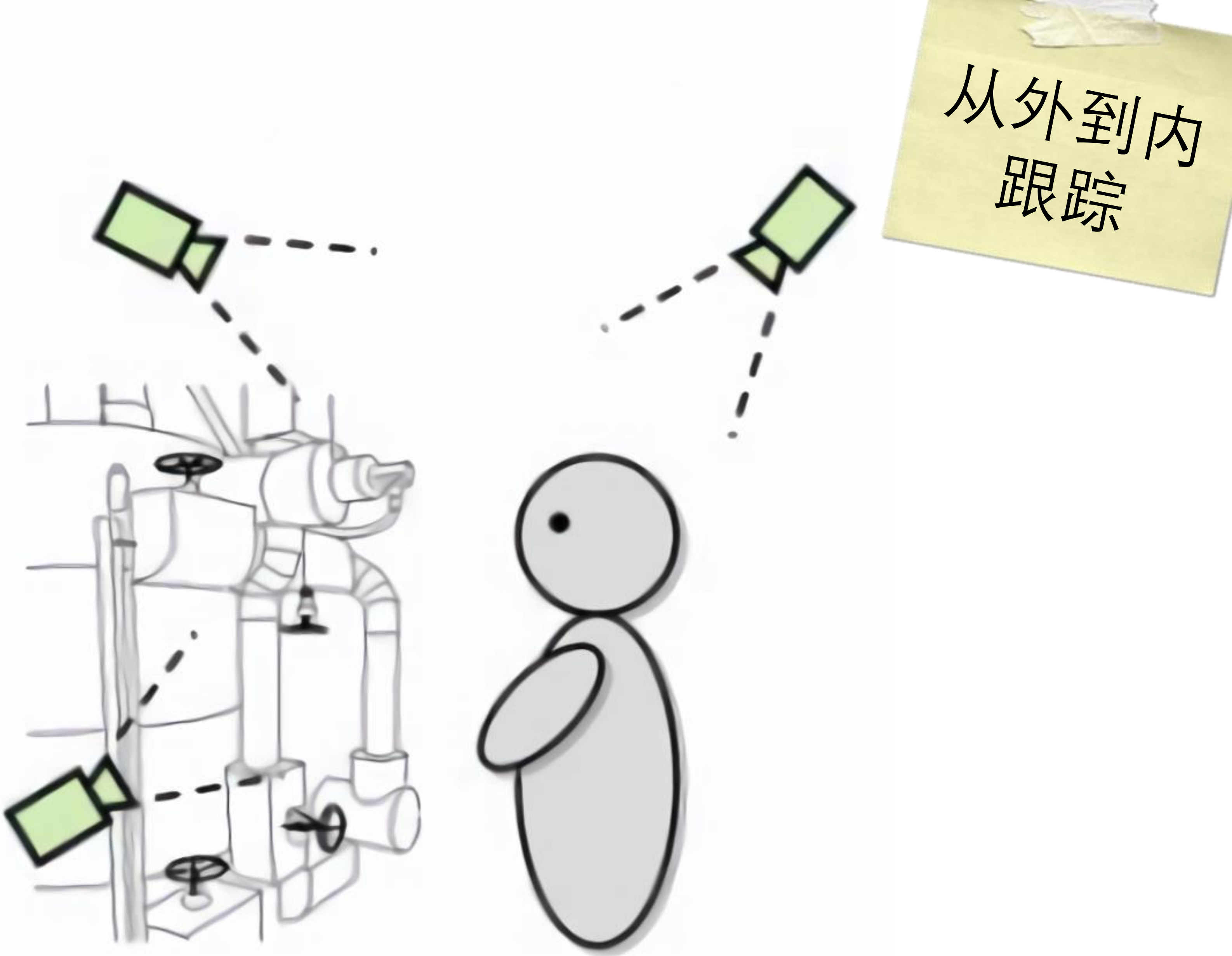
如何估计相机的姿态？

3D位置和朝向

如何估计相机的姿态？

3D位置和朝向

世界坐标下平移和旋转


从外到内
跟踪

鸣谢：Ryerson Multimedia Research Lab

HTC Vive

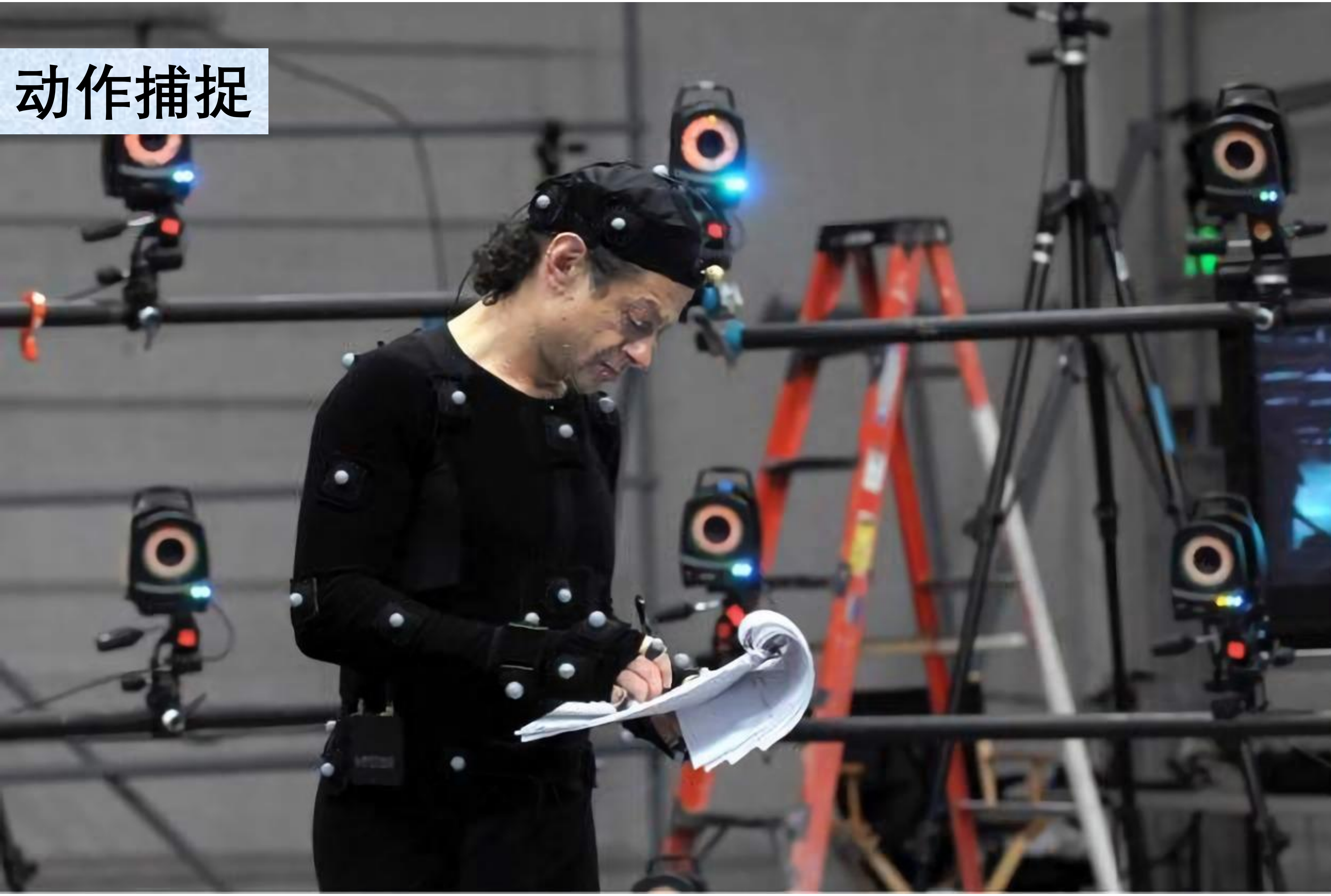

htc
EVEREST VR
OVERKILL VR
VIVEPORT
SUBSCRIPTION
Tilt Brush
by Google

HTC灯塔

CAVE

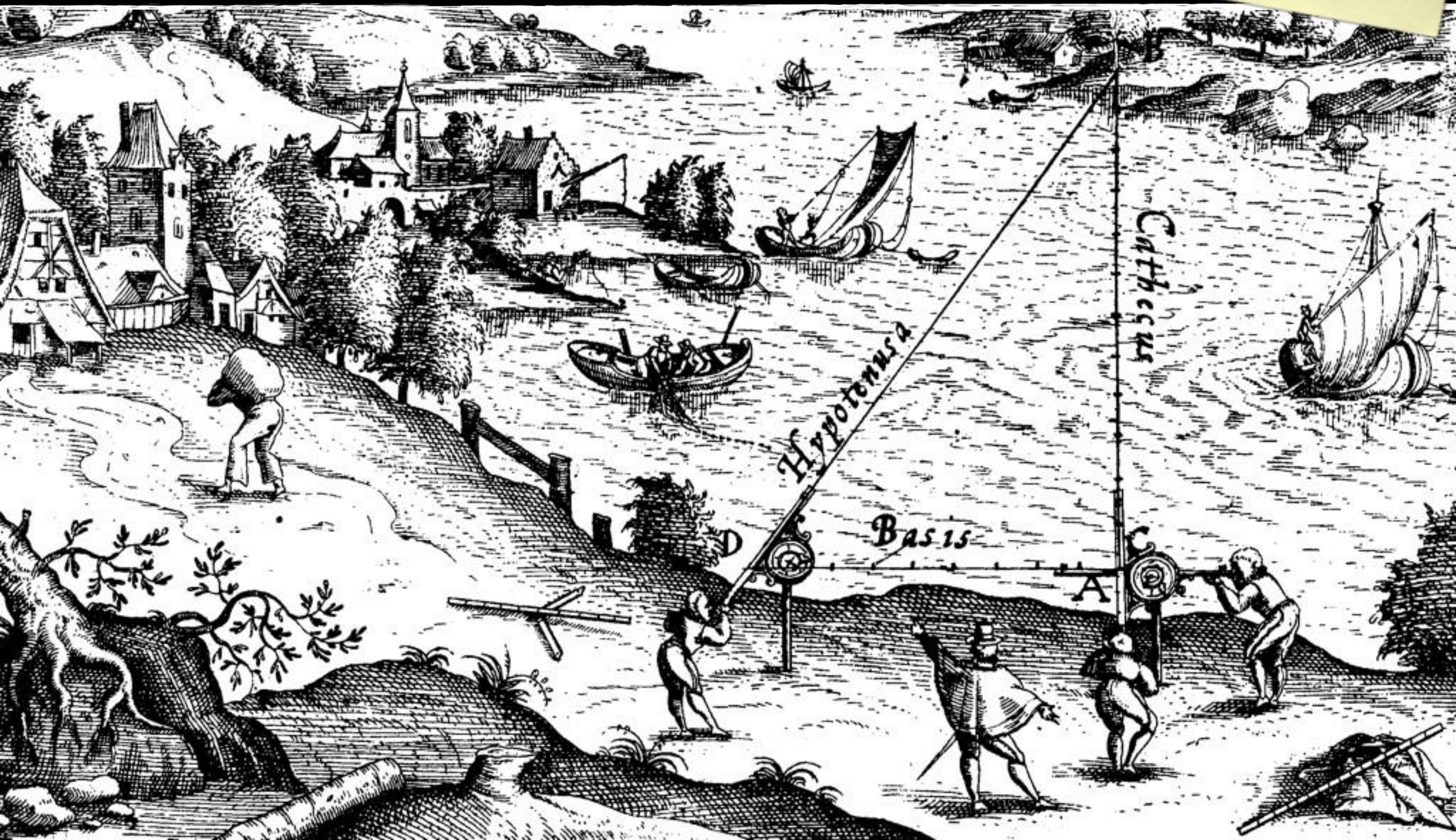
鸣谢：Ryerson Multimedia Research Lab

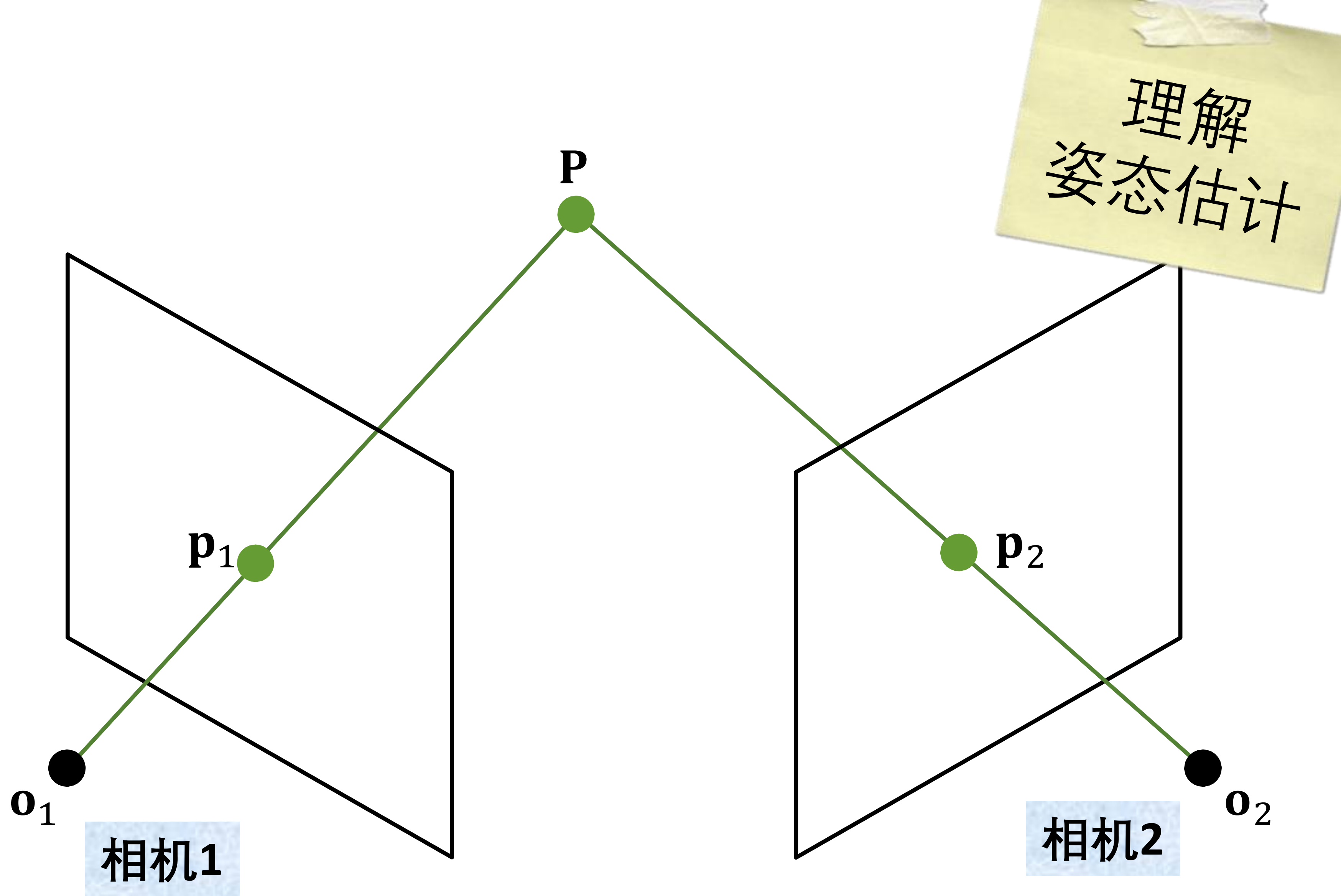
动作捕捉



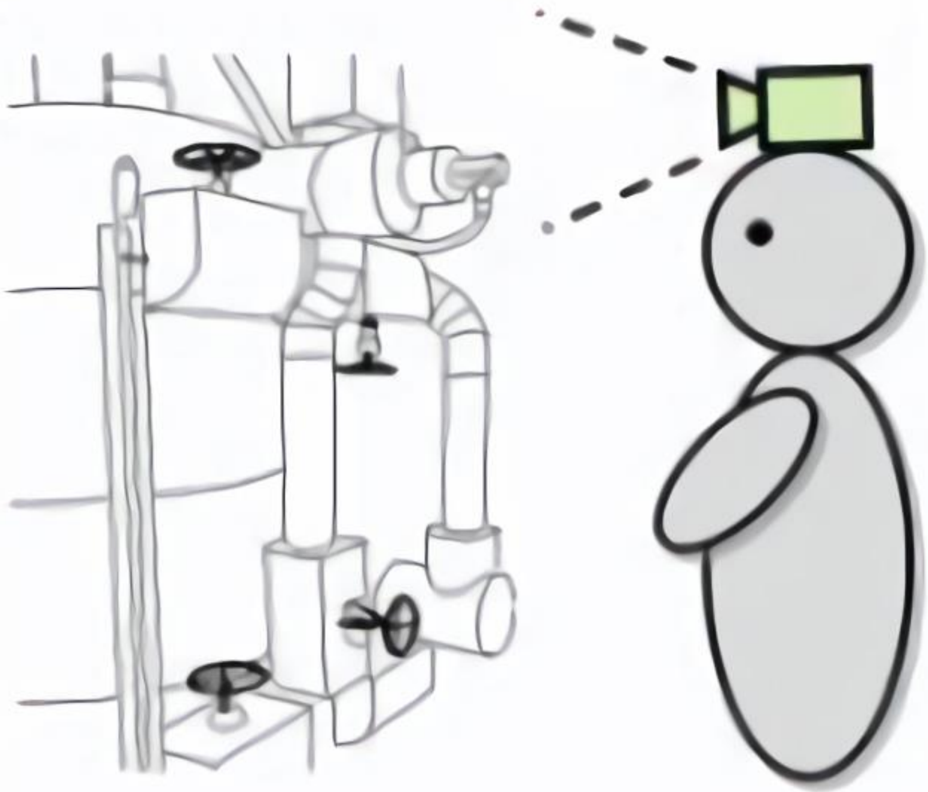
三角测量

理解
姿态估计




理解
姿态估计
P
p1
p2
o1
o2
相机1
相机2

从内到外
跟踪



Power Painter.EASM
AR
POWERPAINTER
5GPH
1
2
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8
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10
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12
13

...of coordinated covers
...ook. Cover: 50% viscose/
...4cm. Sågmyra grey/check.
...**chair £80** Hand made, gives soft
...rns. Each piece of furniture is unique.
..., H100cm. 202.016.82

Scan page to see more

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

问题陈述

3D空间中的点

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

图像中的点

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$
和相机模型$\mathbf{p} = \Pi(\mathbf{P})$

问题陈述

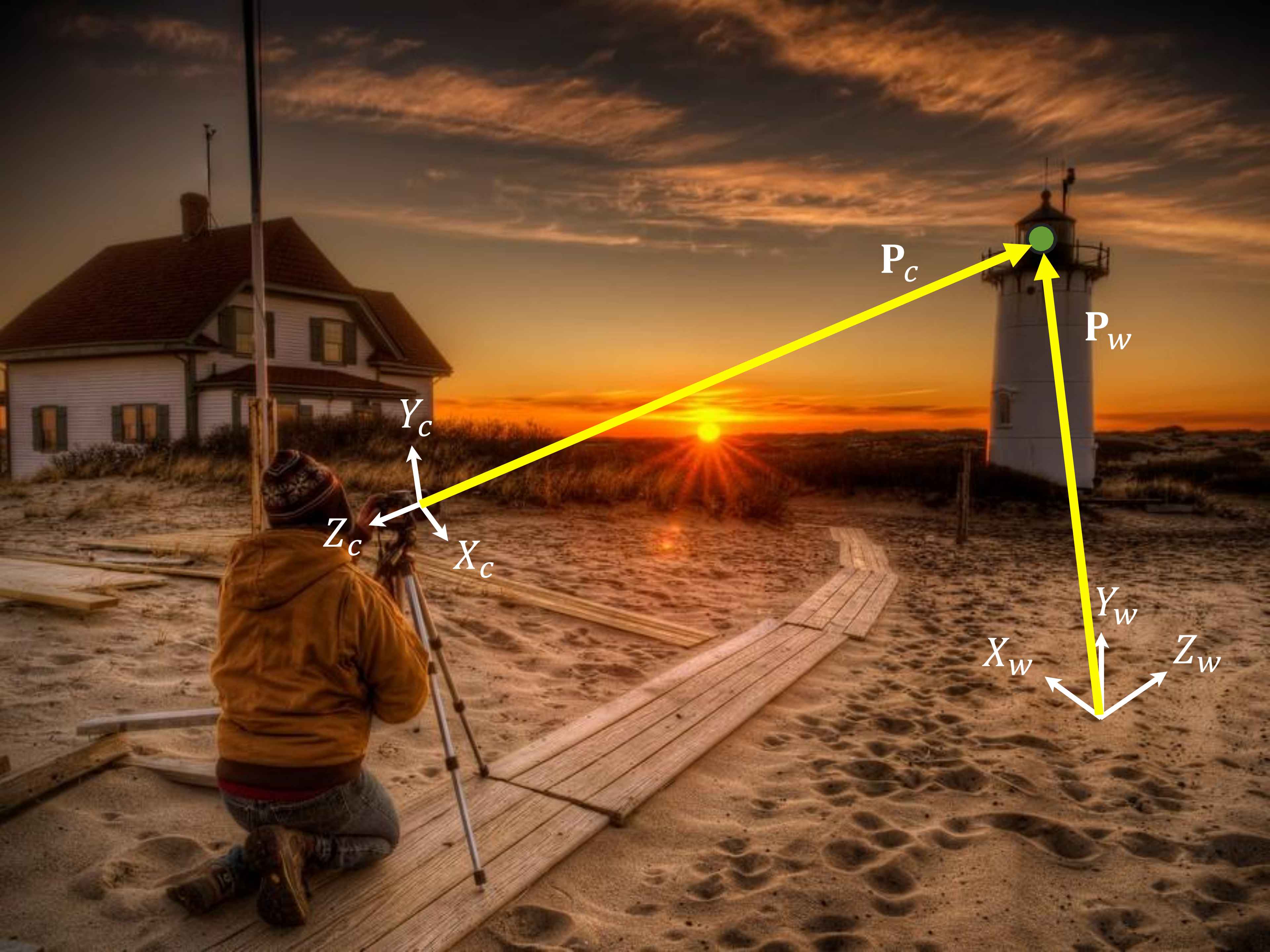
给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$
和相机模型$\mathbf{p} = \Pi(\mathbf{P})$

估计相机的“姿态”

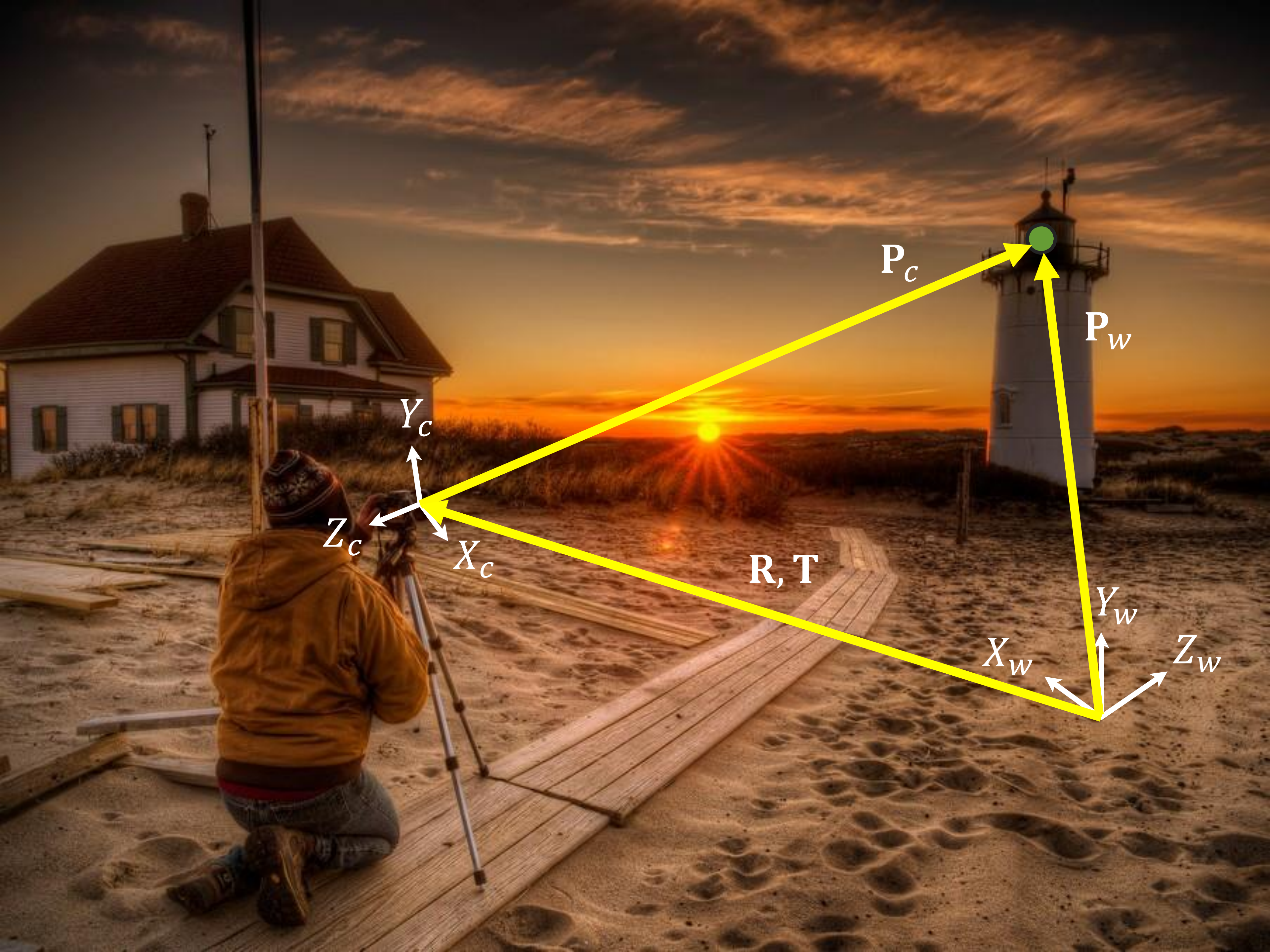
问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$
和相机模型$\mathbf{p} = \Pi(\mathbf{P})$

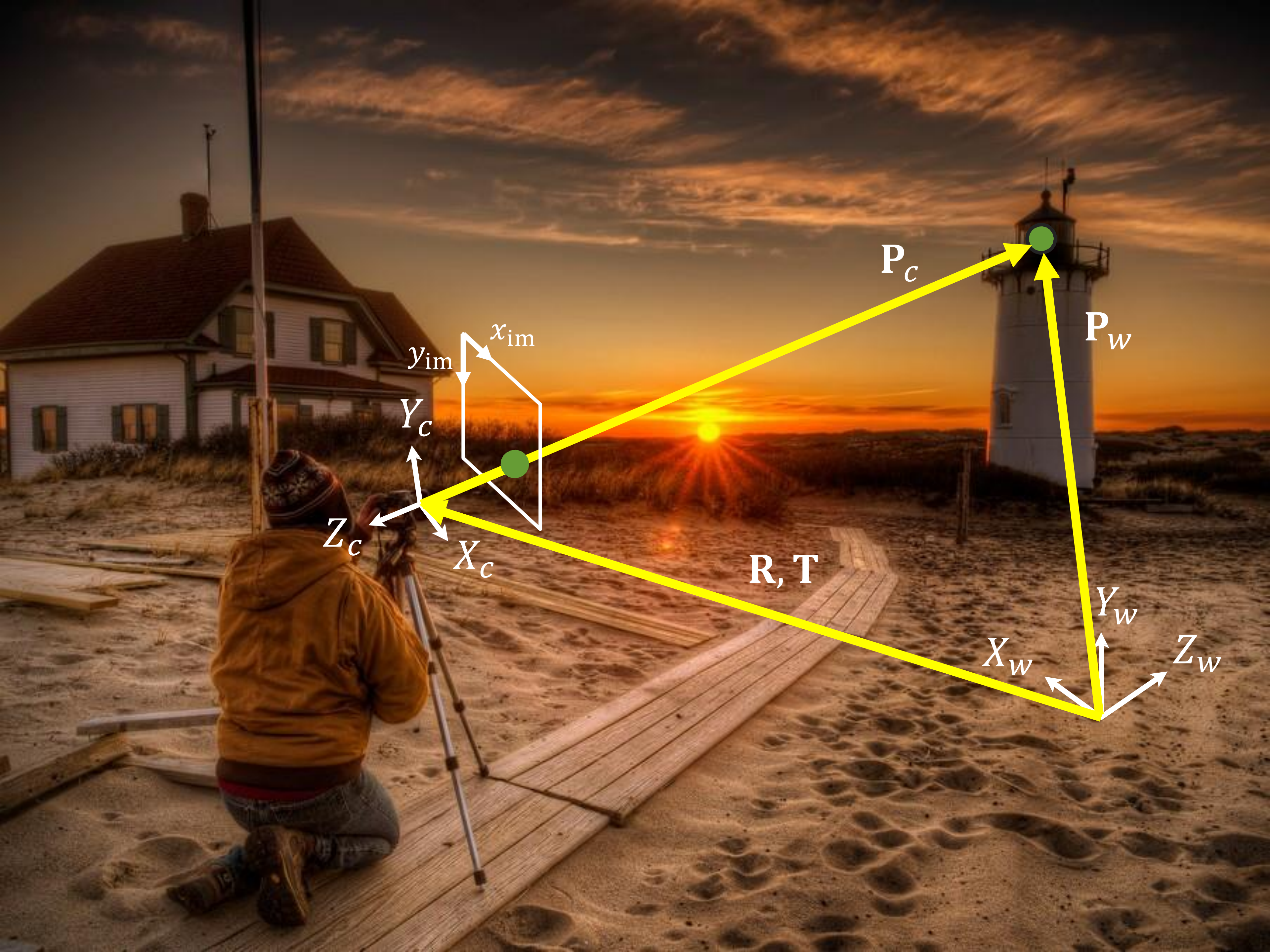
估计相机的“姿态”，即相机模型参数



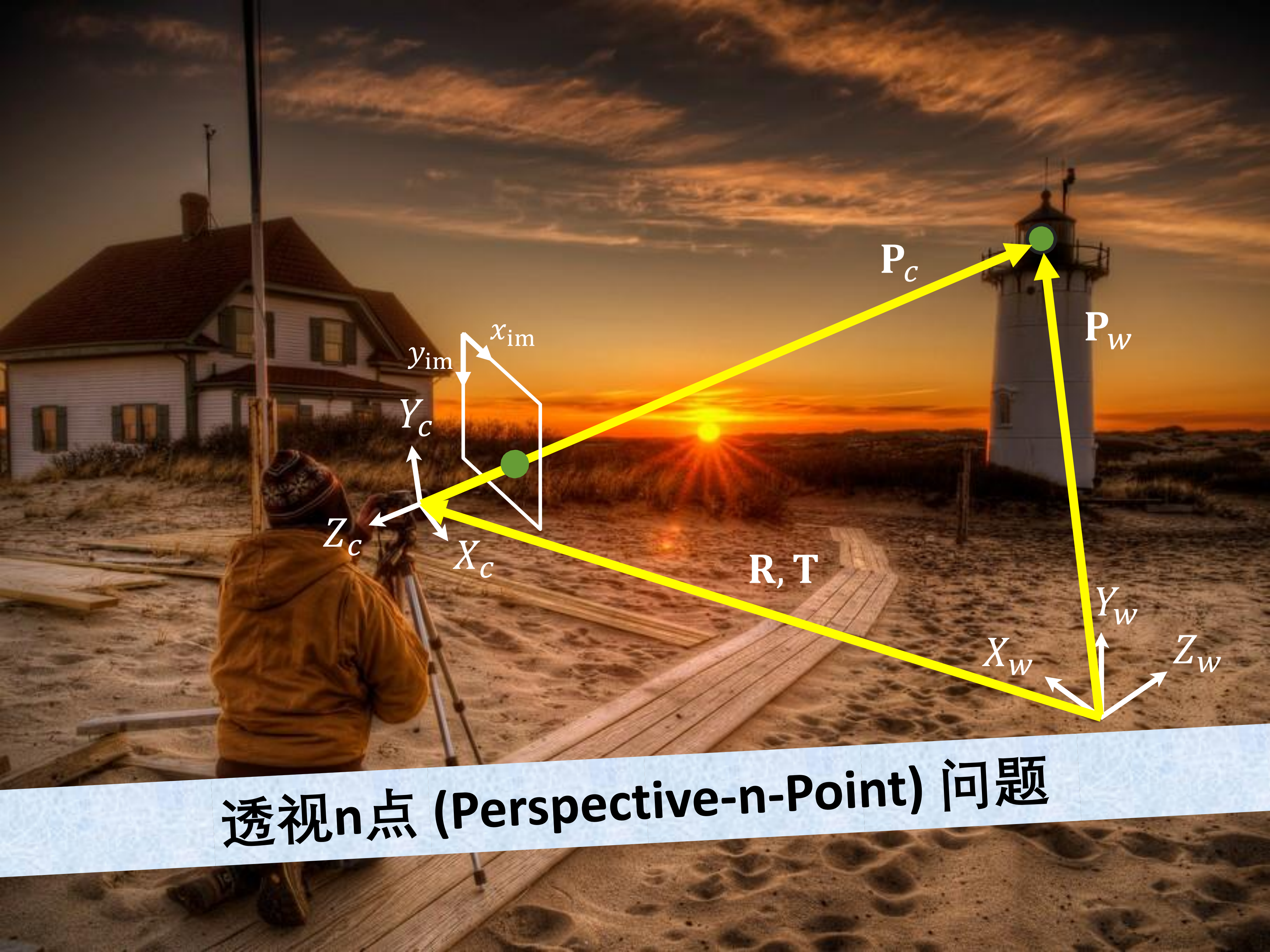
$\mathbf{P}_c$
$\mathbf{P}_w$
Y_c
Z_c
X_c
Y_w
X_w
Z_w



$\mathbf{P}_c$
$\mathbf{P}_w$
Y_c
Z_c
X_c
$\mathbf{R}, \mathbf{T}$
Y_w
X_w
Z_w


$\mathbf{P}_c$
$\mathbf{P}_w$
x_{im}
y_{im}
Y_c
Z_c
X_c
$\mathbf{R}, \mathbf{T}$
Y_w
X_w
Z_w

透视n点 (Perspective-n-Point) 问题



$$\begin{pmatrix} \alpha x_{im} \\ \alpha y_{im} \\ \alpha \end{pmatrix} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

内参矩阵 $\mathbf{M}_{\mathrm{int}}$

外参矩阵 $\mathbf{M}_{\mathrm{ext}}$

回顾

$$\begin{pmatrix} \alpha x_{im} \\ \alpha y_{im} \\ \alpha \end{pmatrix} = \underbrace{\begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{内参矩阵 } \mathbf{M}_{\text{int}}} \underbrace{\begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}}_{\text{外参矩阵 } \mathbf{M}_{\text{ext}}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

内参矩阵 $\mathbf{M}_{\text{int}}$

外参矩阵 $\mathbf{M}_{\text{ext}}$

内参矩阵是可逆的

回顾

$$\begin{pmatrix}\alpha x_{im}\\ \alpha y_{im}\\ \alpha\end{pmatrix}=\underbrace{\begin{pmatrix}-f/s_x & 0 & o_x\\ 0 & -f/s_y & o_y\\ 0 & 0 & 1\end{pmatrix}}_{\text{内参矩阵 } \mathbf{M}_{\text{int}}}\underbrace{\begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T}\\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T}\\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T}\end{pmatrix}}_{\text{外参矩阵 } \mathbf{M}_{\text{ext}}}\begin{pmatrix}X_w\\ Y_w\\ Z_w\\ 1\end{pmatrix}$$

除以α

内参矩阵 $\mathbf{M}_{\text{int}}$

外参矩阵 $\mathbf{M}_{\text{ext}}$

回顾

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & \begin{matrix} -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{matrix} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]$$

$$
\begin{aligned}
\boldsymbol{\Pi} &= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix} \\
&= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}] \\
&= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}
\end{aligned}
$$

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

如何求解相机模型参数?

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

$$
\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}
$$

改写

$$
\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}
$$

展开

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

展开

$$x_{\mathrm{im}} = \frac{\boldsymbol{\pi}_1^{\mathrm{T}} \mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X}} \qquad y_{\mathrm{im}} = \frac{\boldsymbol{\pi}_2^{\mathrm{T}} \mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X}}$$

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

展开

$$x_{\mathrm{im}} = \frac{\boldsymbol{\pi}_1^{\mathrm{T}} \mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X}} \qquad y_{\mathrm{im}} = \frac{\boldsymbol{\pi}_2^{\mathrm{T}} \mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X}}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

展开

$$x_{\mathrm{im}} = \frac{\boldsymbol{\pi}_1^{\mathrm{T}} \mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X}} \qquad y_{\mathrm{im}} = \frac{\boldsymbol{\pi}_2^{\mathrm{T}} \mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X}}$$

改写

$$\boldsymbol{\pi}_1^{\mathrm{T}} \mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X} x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}} \mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}} \mathbf{X} y_{\mathrm{im}} = 0$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

3D空间和图像中的一对对应点

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

N对对应点

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\underbrace{\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix}}_{\mathbf{A}_{2N\times 12}} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\underbrace{\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix}}_{\mathbf{A}_{2N\times 12}} \underbrace{\begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix}}_{\mathbf{x}_{12\times 1}} = \mathbf{0}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\underbrace{\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix}}_{\mathbf{A}_{2N\times 12}} \underbrace{\begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix}}_{\mathbf{x}_{12\times 1}} = \mathbf{0}$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\underbrace{\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix}}_{\mathbf{A}_{2N\times 12}} \underbrace{\begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix}}_{\mathbf{x}_{12\times 1}} = \mathbf{0}$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \ \text{ subject to } \|\mathbf{x}\| = 1$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

$$\mathbf{Ax} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

计算A的SVD，$\mathbf{A} = \mathbf{UDV}^{\mathrm{T}}$，解就是V的最后一列

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

计算A的SVD，$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathrm{T}}$，解就是V的最后一列

$$\mathbf{\Pi} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$\mathbf{Ax} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

计算A的SVD，$\mathbf{A} = \mathbf{UDV}^{\mathrm{T}}$，解就是V的最后一列

$$\mathbf{\Pi} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

如何得到内参和外参?

$$
\begin{aligned}
\boldsymbol{\Pi} &= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix} \\
&= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}] \\
&= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}
\end{aligned}
$$

$$
\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}
$$

$$
= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]
$$

$$
= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}
$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]$$

$$= \begin{pmatrix} \pi & \cdots \\ \vdots \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$[\mathbf{R}| - \mathbf{RT}] \begin{pmatrix} \mathbf{T} \\ 1 \end{pmatrix} = \mathbf{RT} - \mathbf{RT} = \mathbf{0}$$

$$\mathbf{\Pi}\begin{pmatrix}\mathbf{T}\\1\end{pmatrix}=\mathbf{0}$$

$$\mathbf{\Pi}\begin{pmatrix}\mathbf{T}\\1\end{pmatrix}=\mathbf{0}$$

是不是很眼熟?

$$\mathbf{\Pi}\begin{pmatrix}\mathbf{T}\\1\end{pmatrix}=\mathbf{0}$$

是不是很眼熟?

齐次最小二乘问题

$$\Pi \begin{pmatrix} T \\ 1 \end{pmatrix} = 0$$

是不是很眼熟?

齐次最小二乘问题

计算Π的SVD，$\Pi = UDV^T$，解就是V的最后一列

$$
\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & \begin{matrix} -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{matrix} \end{pmatrix}
$$

$$
= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]
$$

$$
= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}
$$

$$
\begin{aligned}
\mathbf{\Pi} &= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix} \\
&= [\mathbf{M}_{\mathrm{int}}\mathbf{R} | -\mathbf{M}_{\mathrm{int}}\mathbf{R}\mathbf{T}] \\
&= \left(\begin{array}{ccc|c} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{M}_{\text{int}}\mathbf{R} &= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix} \\
&= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{M}_{\text{int}}\mathbf{R} &= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix} \\
&= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}
\end{aligned}
$$

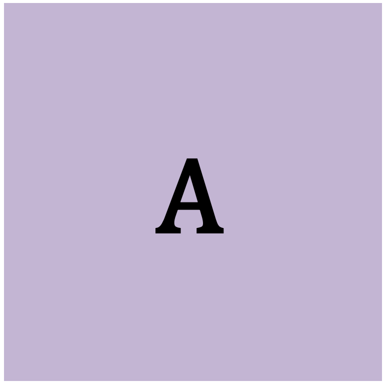
如何求解?

QR分解

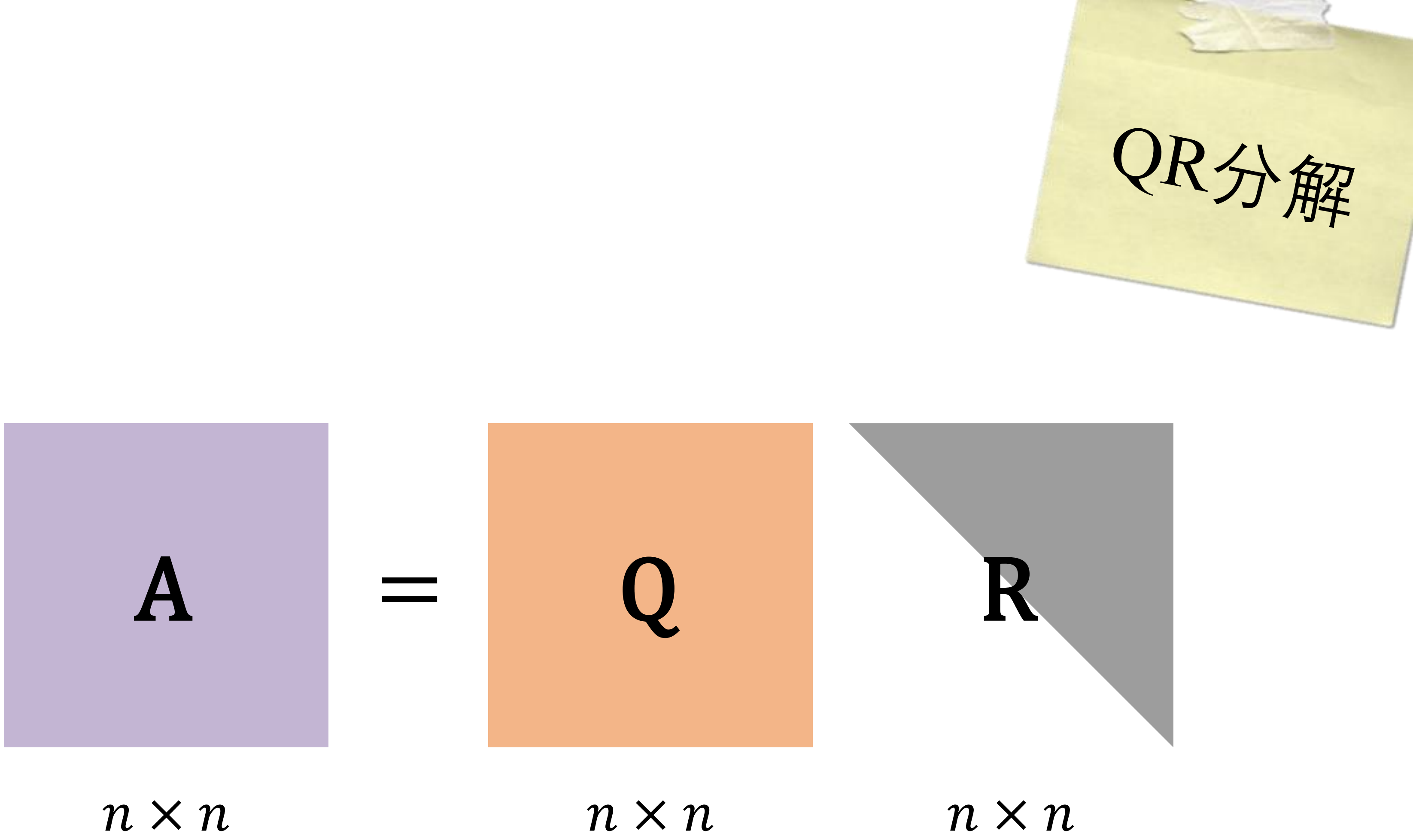
(QR Decomposition)

回顾： QR分解

QR分解



$n \times n$


QR分解
A
=
Q
R
$n \times n$
$n \times n$
$n \times n$

定义： 对于任意给定的实数方阵$\mathbf{A} \in \mathbb{R}^{n \times n}$，它的**QR分解（QR Decomposition）**定义为

$$\mathbf{A} = \mathbf{QR}$$

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使得

$\mathbf{Q}$是具有正交列向量的$n \times n$的正交矩阵

定义： 对于任意给定的实数方阵$\mathbf{A} \in \mathbb{R}^{n\times n}$，它的**QR分解（QR Decomposition）**定义为

$$\mathbf{A} = \mathbf{QR}$$

使得

$\mathbf{Q}$是具有正交列向量的 $n \times n$的正交矩阵

$\mathbf{R}$是$n \times n$的上三角矩阵

由QR到RQ分解

由QR到RQ分解

$$E = \begin{pmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{pmatrix}$$

由QR到RQ分解

$$\mathbf{E}^{-1} = \mathbf{E} = \mathbf{E}^{\mathrm{T}}$$

由QR到RQ分解

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

由QR到RQ分解

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{pmatrix}$$

由QR到RQ分解

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \underline{\begin{pmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{pmatrix}}$$

上下反转

由QR到RQ分解

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$$

由QR到RQ分解

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix} = \begin{pmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{pmatrix}$$

由QR到RQ分解

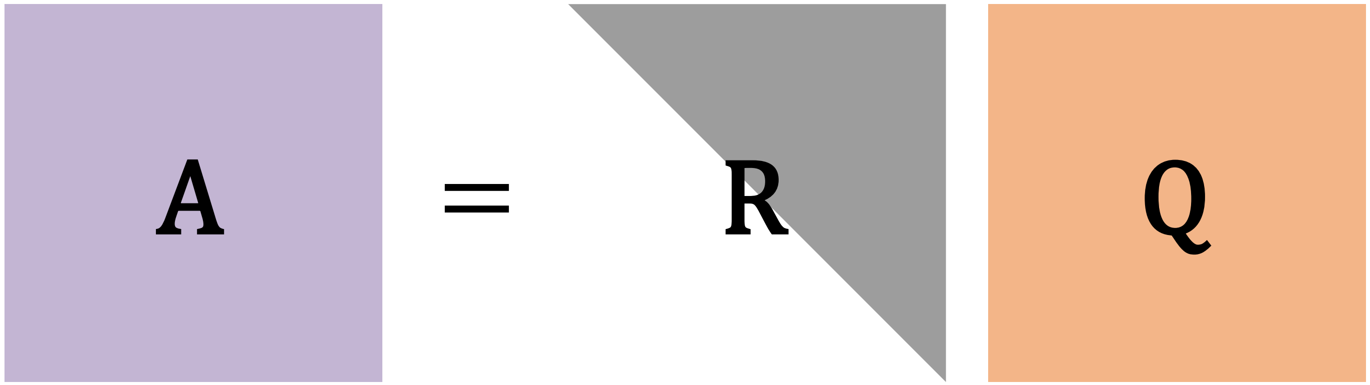
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix} = \begin{pmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{pmatrix}$$

左右反转

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

由QR到RQ分解



由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

由QR到RQ分解

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}} \mathbf{R}^{\mathrm{T}}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A}^{T}\mathbf{E} = \mathbf{Q}^{T}\mathbf{R}^{T}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

$$\mathbf{E}\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{EQ}\,\mathbf{A}^{T}\mathbf{E} = \mathbf{E}\mathbf{R}^{T}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

$$\mathbf{E}\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

由QR到RQ分解

$$\mathbf{A} = \mathbf{R}\mathbf{Q}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

$$\mathbf{E}\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \left(\mathbf{Q}^{\mathrm{T}}\mathbf{E}\right)\left(\mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}\right)$$

由QR到RQ分解

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{Q}^{\mathrm{T}}\,\mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

$n \times n$ $\quad$ $n \times n$ $\quad$ $n \times n$

3

主要步骤

步骤1

转置并左右反转

$$\tilde{\mathbf{A}} = \mathbf{A}^{\mathrm{T}}\mathbf{E}$$

步骤2

计算$\tilde{A}$的QR分解

$$\tilde{\mathbf{A}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}$$

步骤3

计算R和Q

$$\mathbf{R} = \mathbf{E}\widetilde{\mathbf{R}}^{\mathrm{T}}\mathbf{E}$$

$$\mathbf{Q} = \mathbf{E}\widetilde{\mathbf{Q}}^{\mathrm{T}}$$

回顾：QR分解

$$
\begin{aligned}
\mathbf{M}_{\text{int}}\mathbf{R} &= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix} \\
&= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}
\end{aligned}
$$

如何求解?

$$
\begin{aligned}
\mathbf{M}_{\mathrm{int}}\mathbf{R} &= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix} \\
&= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}
\end{aligned}
$$

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

上三角矩阵

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

正交矩阵

上三角矩阵

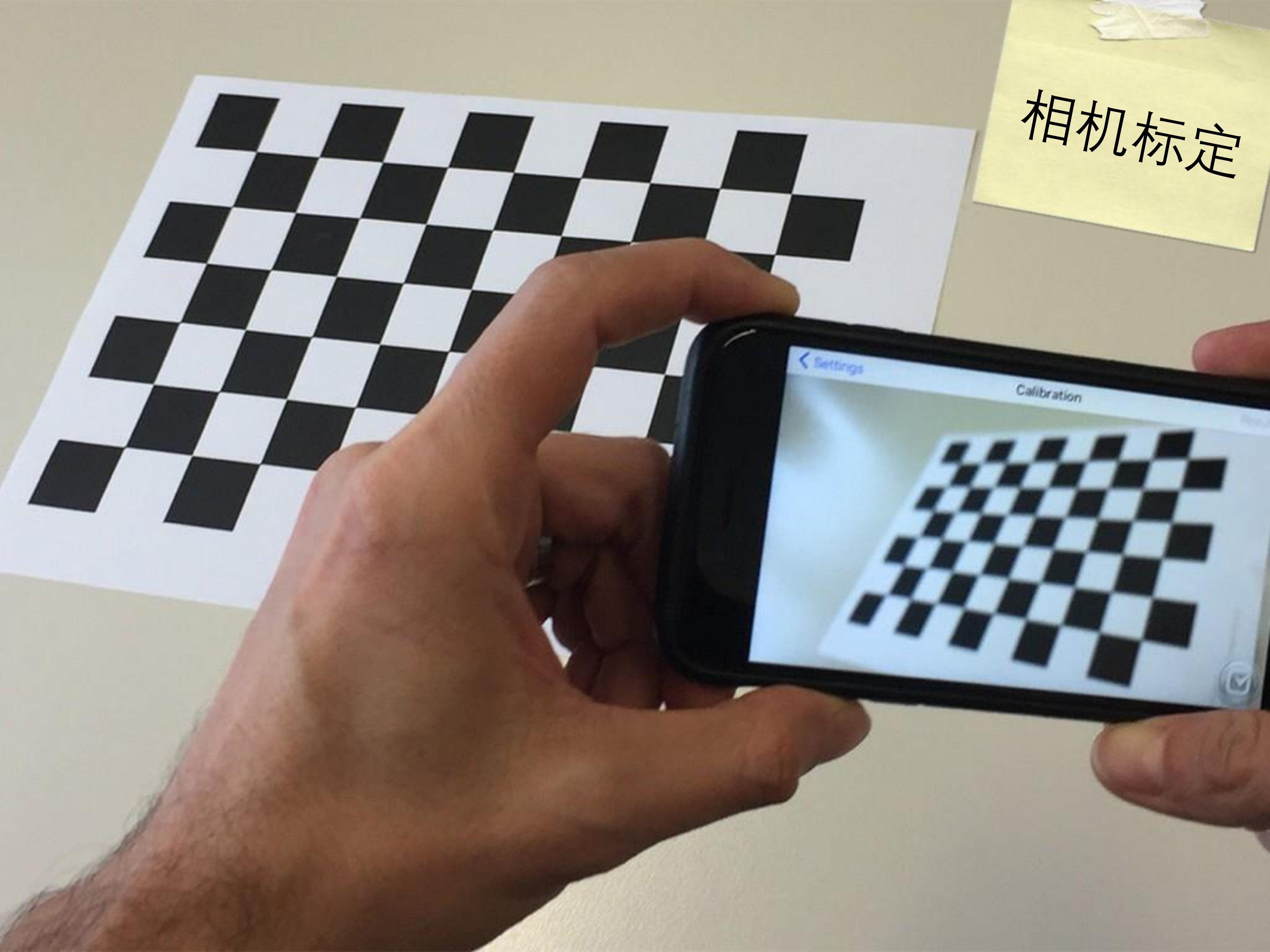
$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

正交矩阵

上三角矩阵

计算RQ分解，R是内参矩阵，Q是旋转矩阵


相机标定
Settings
Calibration

A Flexible New Technique for Camera Calibration

Zhengyou Zhang, *Senior Member, IEEE*

Abstract—We propose a flexible new technique to easily calibrate a camera. It only requires the camera to observe a planar pattern shown at a few (at least two) different orientations. Either the camera or the planar pattern can be freely moved. The motion need not be known. Radial lens distortion is modeled. The proposed procedure consists of a closed-form solution, followed by a nonlinear refinement based on the maximum likelihood criterion. Both computer simulation and real data have been used to test the proposed technique and very good results have been obtained. Compared with classical techniques which use expensive equipment such as two or three orthogonal planes, the proposed technique is easy to use and flexible. It advances 3D computer vision one more step from laboratory environments to real world use. The corresponding software is available from the author's Web page.

Index Terms—Camera calibration, calibration from planes, 2D pattern, flexible plane-based calibration, absolute conic, projective mapping, lens distortion, closed-form solution, maximum likelihood estimation, flexible setup.

Our current research is focused on a desktop vision system (DVS) since the potential for using DVSs is large. Cameras are becoming inexpensive and ubiquitous. A DVS aims at the general public who are not experts in computer vision. A typical computer user will perform vision tasks only from time to time, so they will not be willing to invest money for expensive equipment. Therefore, flexibility, robustness, and low cost are important. The camera calibration technique described in this paper was developed with these considerations in mind.

The proposed technique only requires the camera to observe a planar pattern shown at a few (at least two) different orientations. The pattern can be printed on a laser printer and attached to a "reasonable" planar surface (e.g., a hard book cover). Either the camera or the planar pattern can be moved by hand. The motion need not be known. The proposed approach, which uses 2D metric information, lies between the photogrammetric calibration, which uses explicit 3D model, and self-calibration, which uses motion rigidity or equivalently implicit 3D information. Both computer simulation and real data have been used to test the proposed technique and very good results have been obtained. Compared with classical techniques, the proposed technique is considerably more flexible: Anyone can make a calibration pattern by him/herself and the setup is very easy. Compared with self-calibration, it

TPAMI, 2000

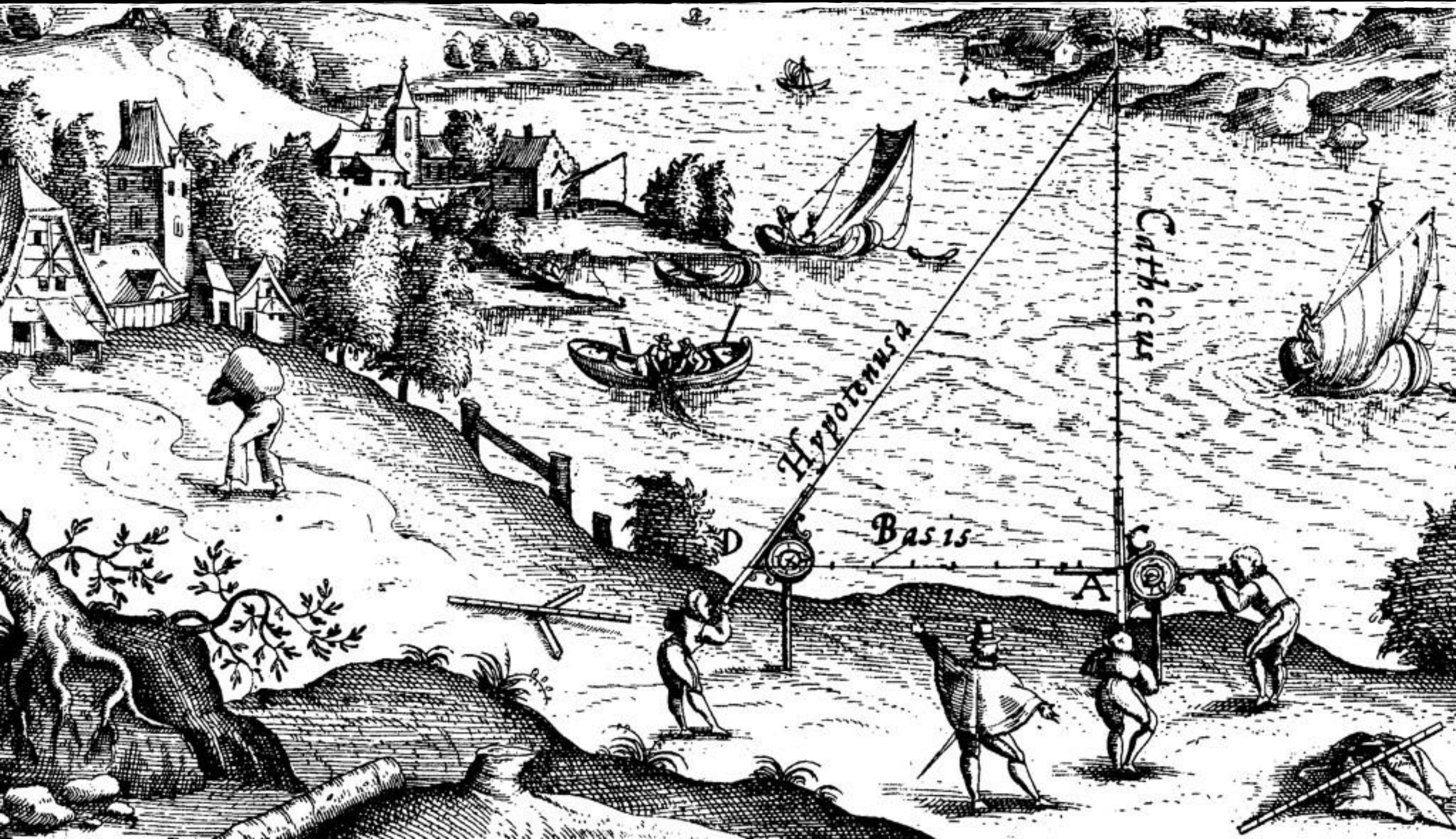
鸣谢： Pepsi Max

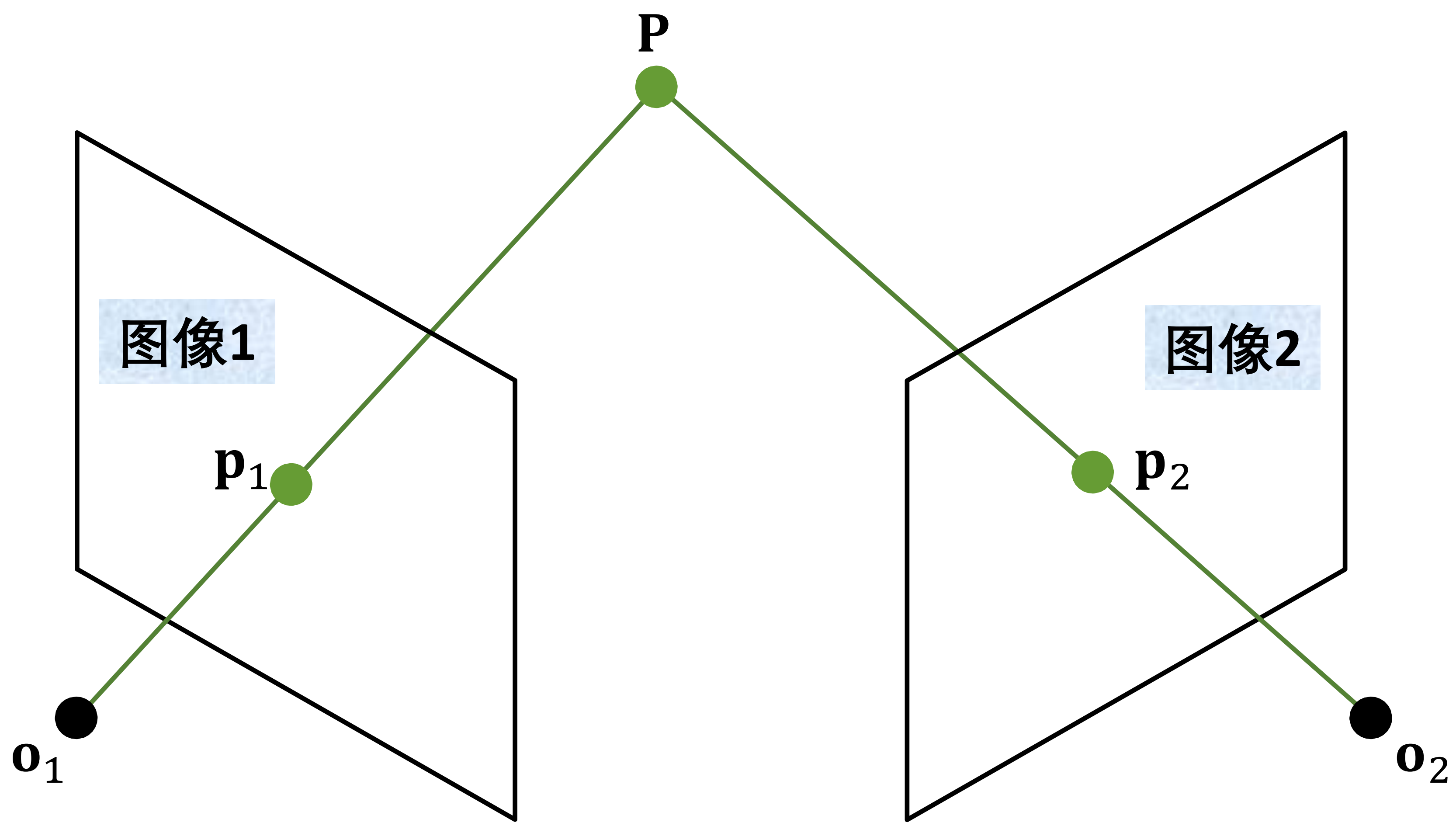
如果没有3D空间对应点呢？

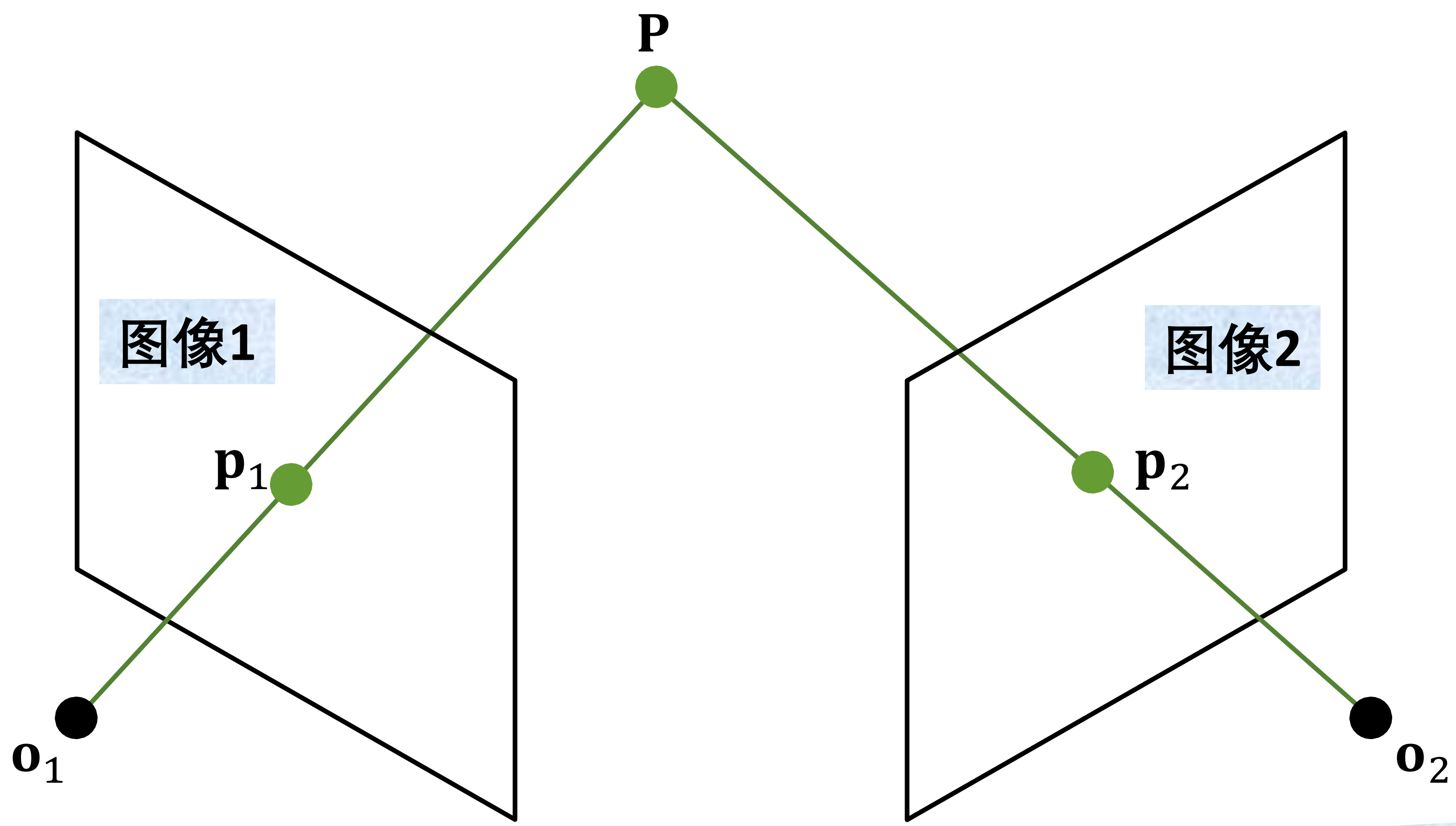
如果没有3D空间对应点呢？

找到多幅图像点对应关系，求解相机模型参数

三角测量




P
图像1
图像2
p1
p2
o1
o2

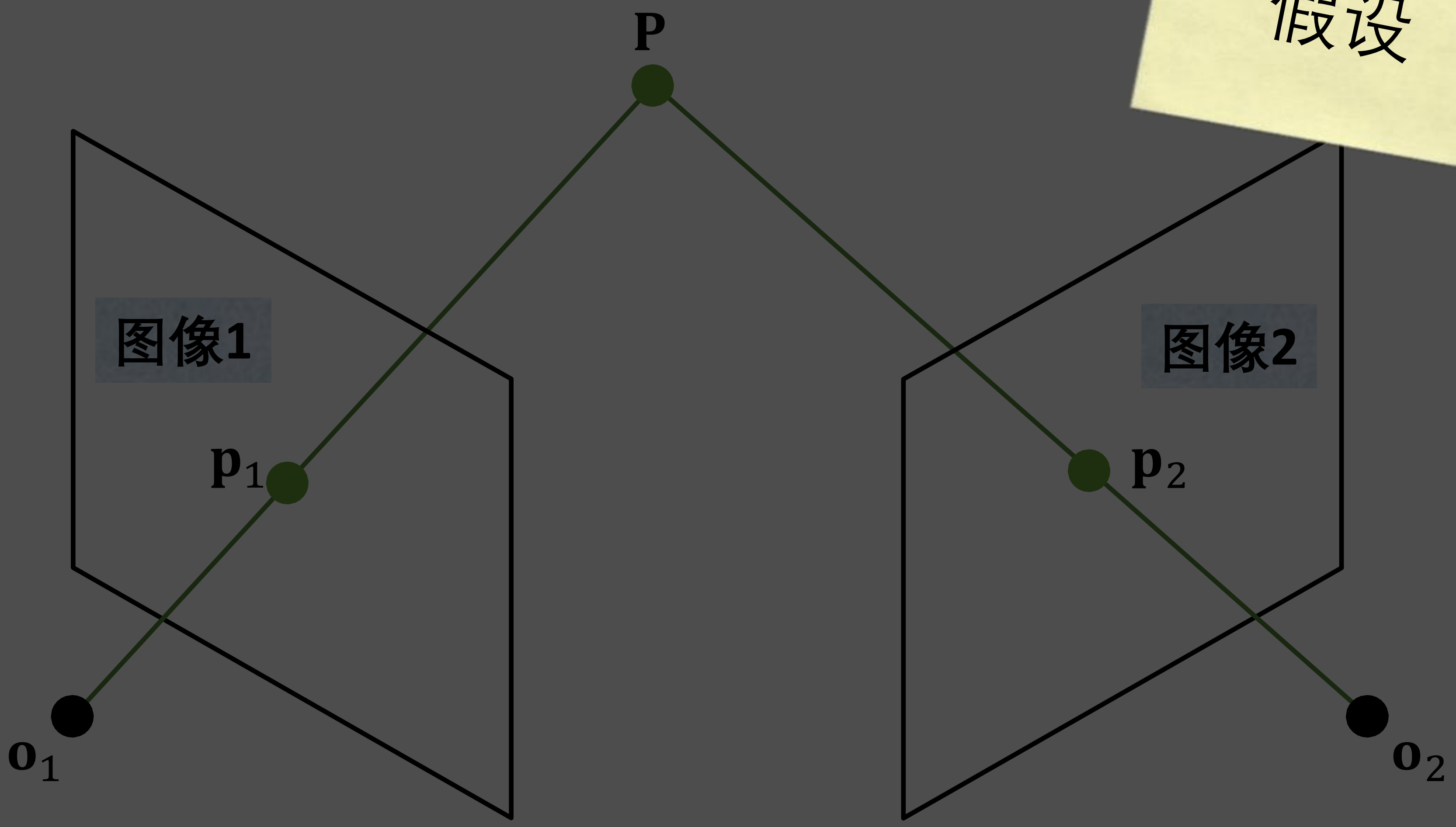


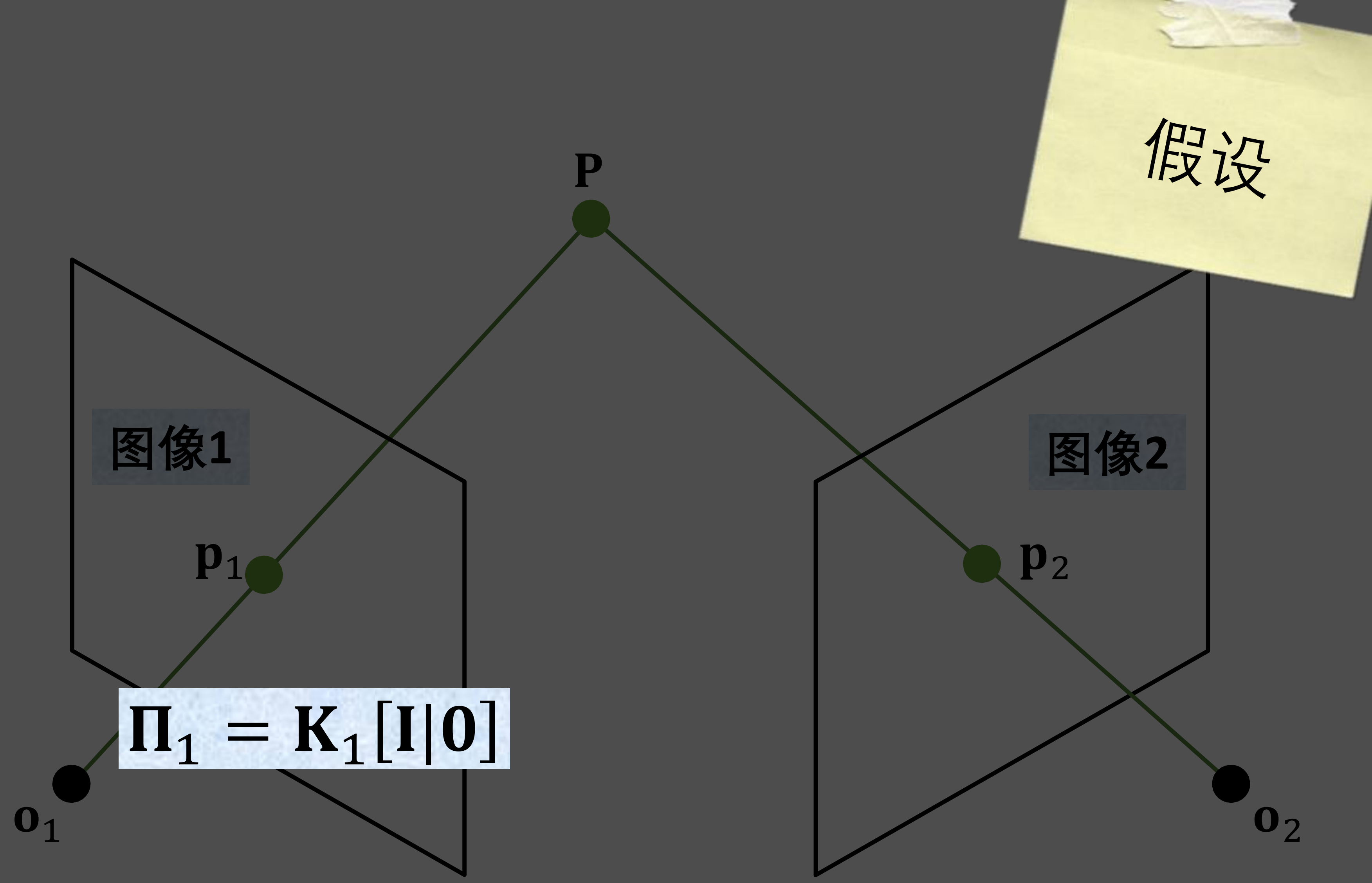
如何关联2D-2D对应点？

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \mathbf{F} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = 0$$

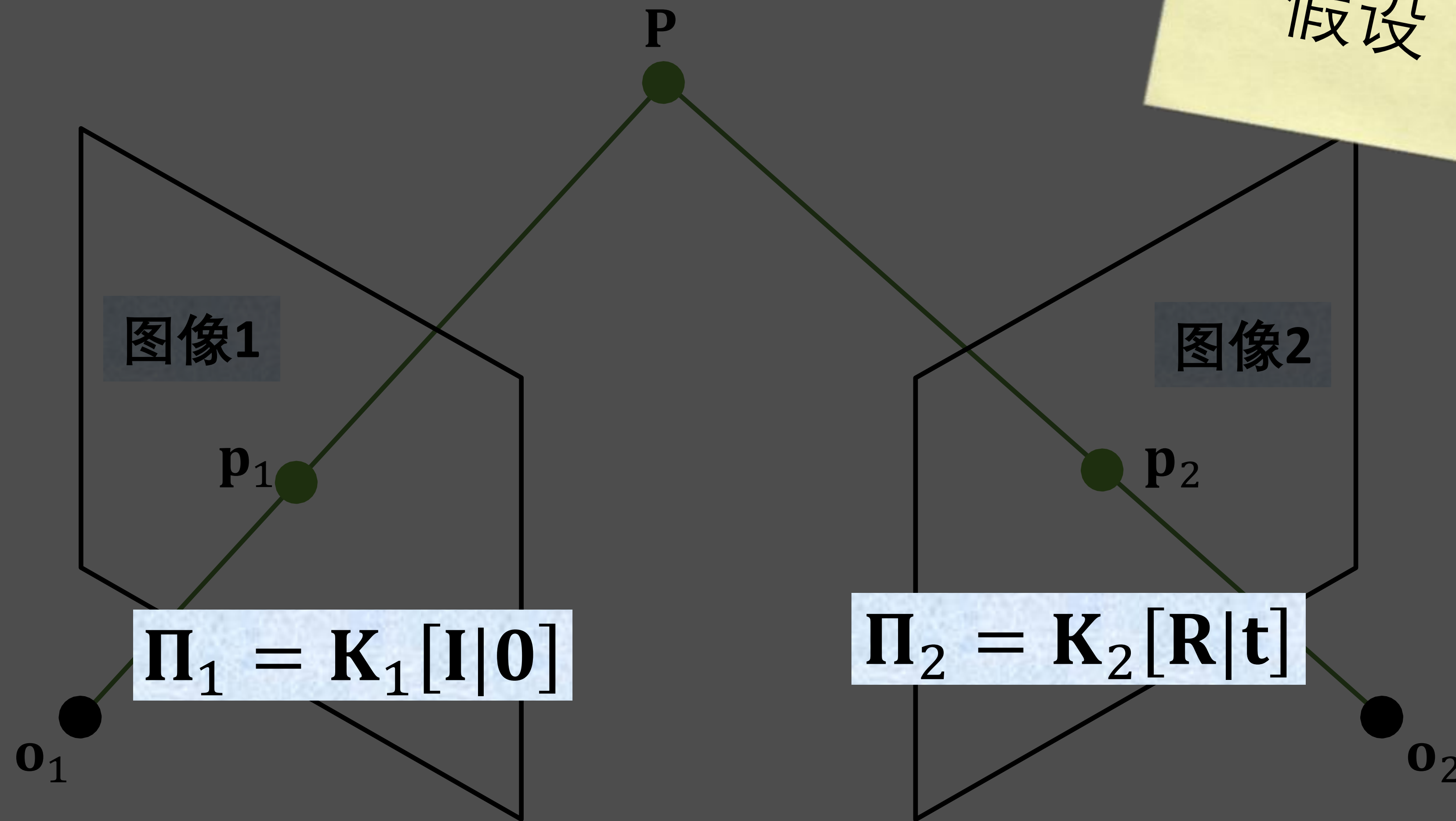
基础矩阵

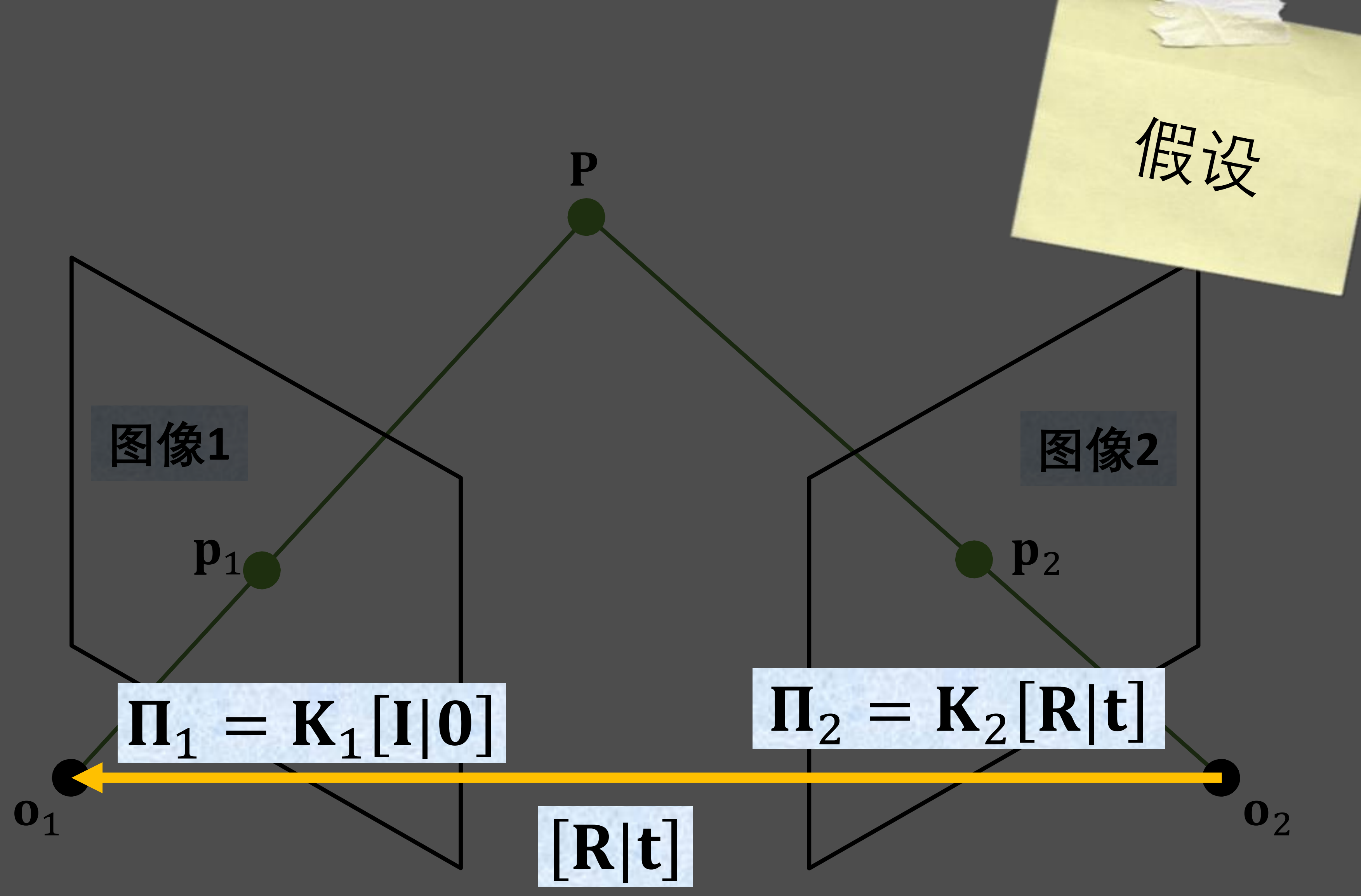
假设

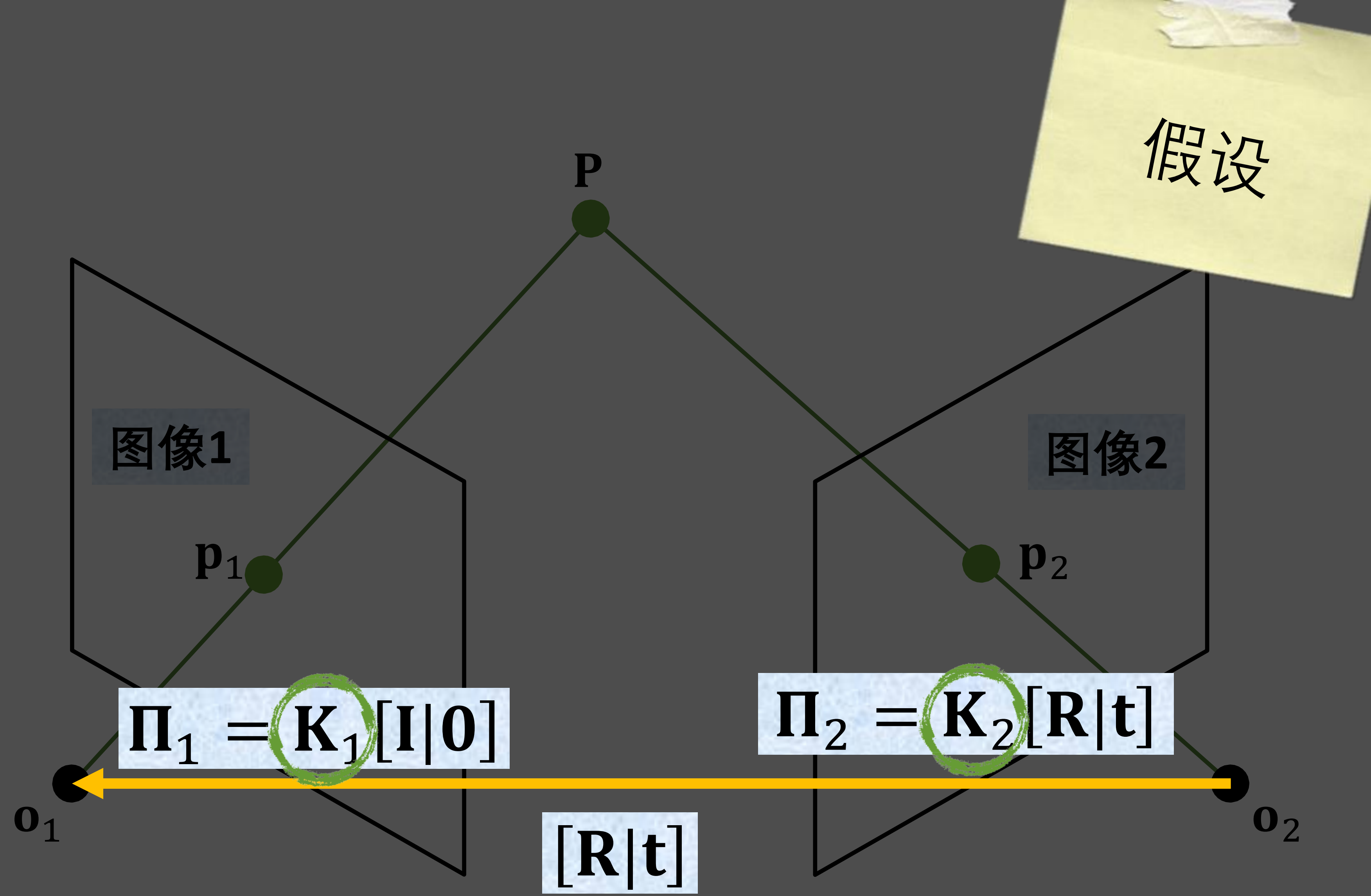



假设
P
图像1
图像2
p1
p2
o1
o2
$\mathbf{\Pi}_1 = \mathbf{K}_1[\mathbf{I}|\mathbf{0}]$

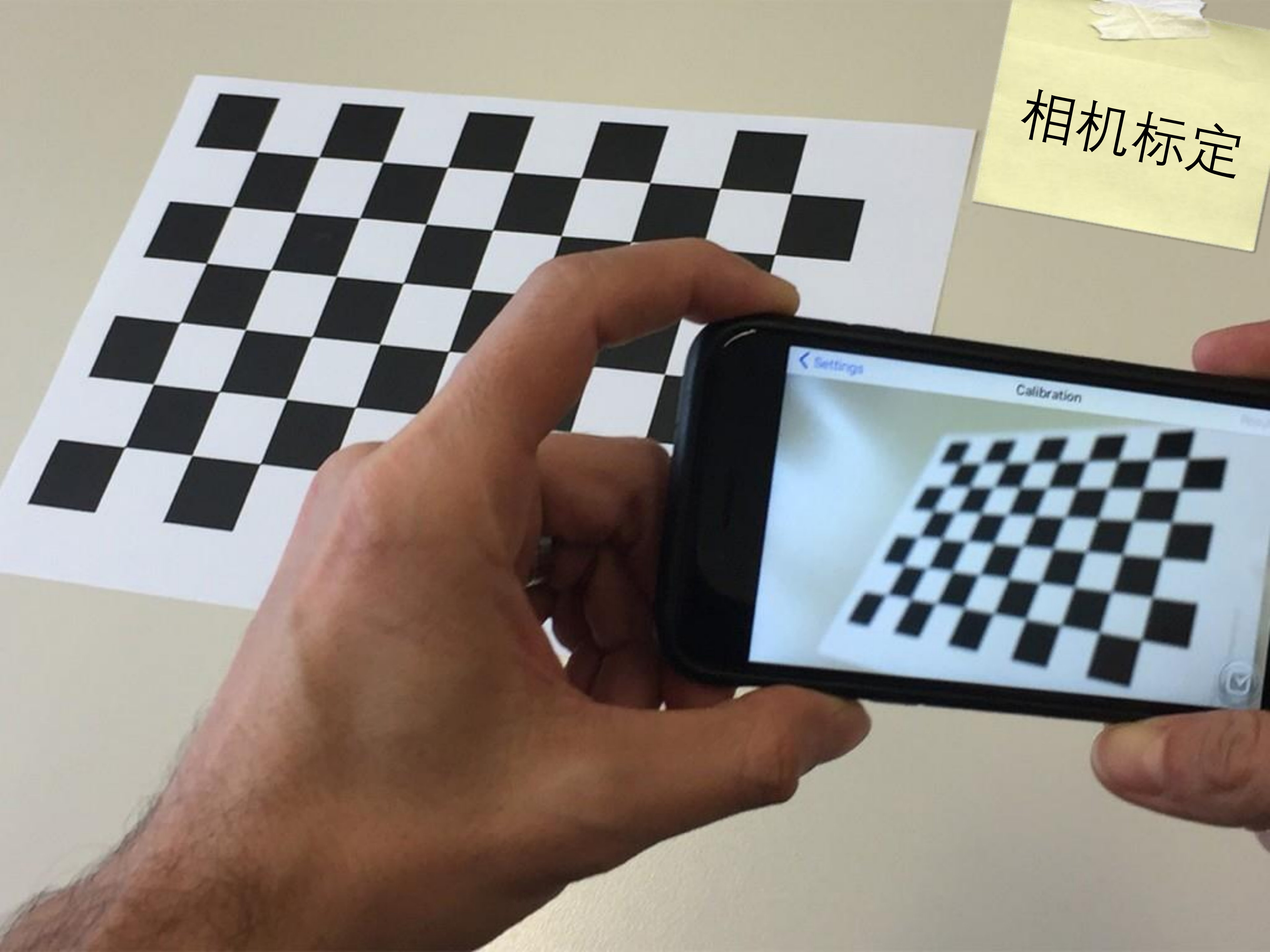
假设


P
图像1
图像2
$\mathbf{p}_1$
$\mathbf{p}_2$
$\mathbf{o}_1$
$\mathbf{o}_2$
$\boldsymbol{\Pi}_1 = \mathbf{K}_1[\mathbf{I}|\mathbf{0}]$
$\boldsymbol{\Pi}_2 = \mathbf{K}_2[\mathbf{R}|\mathbf{t}]$


假设
P
图像1
图像2
$\mathbf{p}_1$
$\mathbf{p}_2$
$\mathbf{\Pi}_1 = \mathbf{K}_1[\mathbf{I}|\mathbf{0}]$
$\mathbf{\Pi}_2 = \mathbf{K}_2[\mathbf{R}|\mathbf{t}]$
$\mathbf{o}_1$
$\mathbf{o}_2$
$[\mathbf{R}|\mathbf{t}]$



假设
P
图像1
图像2
p_1
p_2
$\Pi_1 = K_1[I|0]$
$\Pi_2 = K_2[R|t]$
o_1
o_2
$[R|t]$


相机标定
Settings
Calibration

Camera Calibration

Goal

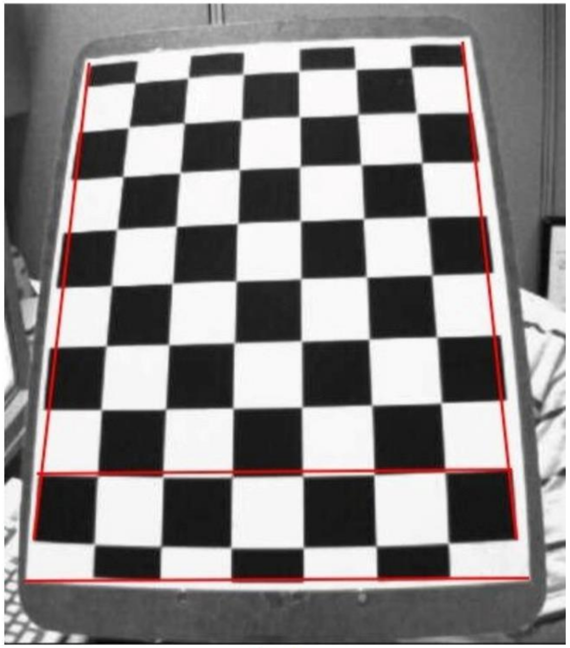
In this section, we will learn about

- types of distortion caused by cameras
- how to find the intrinsic and extrinsic properties of a camera
- how to undistort images based off these properties

Basics

Some pinhole cameras introduce significant distortion to images. Two major kinds of distortion are radial distortion and tangential distortion.

Radial distortion causes straight lines to appear curved. Radial distortion becomes larger the farther points are from the center of the image. For example, one image is shown below in which two edges of a chess board are marked with red lines. But, you can see that the border of the chess board is not a straight line and doesn't match with the red line. All the expected straight lines are bulged out. Visit Distortion (optics) for more details.



image

OpenCV 4.7.0-dev

docs.opencv.org/4.x/dc/dbb/tutorial_py_calibration.html

Camera Calibration

Goal

In this section, we will learn about

- types of distortion caused by cameras
- how to find the intrinsic and extrinsic properties of a camera
- how to undistort images based off these properties

Basics

Some pinhole cameras introduce significant distortion to images. Two major kinds of distortion are radial distortion and tangential distortion.

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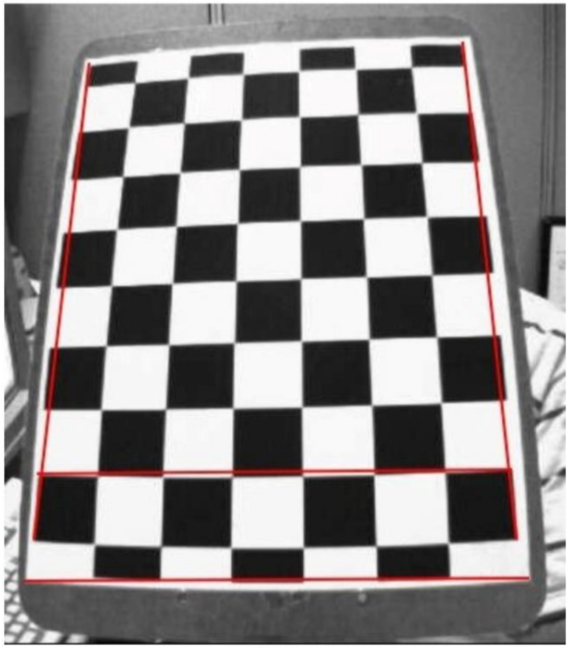

image

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \mathbf{K}_2^{-\mathrm{T}} \mathbf{E} \mathbf{K}_1^{-1} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = 0$$

本质矩阵

$$\mathbf{E} = [\mathbf{t}_\times]\mathbf{R}$$

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本质矩阵

反对称矩阵

回顾： 线性代数

谱分解

定理：令$\mathbf{M}$是一个$d \times d$实反对称矩阵，它具有非零虚特征值$i\lambda_1, \cdots, i\lambda_d$，则存在正交矩阵$\mathbf{U}$，使得：

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$$\mathbf{M} = \mathbf{U} \begin{pmatrix} 0 & \lambda_1 & & & & & & & & \\ -\lambda_1 & 0 & & & & & & & & \\ & & 0 & \lambda_2 & & & & & & \\ & & -\lambda_2 & 0 & & & & & & \\ & & & & \ddots & & & & & \\ & & & & & 0 & \lambda_{\lfloor d/2 \rfloor} & & & \\ & & & & & -\lambda_{\lfloor d/2 \rfloor} & 0 & & & \\ & & & & & & & 0 & & \\ & & & & & & & & \ddots & \\ & & & & & & & & & 0 \end{pmatrix} \mathbf{U}^T$$

定义： 矩阵的初等行变换指

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1. 互换矩阵中两行的位置

初等行变换

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1. 互换矩阵中两行的位置
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增广矩阵

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增广矩阵

对增广矩阵进行初等变换不影响方程组的解

回顾：线性代数

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谱分解

$$[\mathbf{t}_\times] = \lambda \mathbf{U} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{U}^\mathrm{T}$$

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λ去哪了？

$$\mathbf{P}_r^{\mathrm{T}}\mathbf{E}\mathbf{P}_l = 0$$

在齐次坐标系中是确定的，且具有尺度不变性

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我们有：

$$\mathbf{E} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{T}}$$

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比较两个式子，为了把$\mathbf{E}$写成SVD的形式，需要把Z构造成对角矩阵和正交矩阵相乘的形式

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初等矩阵变换

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$$\mathbf{W} = \mathbf{R}_Z\left(\frac{\pi}{2}\right) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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或

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正交矩阵

改写

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正交矩阵

改写

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{ZW} = \mathrm{diag}(1,1,0)$$

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正交矩阵

改写

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

这意味着什么?

$$\mathbf{ZW} = \mathrm{diag}(1,1,0)$$

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这意味着什么?

本质矩阵秩为2，且奇异值相等

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

旋转矩阵

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代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

$$\mathbf{E} = \mathbf{UZWV}^{\mathrm{T}}$$

旋转矩阵

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代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

$$\mathbf{E} = \mathbf{UZWV}^{\mathrm{T}}$$

我们有：

$$\mathbf{E} = [\mathbf{t}_{\times}]\mathbf{R} = \left(\mathbf{UZU}^{\mathrm{T}}\right)\mathbf{R}$$

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对比

$$\mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

$$\mathbf{E} = \mathbf{UZWV}^{\mathrm{T}}$$

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旋转矩阵

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$$\mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$$

我们又有：

$$\mathbf{ZW}^{\mathrm{T}} = -\,\mathrm{diag}(1,1,0)$$

并服从如下约束：

$$\det \mathbf{R} = 1$$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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同理

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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对比

$$\mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$$

我们又有：

$$\mathbf{ZW}^{\mathrm{T}} = -\,\mathrm{diag}(1,1,0)$$

并服从如下约束：

$$\det\mathbf{R} = 1$$

同理

$$\mathbf{R_2} = \mathbf{UW}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

旋转矩阵

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并服从如下约束：

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旋转矩阵

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^\mathrm{T}$$

平移向量

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^\mathrm{T}$$

我们有：

$$[\mathbf{t}_\times]\mathbf{t} = \mathbf{0}$$

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^\mathrm{T}$$

我们有：

$$[\mathbf{t}_\times]\mathbf{t} = \mathbf{0}$$

$\mathbf{t}$是$[\mathbf{t}_\times]$的零空间

平移向量

$$[\mathbf{t}_{\times}] = \mathbf{UZU}^{\mathrm{T}}$$

我们有：

$$[\mathbf{t}_{\times}]\mathbf{t} = \mathbf{0}$$

$\mathbf{t}$是$[\mathbf{t}_{\times}]$的零空间

$\mathbf{t}$是$\mathbf{U}$的最后一列$\mathbf{u}_3$

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考虑一个尺度因子$\lambda = \pm 1$：

平移向量

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我们有：

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考虑一个尺度因子$\lambda = \pm 1$：

$$\mathbf{t}_1 = \mathbf{u}_3$$

$$\mathbf{t}_2 = -\mathbf{u}_3$$

平移向量

$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$$

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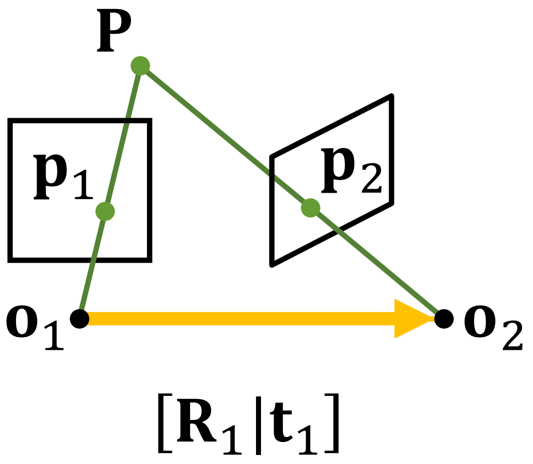
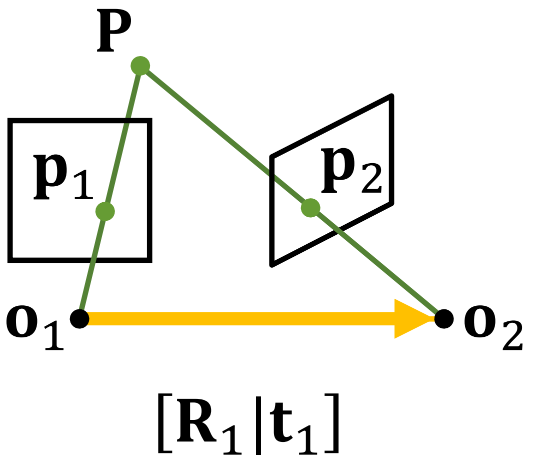
$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$



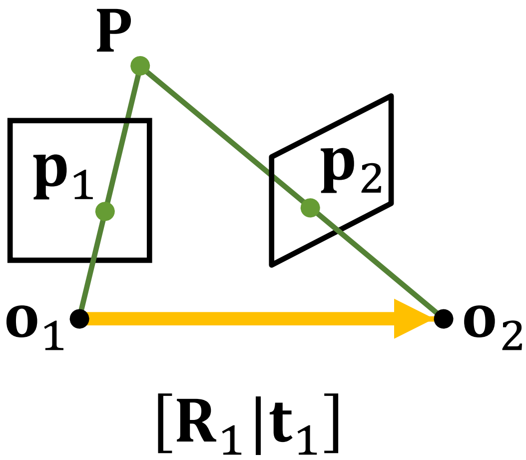
$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$



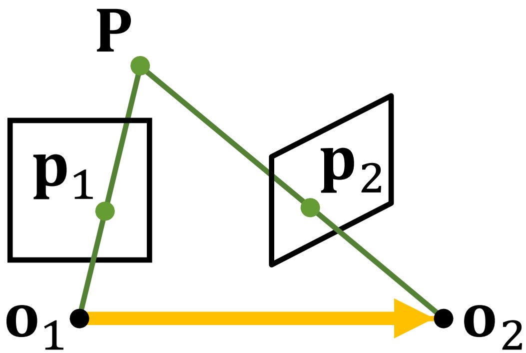
$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{UW}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

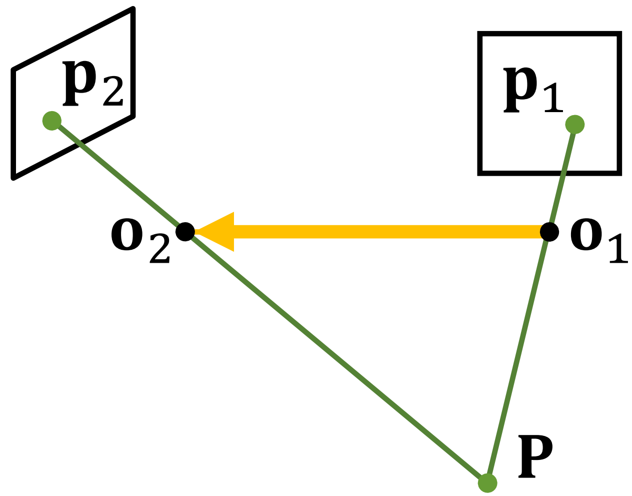


$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{UWV}^T$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{UW}^T\mathbf{V}^T$$



$[\mathbf{R}_1|\mathbf{t}_1]$



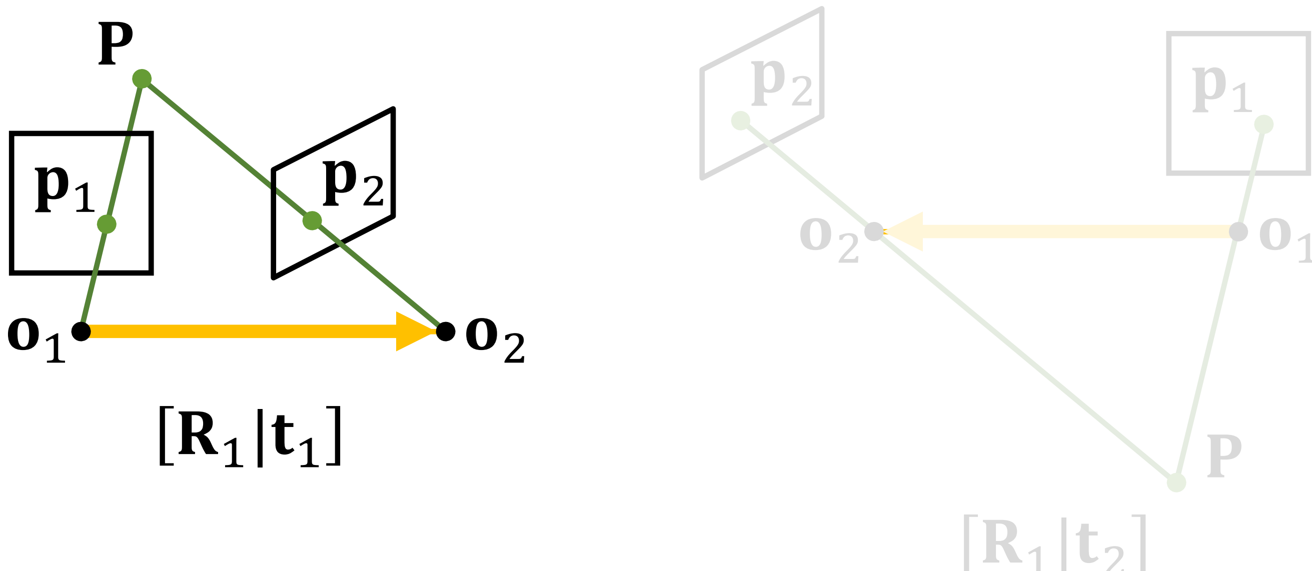
$[\mathbf{R}_1|\mathbf{t}_2]$

$$\mathbf{t}_1 = \mathbf{u}_3$$

$$\mathbf{t}_2 = -\mathbf{u}_3$$

$$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

$$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$



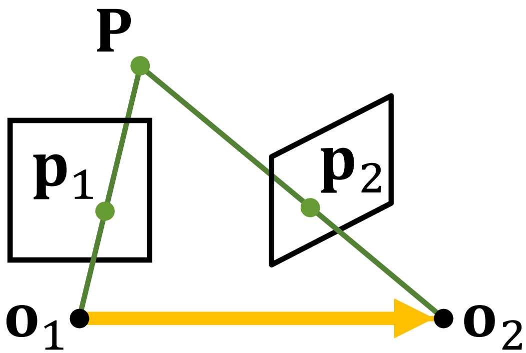
$\mathbf{t}_1 = \mathbf{u}_3$

$\mathbf{t}_2 = -\mathbf{u}_3$

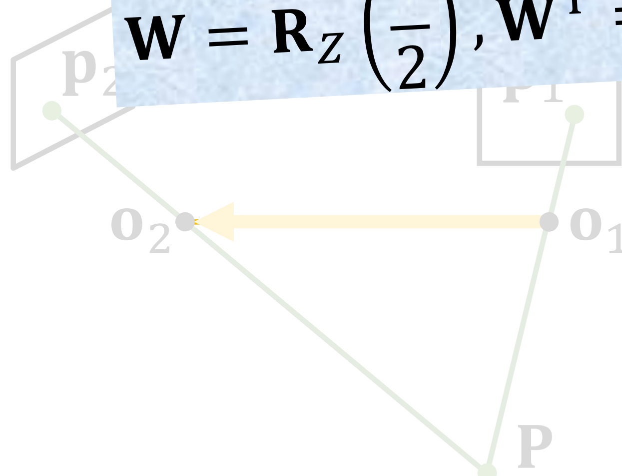
$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$

$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$

$$\mathbf{W} = \mathbf{R}_Z\left(\frac{\pi}{2}\right), \mathbf{W}^{\mathrm{T}} = \mathbf{R}_Z\left(-\frac{\pi}{2}\right)$$



$[\mathbf{R}_1|\mathbf{t}_1]$



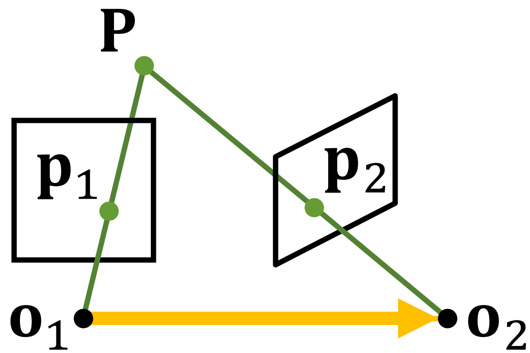
$[\mathbf{R}_1|\mathbf{t}_2]$

$$\mathbf{t}_1 = \mathbf{u}_3$$

$$\mathbf{t}_2 = -\mathbf{u}_3$$

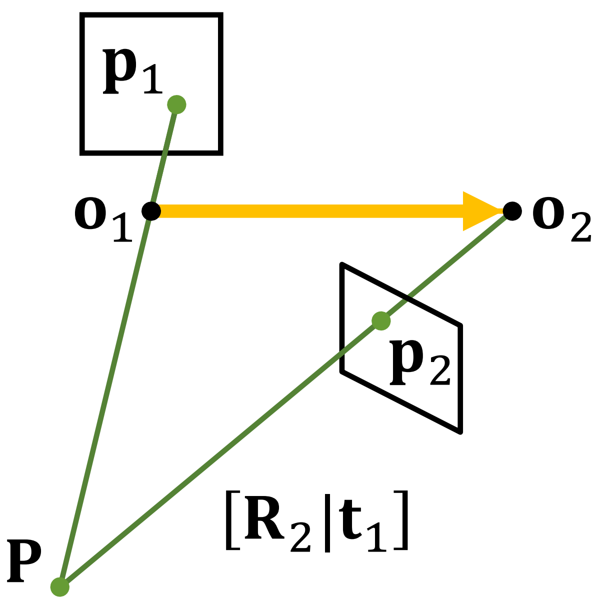
$$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

$$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$



$[\mathbf{R}_1|\mathbf{t}_1]$

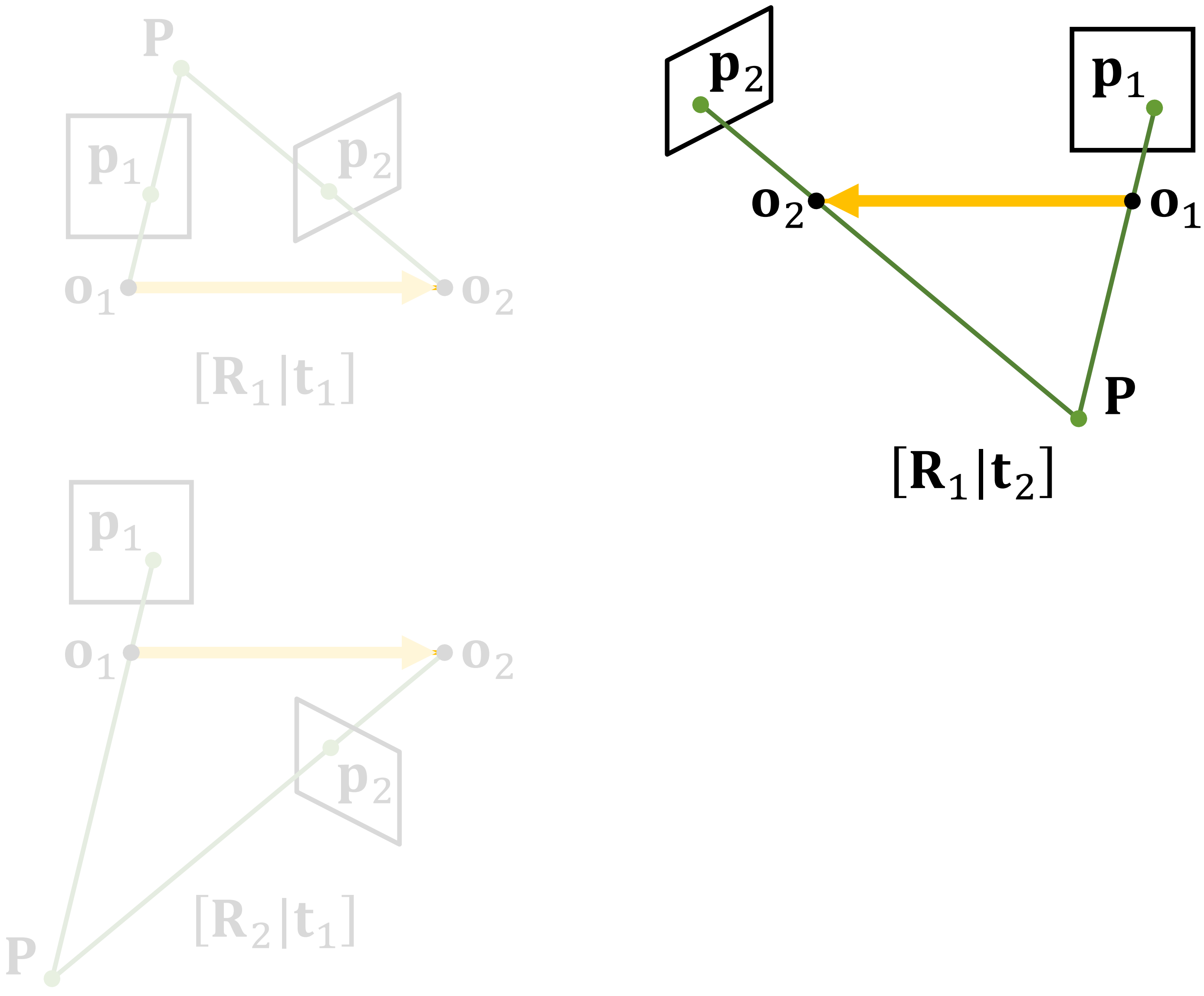
$[\mathbf{R}_1|\mathbf{t}_2]$



$[\mathbf{R}_2|\mathbf{t}_1]$

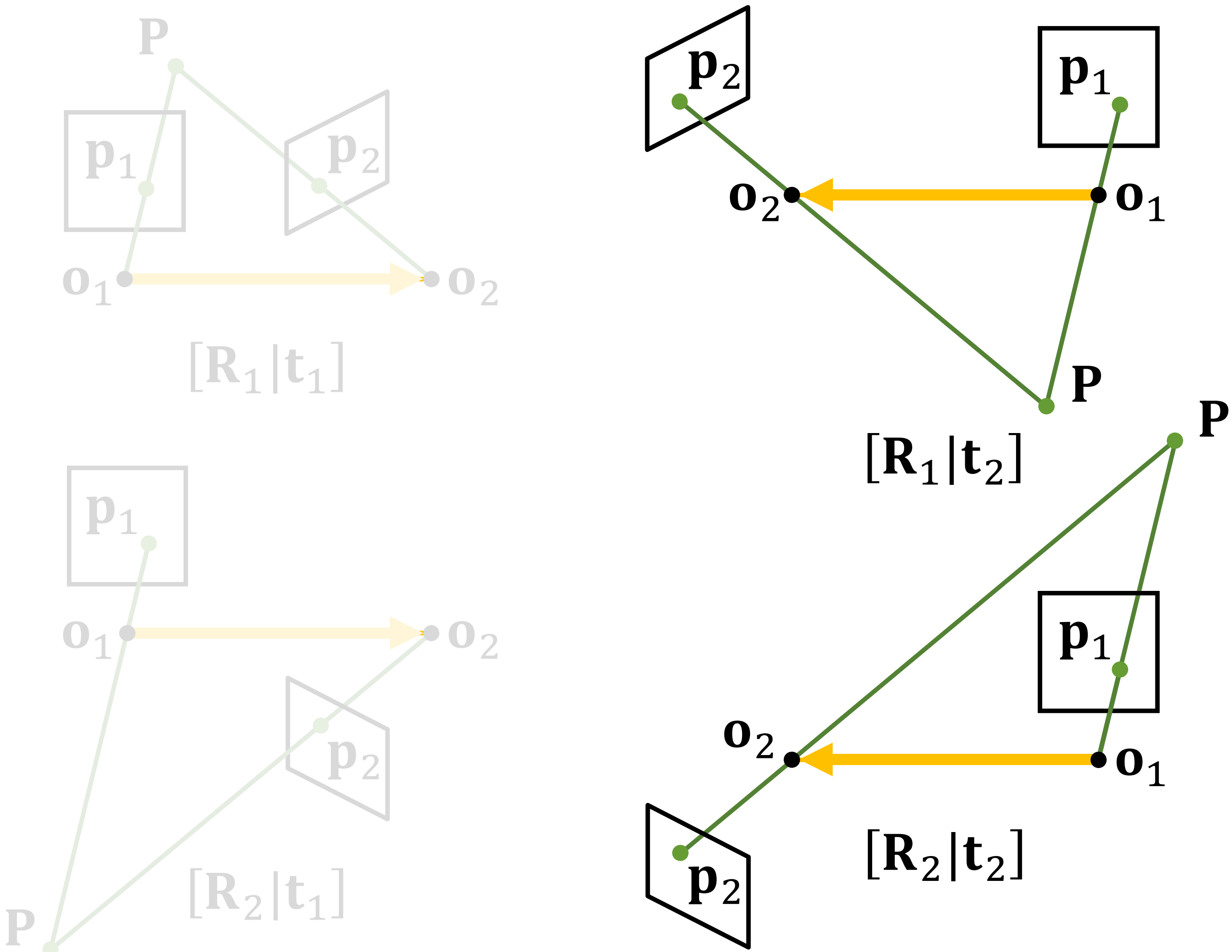
$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$



$$t_1 = u_3 \qquad R_1 = UWV^T$$

$$t_2 = -u_3 \qquad R_2 = UW^TV^T$$



$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

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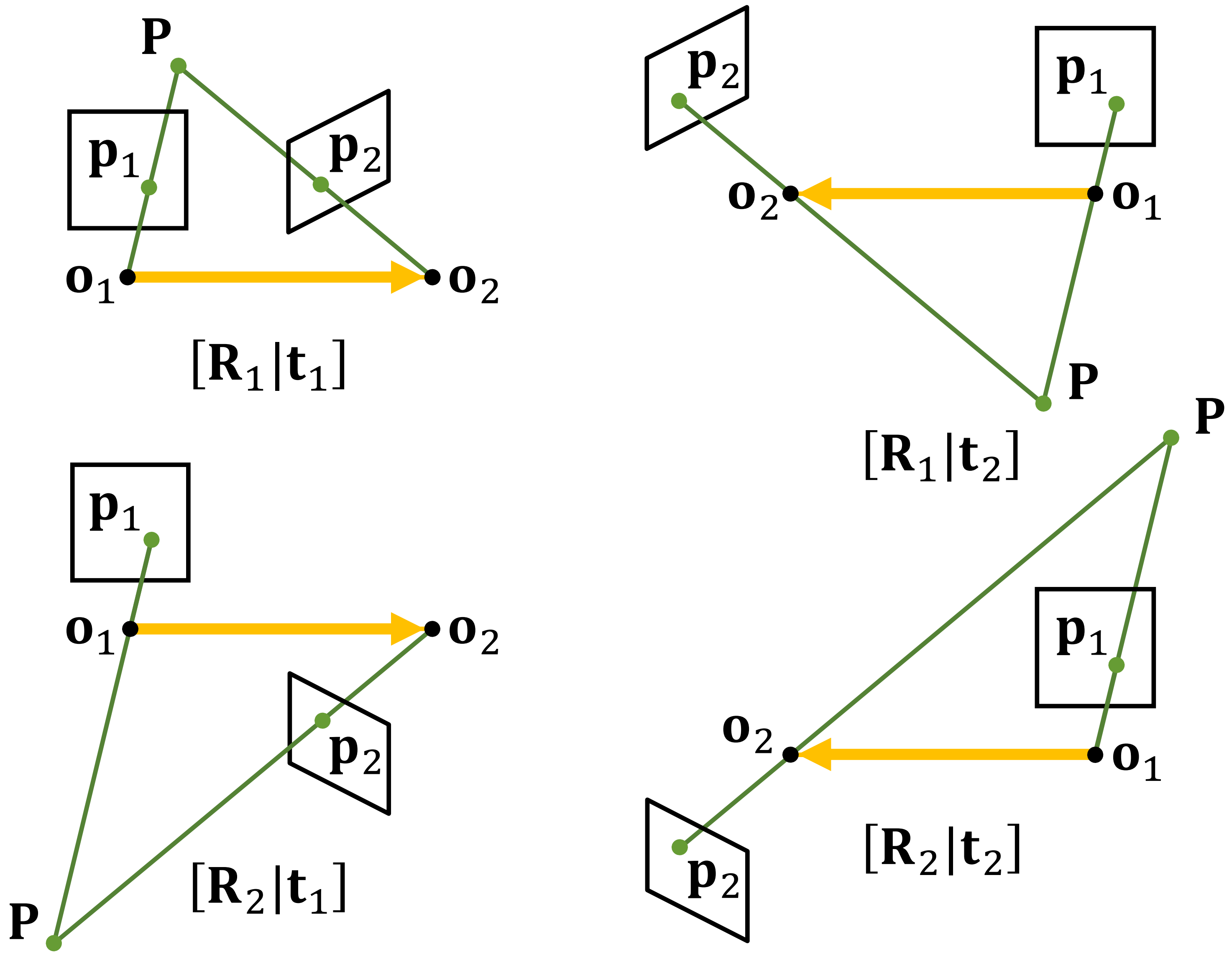

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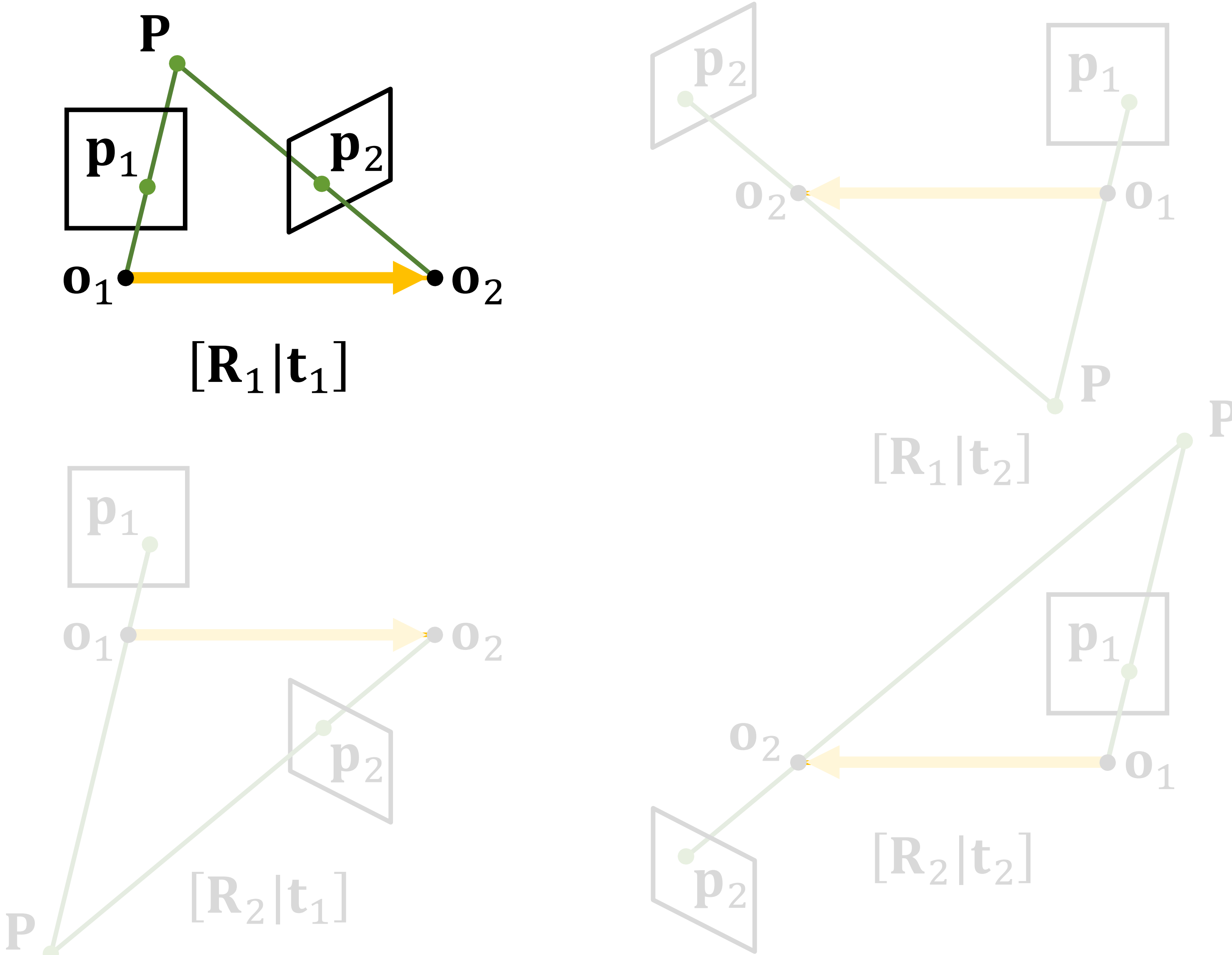
$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$



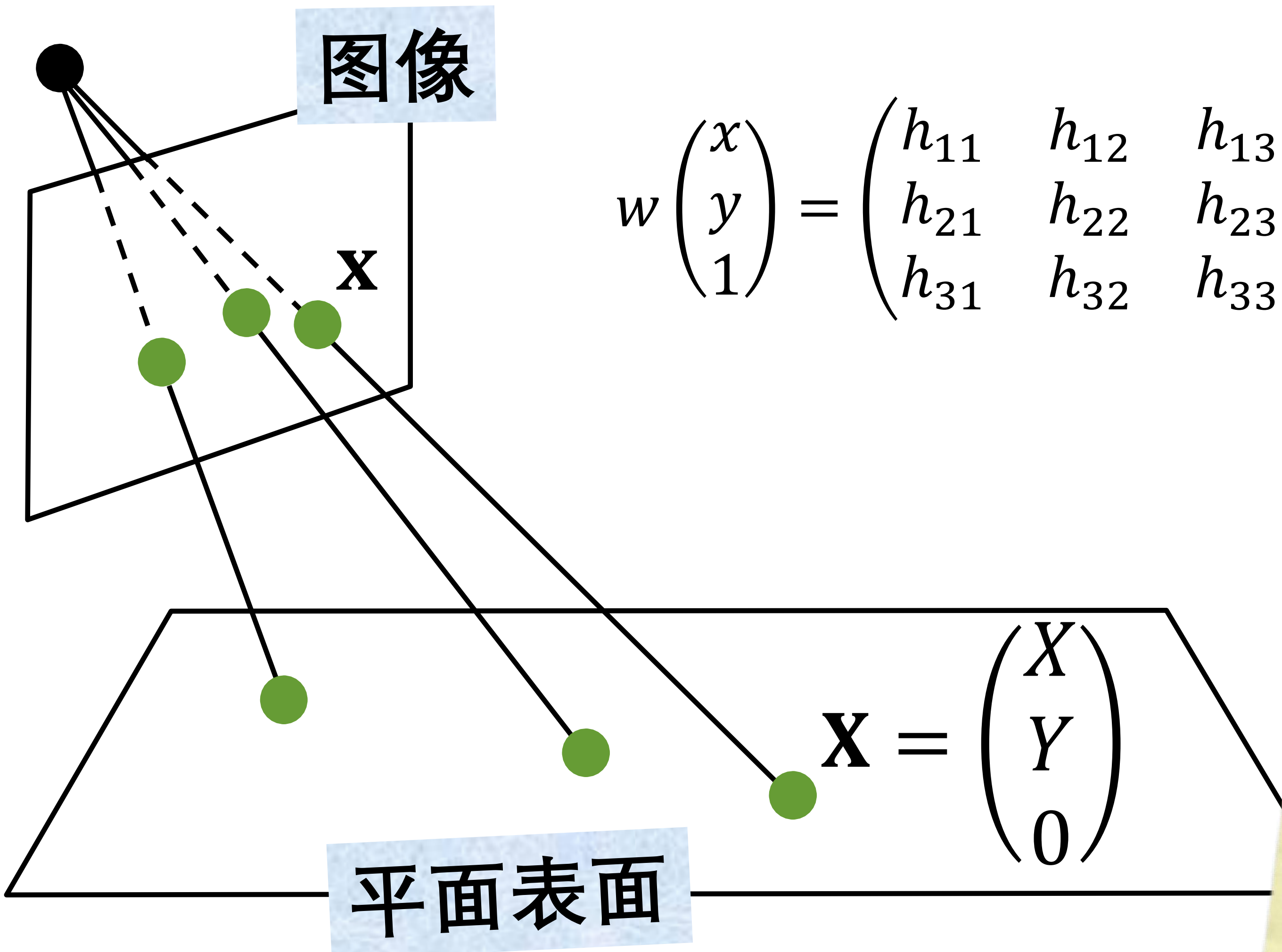
如果世界表面为平面，或平移分量太小，本质/基础矩阵退化

如果世界表面为平面，或平移分量太小，
本质/基础矩阵退化

约束是线性相关的

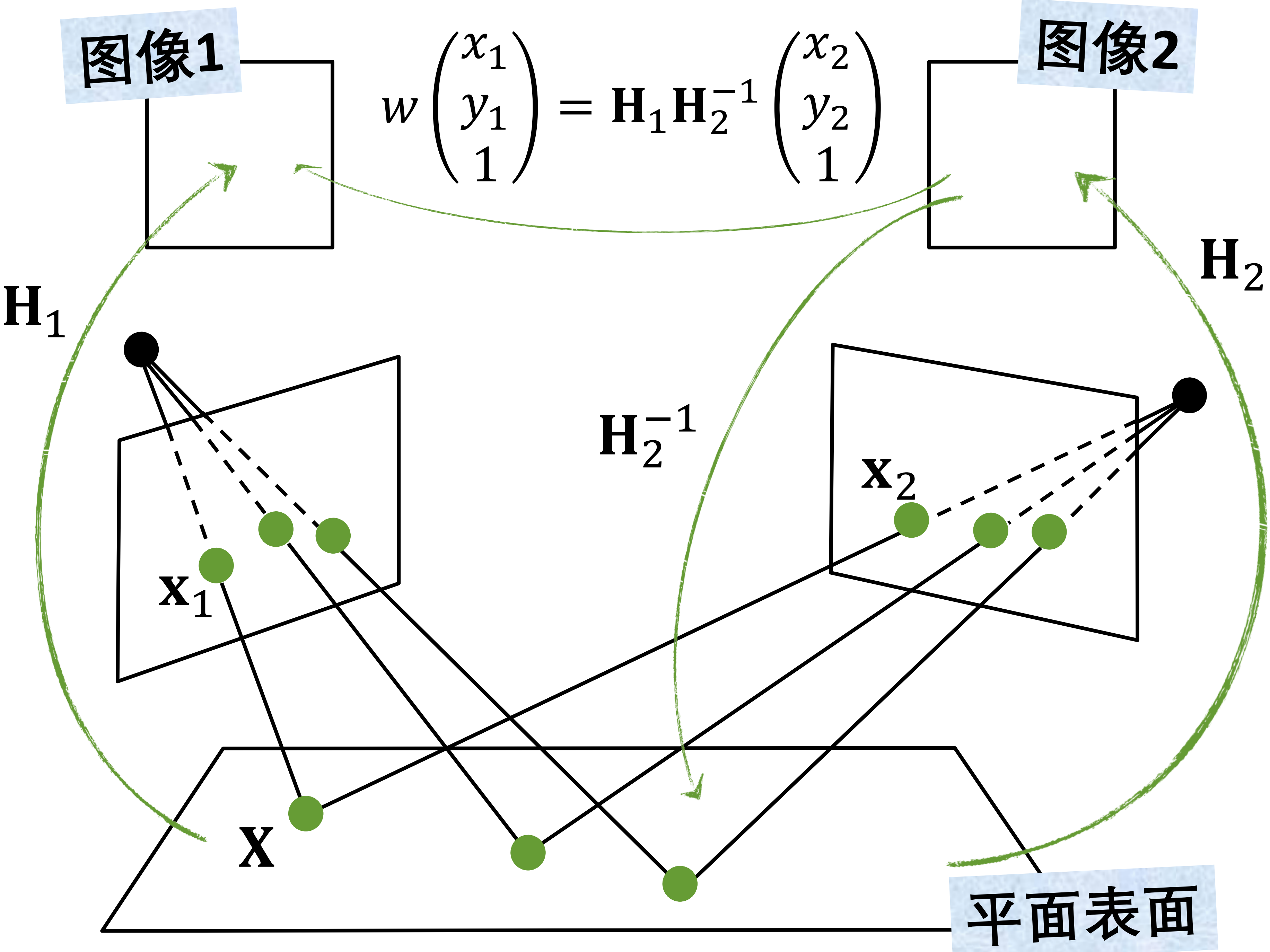
$$\omega \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

单应矩阵

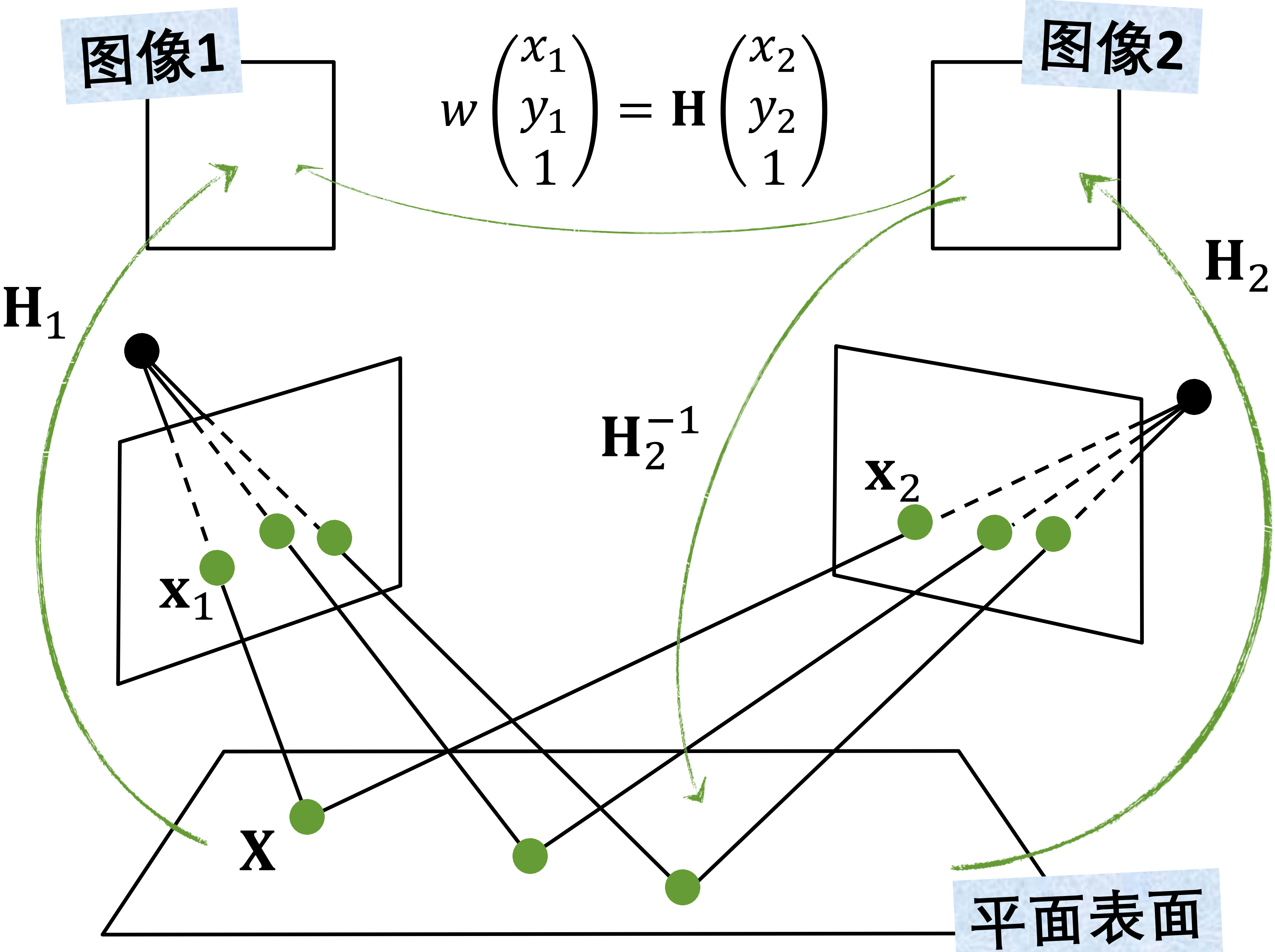


$$w\begin{pmatrix}x\\y\\1\end{pmatrix}=\begin{pmatrix}h_{11}&h_{12}&h_{13}\\h_{21}&h_{22}&h_{23}\\h_{31}&h_{32}&h_{33}\end{pmatrix}\begin{pmatrix}X\\Y\\1\end{pmatrix}$$

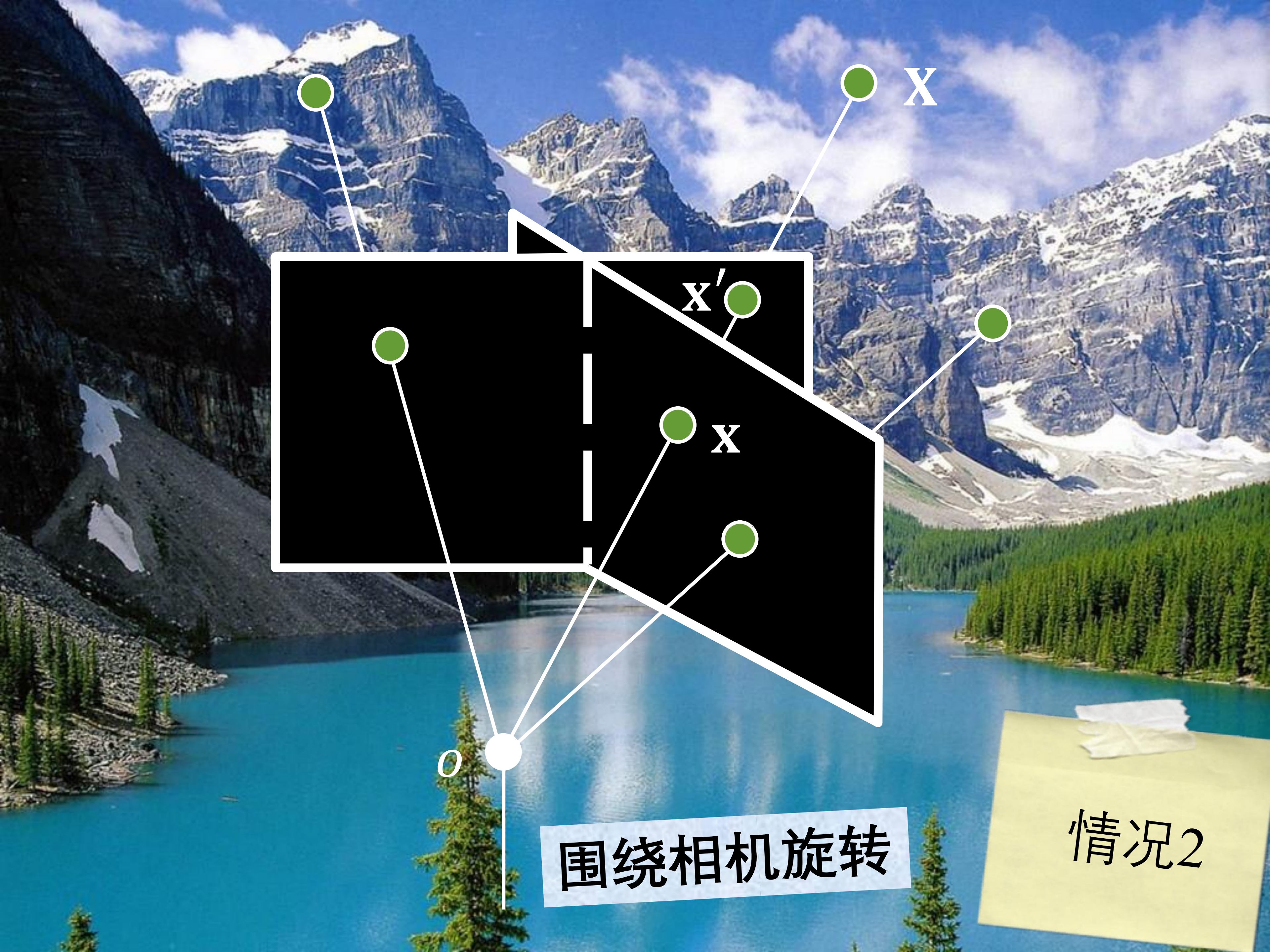
情况1



图像1
图像2
$w\begin{pmatrix}x_1\\y_1\\1\end{pmatrix} = \mathbf{H}_1\mathbf{H}_2^{-1}\begin{pmatrix}x_2\\y_2\\1\end{pmatrix}$
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_2^{-1}$
$\mathbf{x}_1$
$\mathbf{x}_2$
$\mathbf{X}$
平面表面



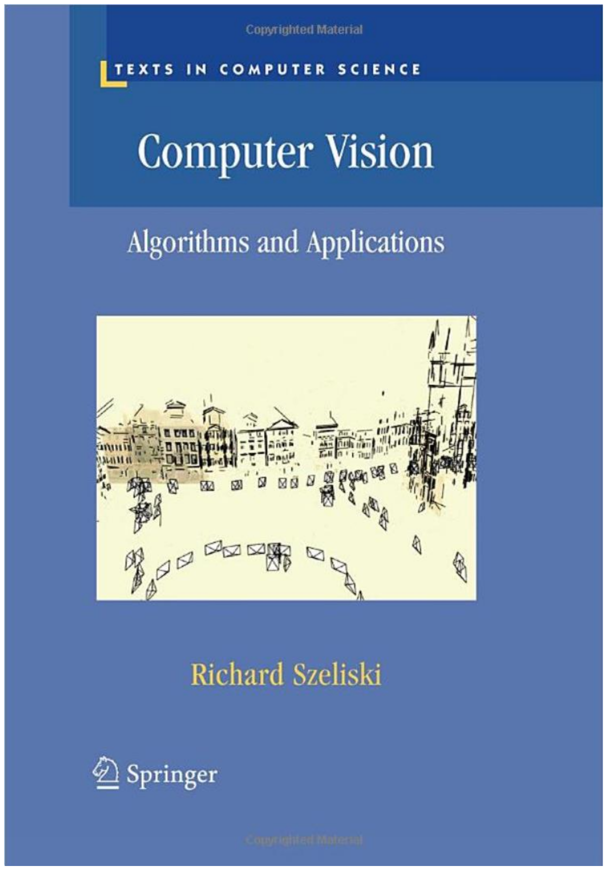
图像1
$$w\begin{pmatrix}x_1\\y_1\\1\end{pmatrix}=\mathbf{H}\begin{pmatrix}x_2\\y_2\\1\end{pmatrix}$$
图像2
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_2^{-1}$
$\mathbf{x}_1$
$\mathbf{x}_2$
$\mathbf{X}$
平面表面


X
x′
x
o
围绕相机旋转
情况2

$$\omega \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

单应矩阵

不能通过伪逆求解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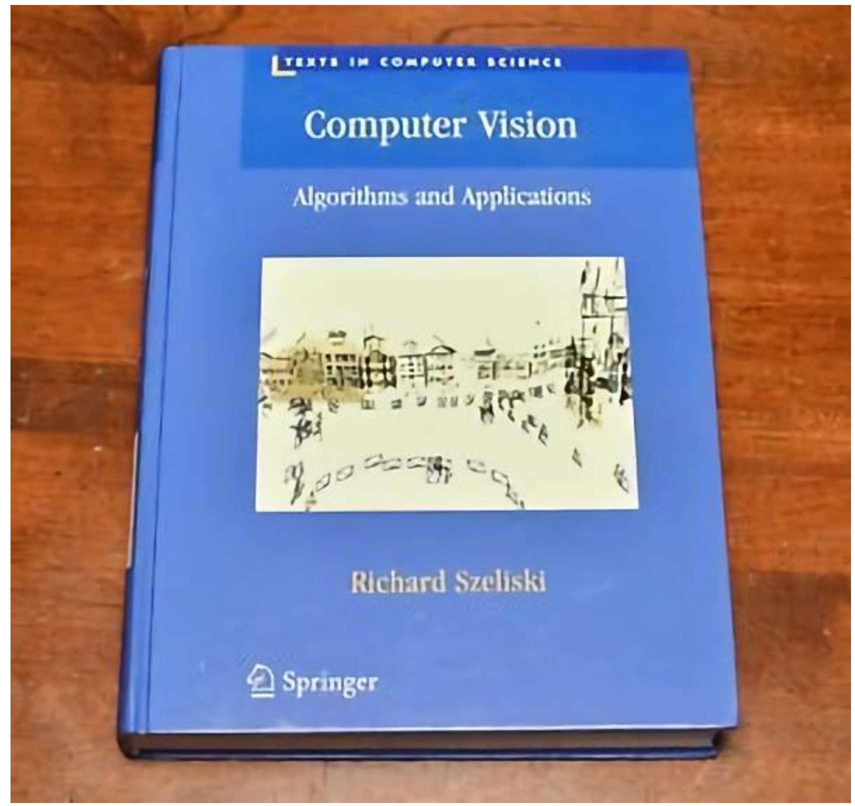
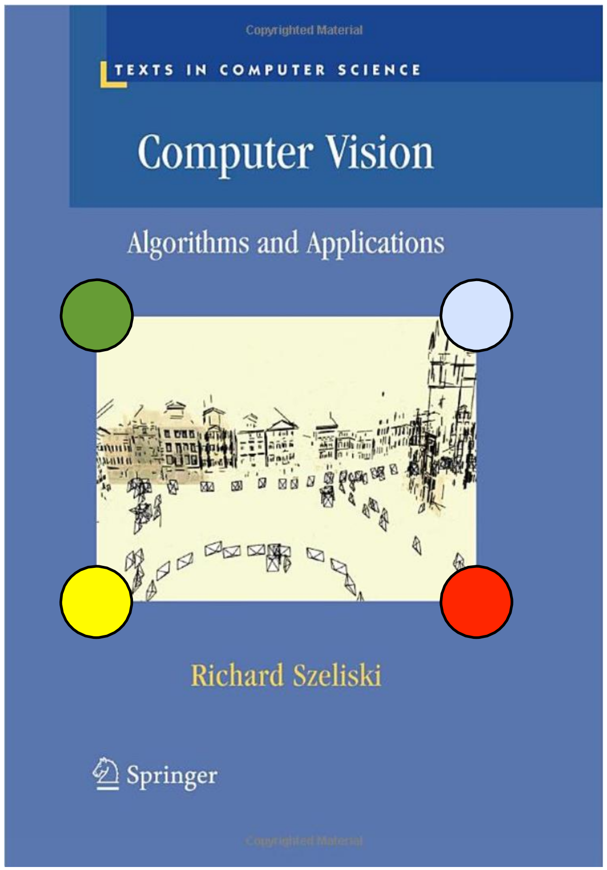
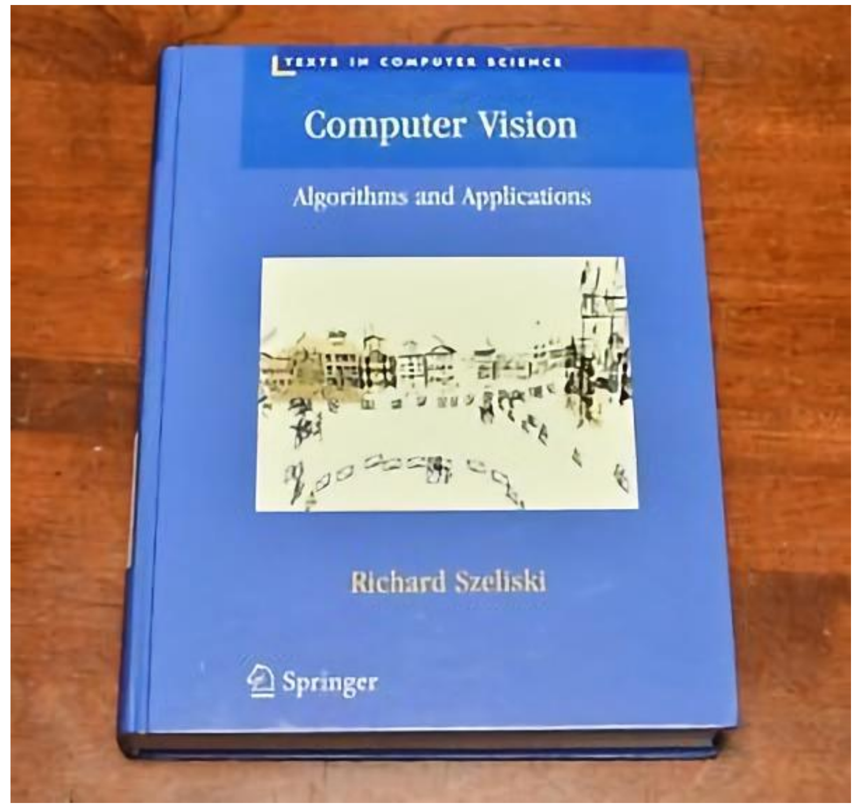
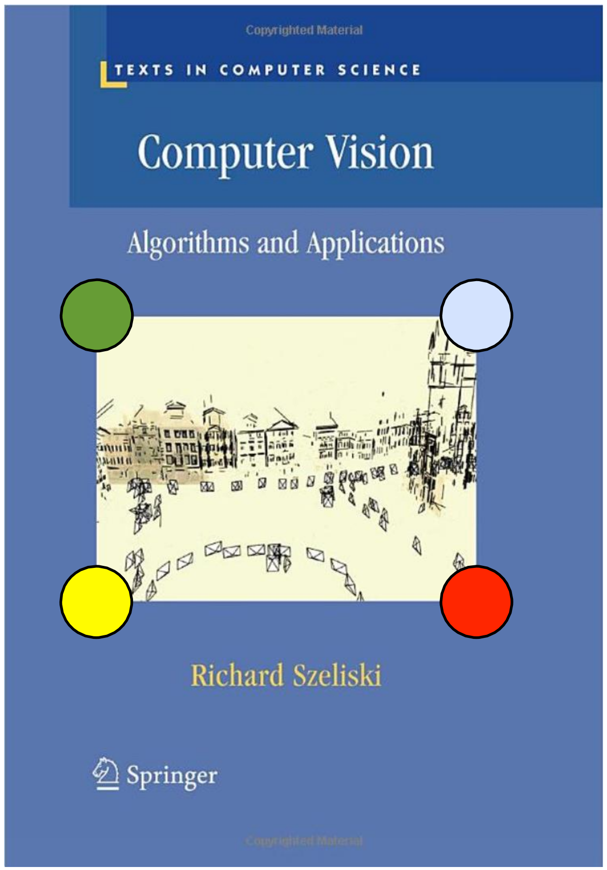

TEXTS IN COMPUTER SCIENCE
Computer Vision
Algorithms and Applications
Richard Szeliski
Springer

$$\mathbf{x}_i = (x_i, y_i)$$

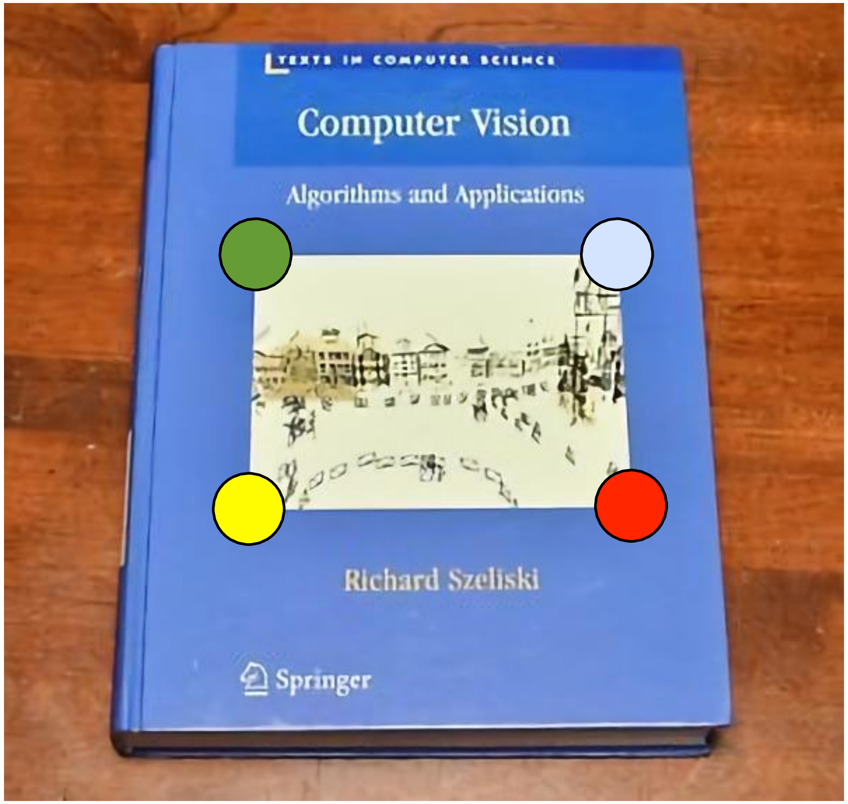




$$\mathbf{x}_i = (x_i, y_i)$$



$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$



$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$
\begin{aligned}
\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} &= \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}
\end{aligned}
$$

展开

$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

展开

$$\mathbf{x}_i' = \begin{pmatrix} \dfrac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}} \\ \dfrac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}} \end{pmatrix}$$

$$\mathbf{x}_i' = \begin{pmatrix} \dfrac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}} \\ \dfrac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}} \end{pmatrix}$$

改写成齐次方程组

$$\mathbf{x}'_i = \begin{pmatrix} \dfrac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}} \\ \dfrac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}} \end{pmatrix}$$

改写成齐次方程组

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 & -y'_1 \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx'_N & -x_Nx'_N & -x'_N \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx'_N & -x_Nx'_N & -y'_N \end{pmatrix} \mathbf{h} = \mathbf{0}$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

至少需要4点来估计h

$$
\begin{pmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\
 & & & & & \vdots & & & \\
x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\
0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N'
\end{pmatrix}
\mathbf{h} = \mathbf{0}
$$

使用最小二乘法估计映射

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix}\mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2$$

不能用伪逆估计

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2 \text{ subject to } \|\mathbf{h}\| = 1$$

$$
\begin{pmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\
 & & & & & \vdots & & & \\
x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\
0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N'
\end{pmatrix}\mathbf{h} = \mathbf{0}
$$

使用最小二乘法估计映射

$$
\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2 \text{ subject to } \|\mathbf{h}\| = 1
$$

计算A的SVD，$\mathbf{h}^*$就是V的最后一列

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} = \mathbf{M}_{\mathrm{int}} \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix}$$

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix}$$

如何恢复相机模型参数?

◆ decomposeHomographyMat()

```
int cv::decomposeHomographyMat ( InputArray            H,
                                 InputArray            K,
                                 OutputArrayOfArrays   rotations,
                                 OutputArrayOfArrays   translations,
                                 OutputArrayOfArrays   normals
                               )
```

Python:

```
cv.decomposeHomographyMat( H, K[, rotations[, translations[, normals]]] ) -> retval, rotations, translations, normals
```

```
#include <opencv2/calib3d.hpp>
```

Decompose a homography matrix to rotation(s), translation(s) and plane normal(s).

Parameters

H	The input homography matrix between two images.
K	The input camera intrinsic matrix.
rotations	Array of rotation matrices.
translations	Array of translation matrices.
normals	Array of plane normal matrices.

This function extracts relative camera motion between two views of a planar object and returns up to four mathematical solution tuples of rotation, translation, and plane normal. The decomposition of the homography matrix H is described in detail in **[166]**.

If the homography H, induced by the plane, gives the constraint

$$s_i \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \sim H \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

on the source image points p_i and the destination image points p'_i, then the tuple of rotations[k] and translations[k] is a change of basis from the source camera's coordinate system to the destination camera's coordinate system. However, by decomposing H, one can only get the translation normalized by the (typically unknown) depth of the scene, i.e. its direction but with normalized length.

If point correspondences are available, at least two solutions may further be invalidated, by applying positive depth constraint, i.e. all points must be in front of the camera.

Examples:

samples/cpp/tutorial_code/features2D/Homography/decompose_homography.cpp.

MATLAB File Help: cv.decompose
https://kyamagu.github.io/mexopencv/matlab/decomposeHomographyMat.html

Output

- **motions** Decomposed `H`. A scalar struct with the following fields:
  - **R** Array of rotation matrices. Cell array of 3x3 rotations.
  - **t** Array of translation matrices. Cell array of 3x1 translations.
  - **n** Array of plane normal matrices. Cell array of 3x1 normals.
- **nsols** number of solutions.

This function extracts relative camera motion between two views observing a planar object from the homography `H` induced by the plane. The intrinsic camera matrix `K` must also be provided. The function may return up to four mathematical solution sets. At least two of the solutions may further be invalidated if point correspondences are available by applying positive depth constraint (all points must be in front of the camera). The decomposition method is described in detail in [Malis].

References

[Malis]:

Ezio Malis, Manuel Vargas, and others. "Deeper understanding of the homography decomposition for vision-based control". 2007.

See also

cv.findHomography, cv.decomposeEssentialMat

MATLAB File Help: cv.decompose

https://kyamagu.github.io/mexopencv/matlab/decomposeHomographyMat.html

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可以通过正深度约束验证

References

[Malis]:

Ezio Malis, Manuel Vargas, and others. "Deeper understanding of the homography decomposition for vision-based control". 2007.

See also

cv.findHomography, cv.decomposeEssentialMat

INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Deeper understanding of the homography decomposition for vision-based control

Ezio Malis and Manuel Vargas

N° 6303

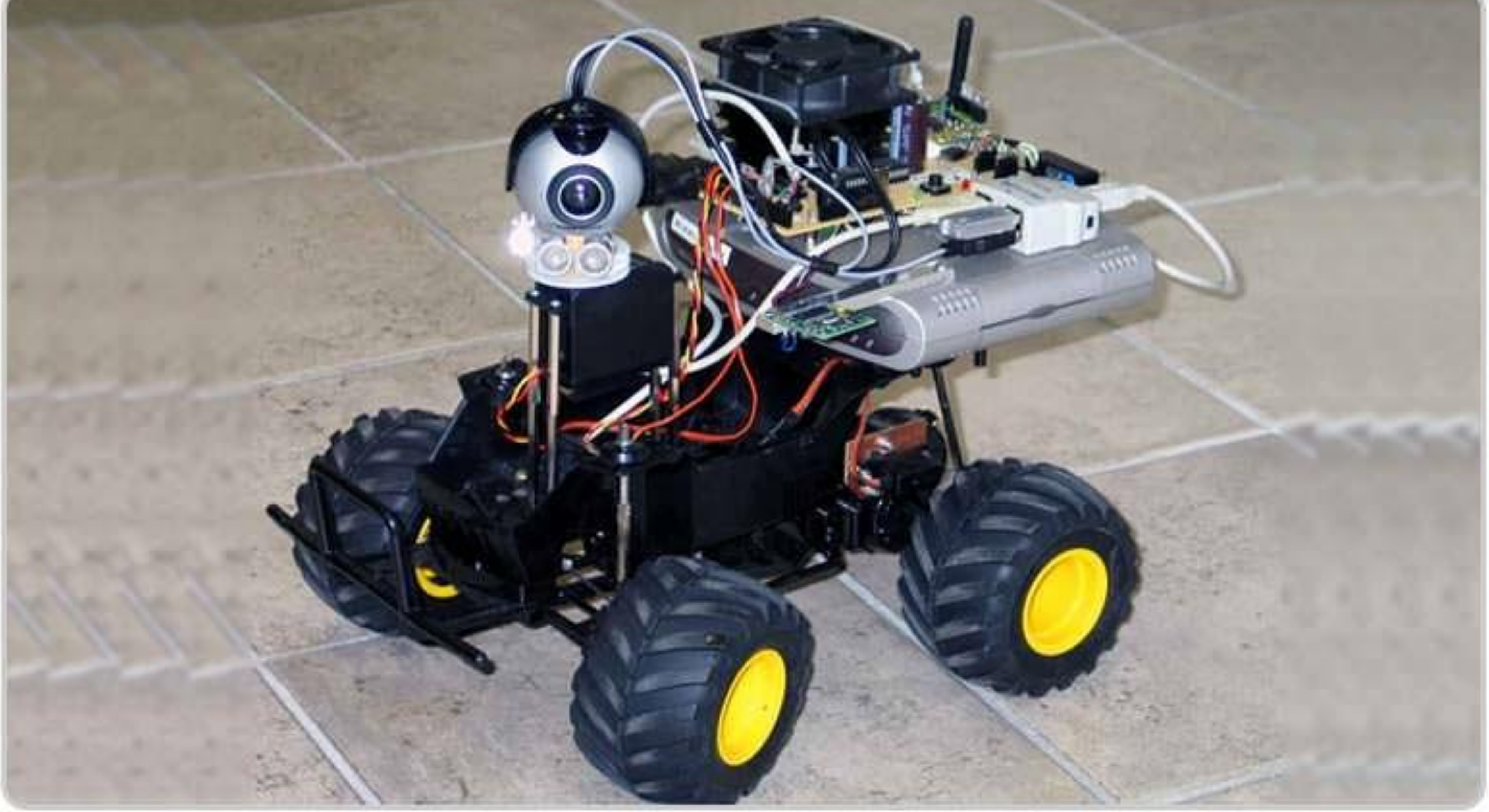
Septembre 2007

INRIA, 2007

SLAM

Simultaneous Localization and Mapping

同步定位与制图



Real-Time 6-DOF Monocular Visual SLAM in a Large-Scale Environment

Hyon Lim, Jongwoo Lim, H. Jin Kim

ICRA 2014 Video

鸣谢： Hypo Lim et al.