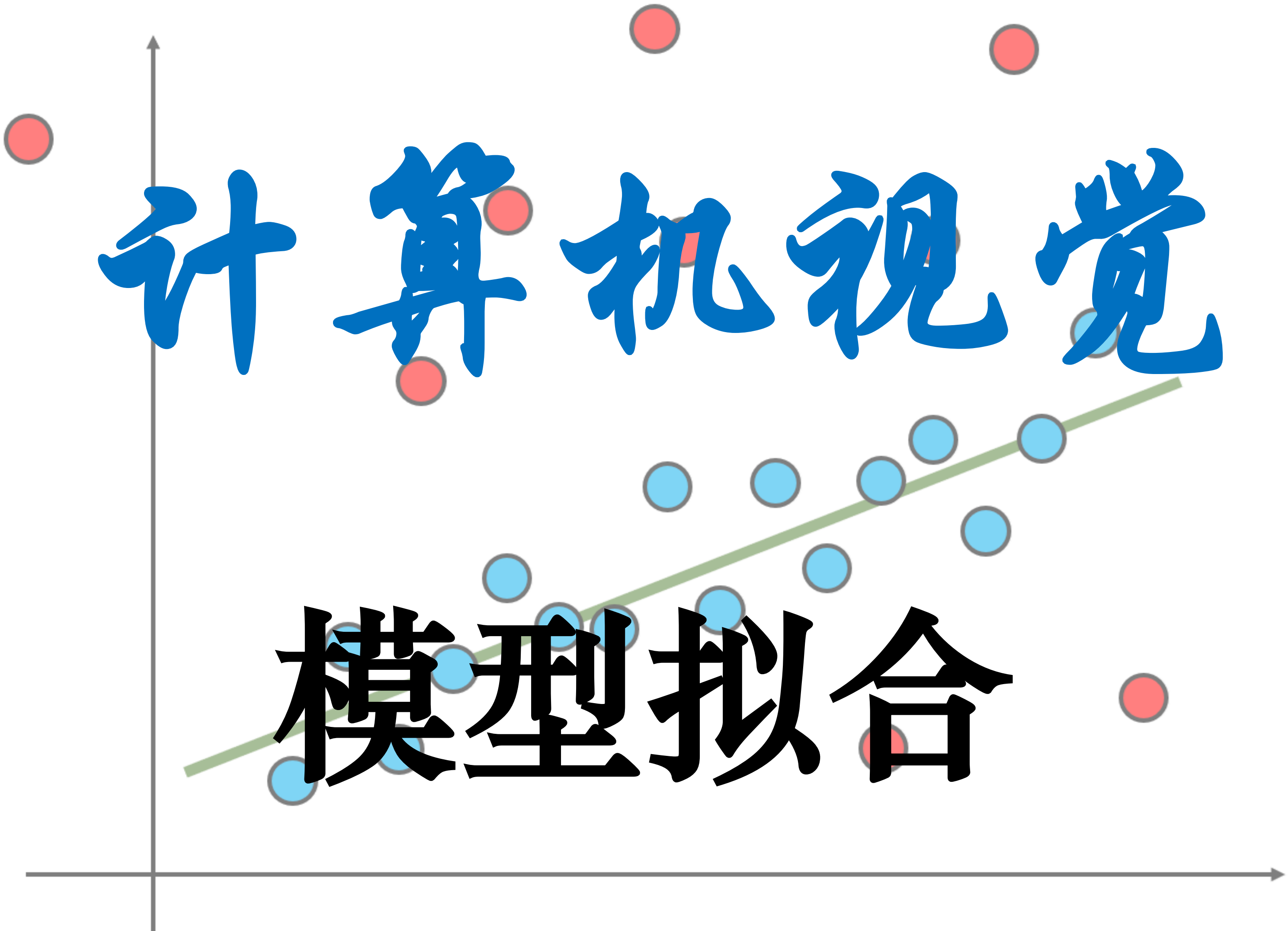


计算机视觉

模型拟合

中国传媒大学
COMMUNICATION UNIVERSITY OF CHINA

本节主题：

最小二乘法拟合

本节主题：

最小二乘法拟合

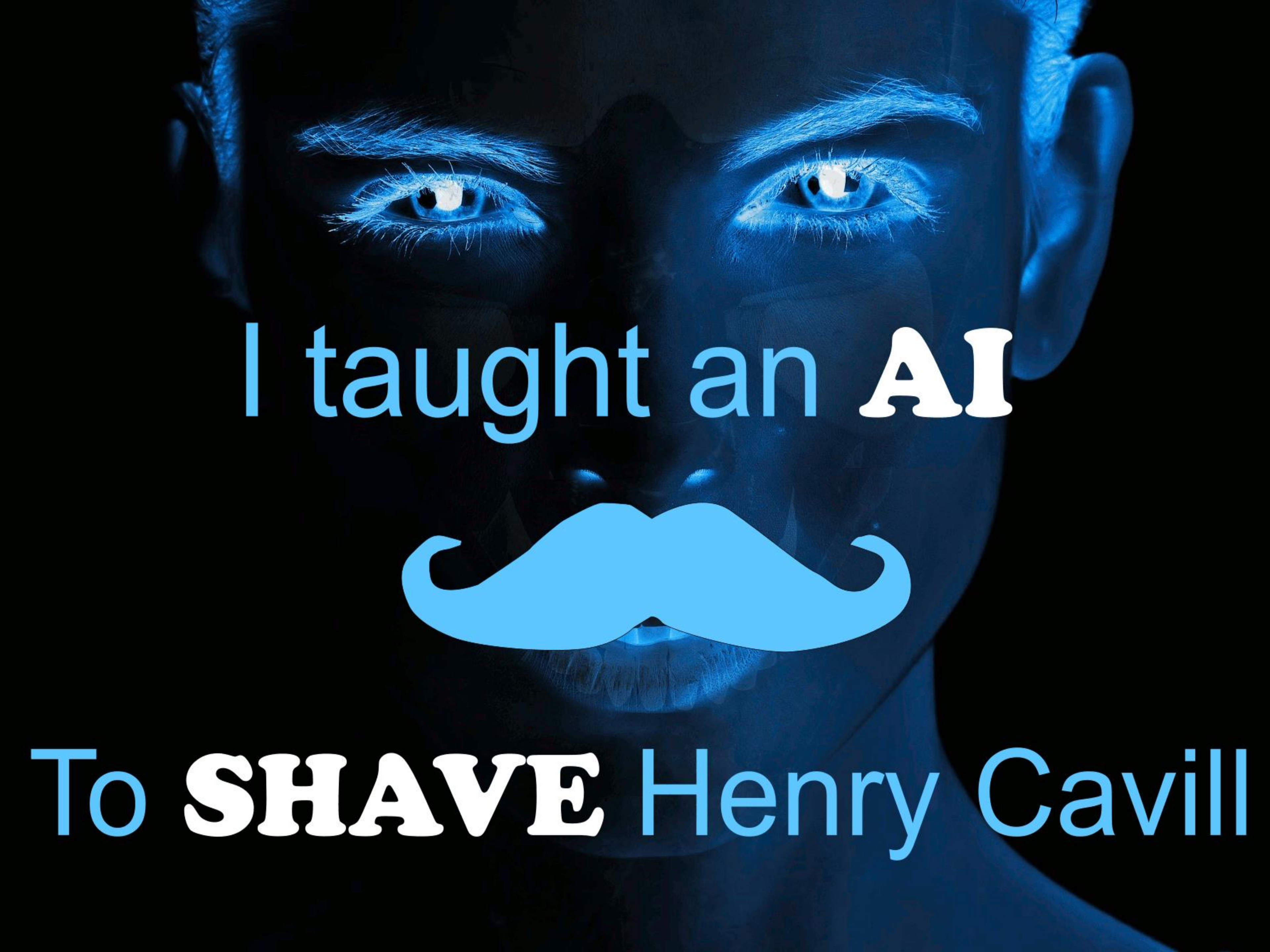
Hough变换

本节主题：

最小二乘法拟合
Hough变换
RANSAC

Henry Cavill


STARLABS

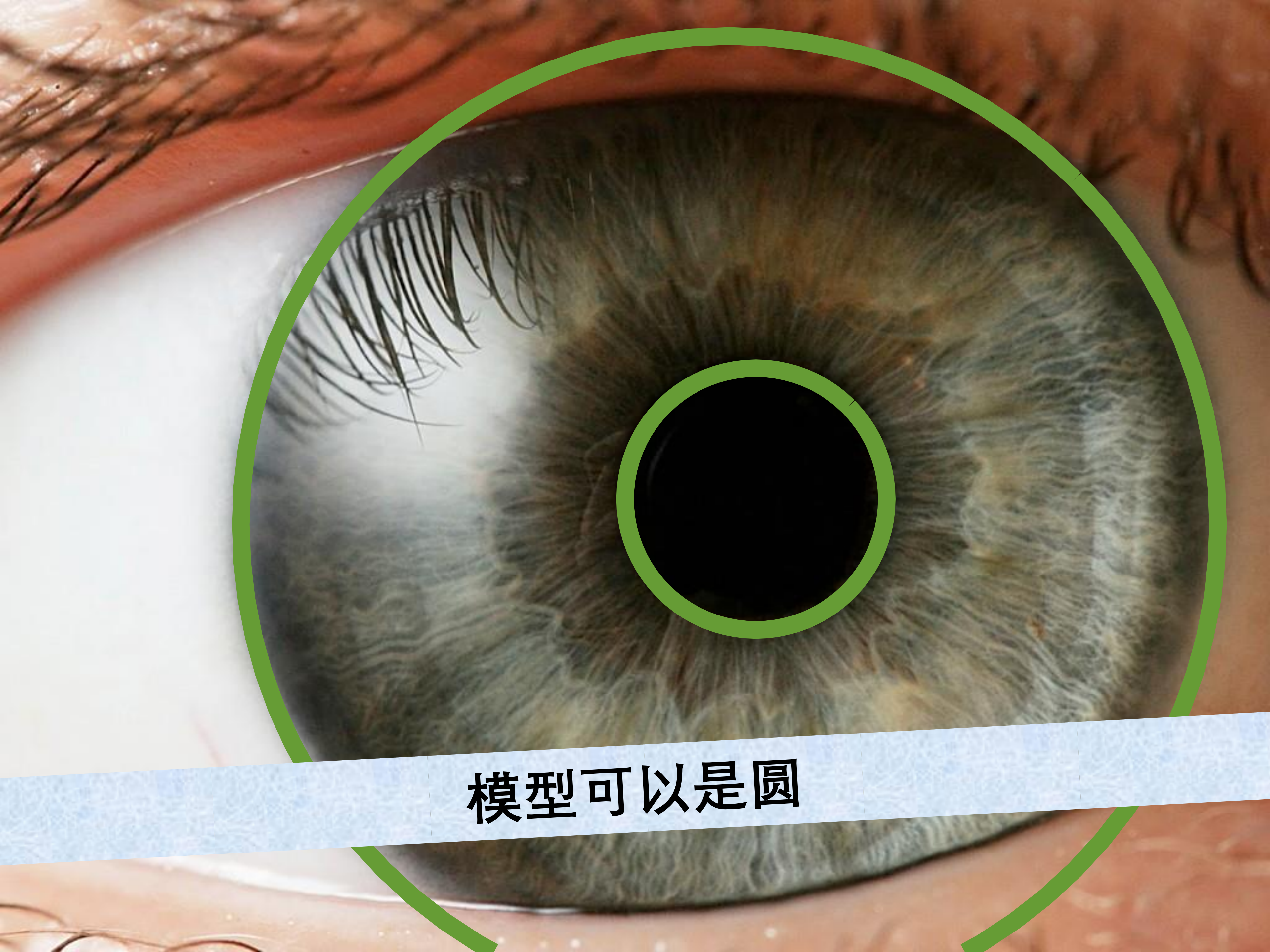

I taught an AI
To SHAVE Henry Cavill


希望将模型与观察到的特征相关联


模型可以是直线


模型可以是直线


模型可以是圆


模型可以是圆

模型可以是任意形状


Oper für das 21.Jahrhundert
模型可以是任意形状


模型可以是变换

模型可以是变换

主要问题：

主要问题：

哪种模型最能代表这一组特征？

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哪种模型最能代表这一组特征?

有多少个模型实例?

主要问题：

哪种模型最能代表这一组特征?

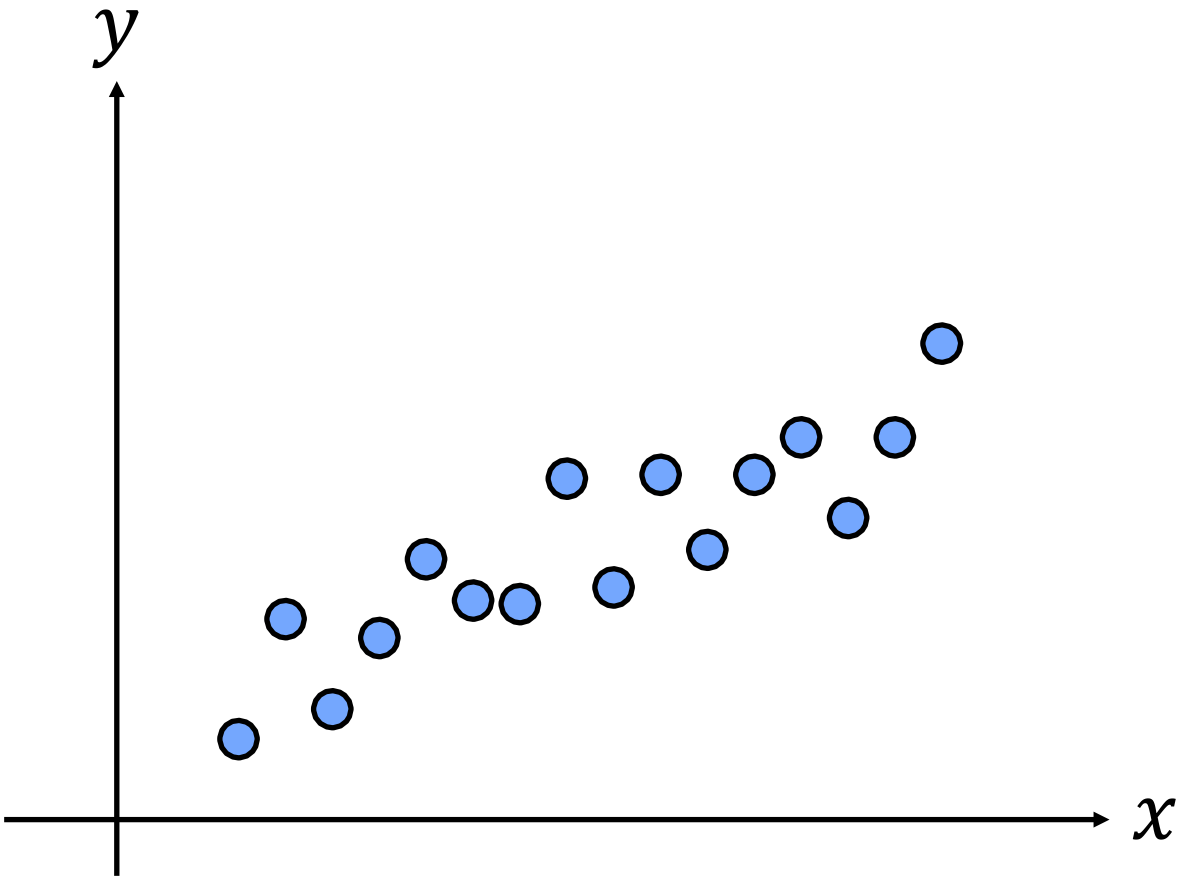
有多少个模型实例?

这些模型实例中的哪一个获得了哪些特征?

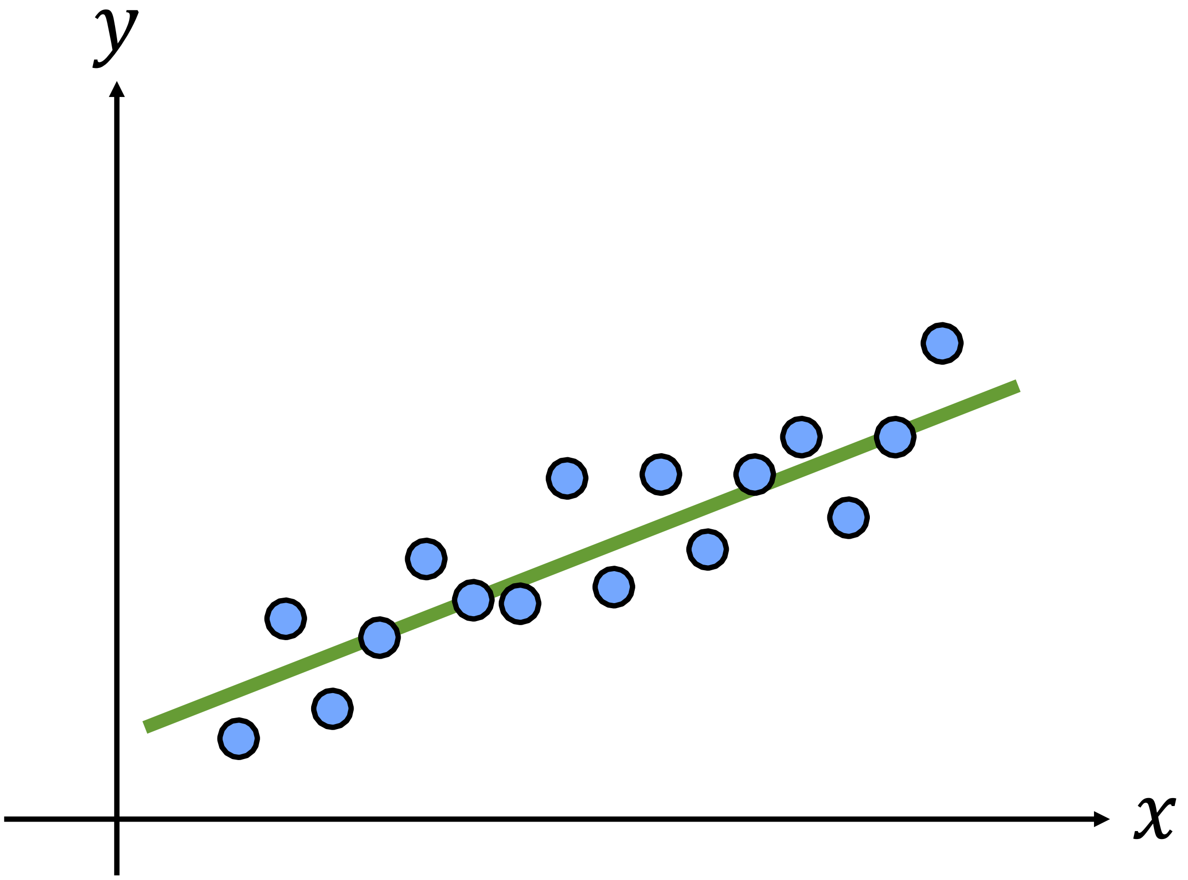
挑战

带噪声数据

带噪声数据



带噪声数据

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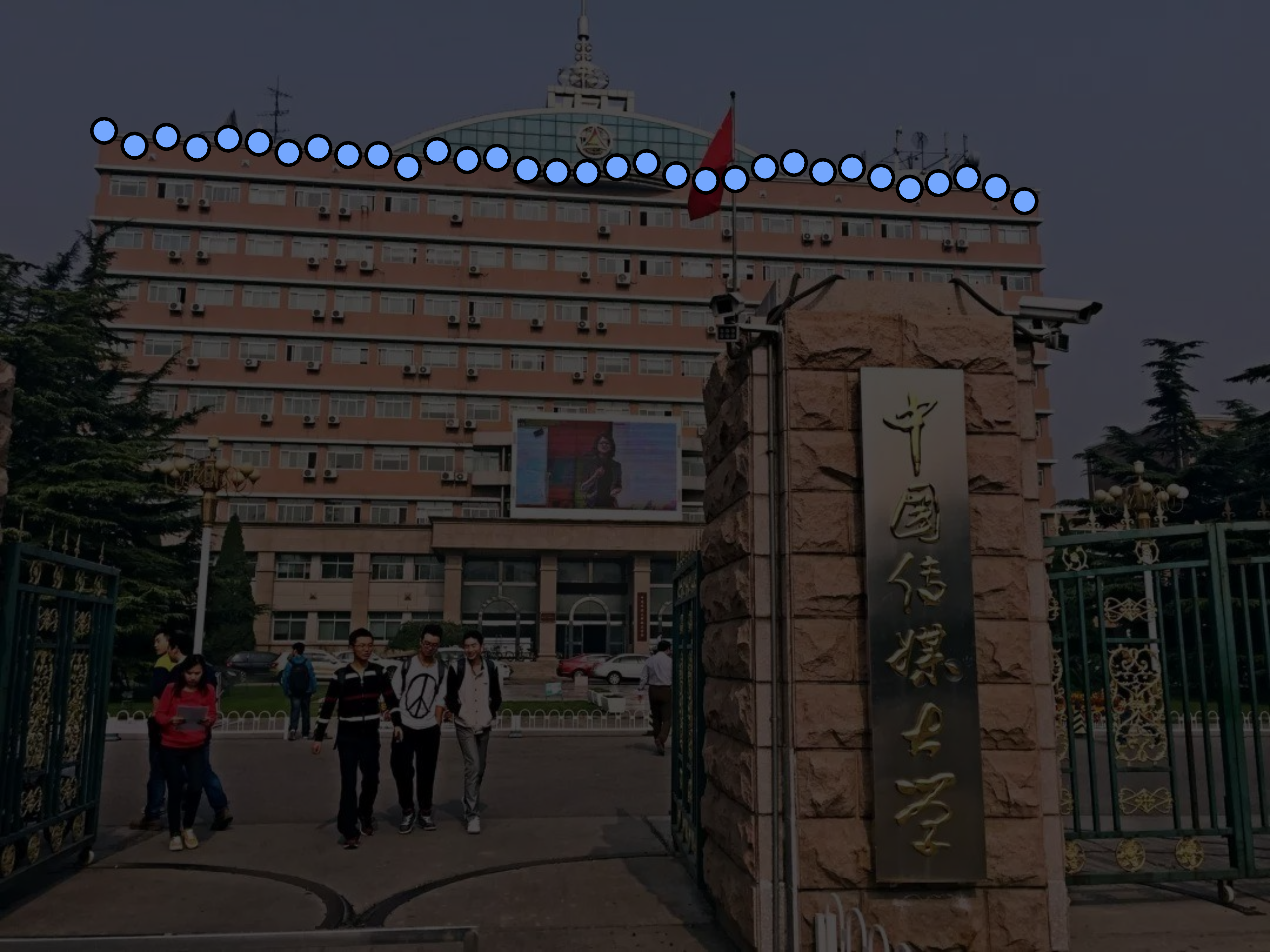
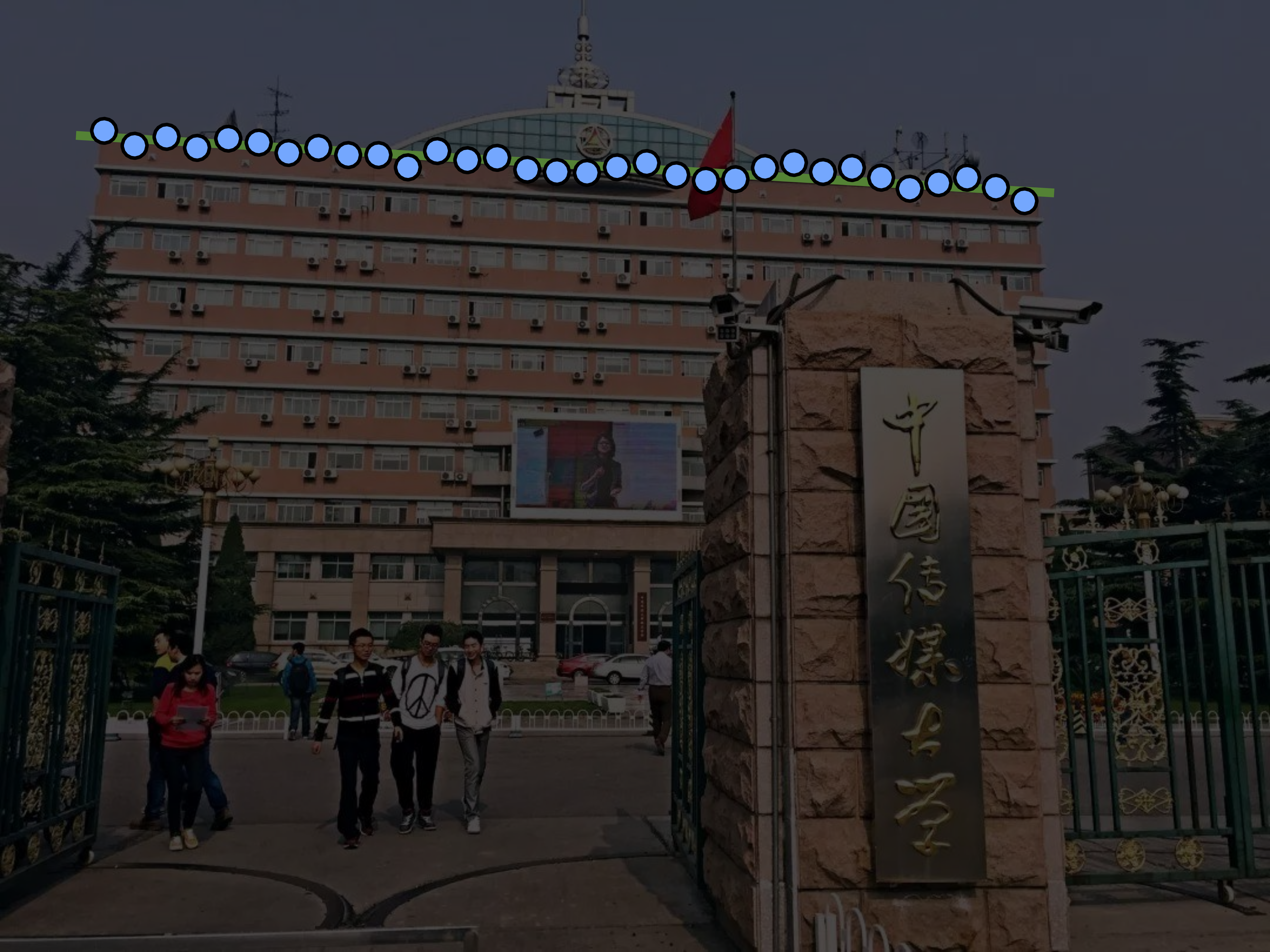

中国传媒大学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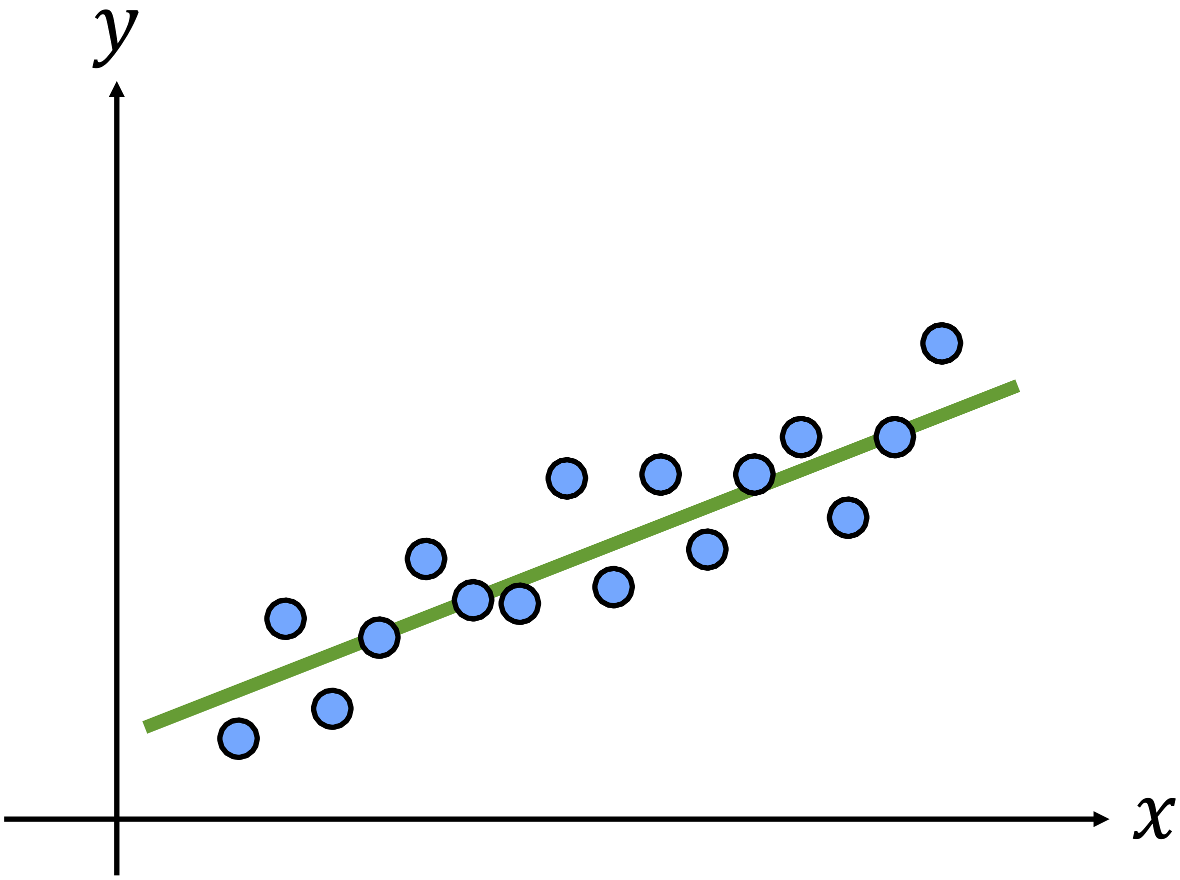
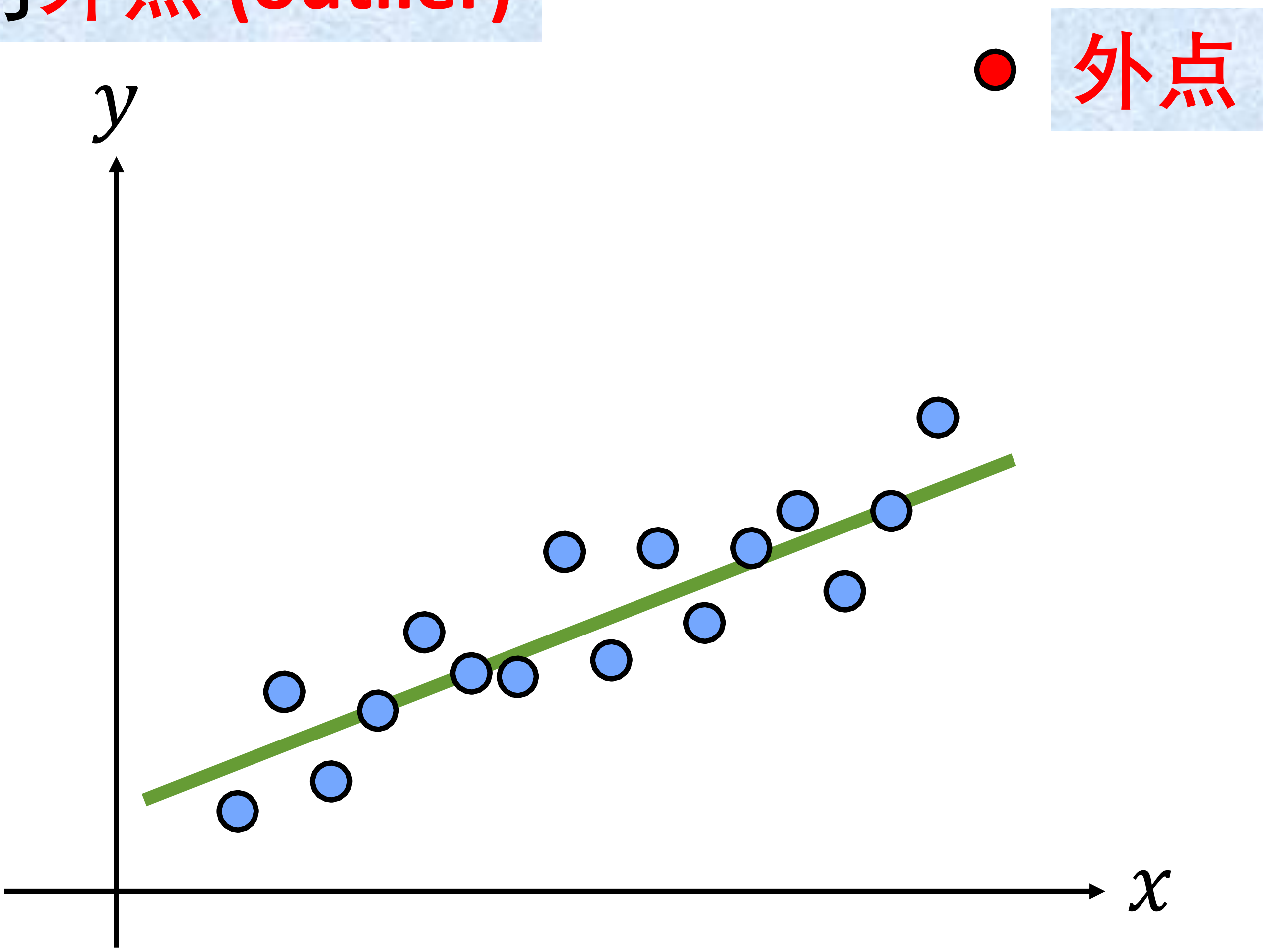
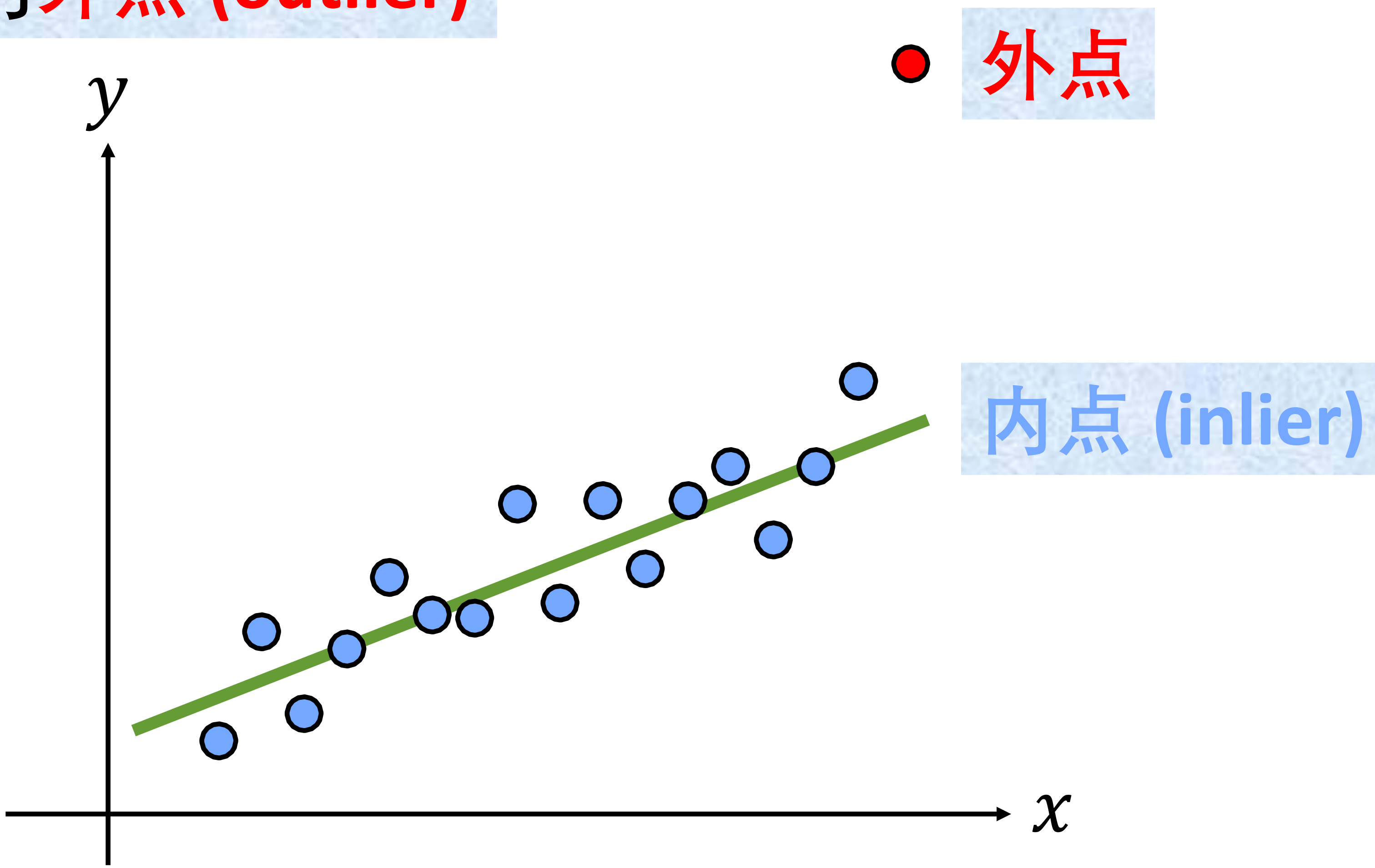
带噪声数据



数据中的外点 (outlier)



数据中的外点 (outlier)

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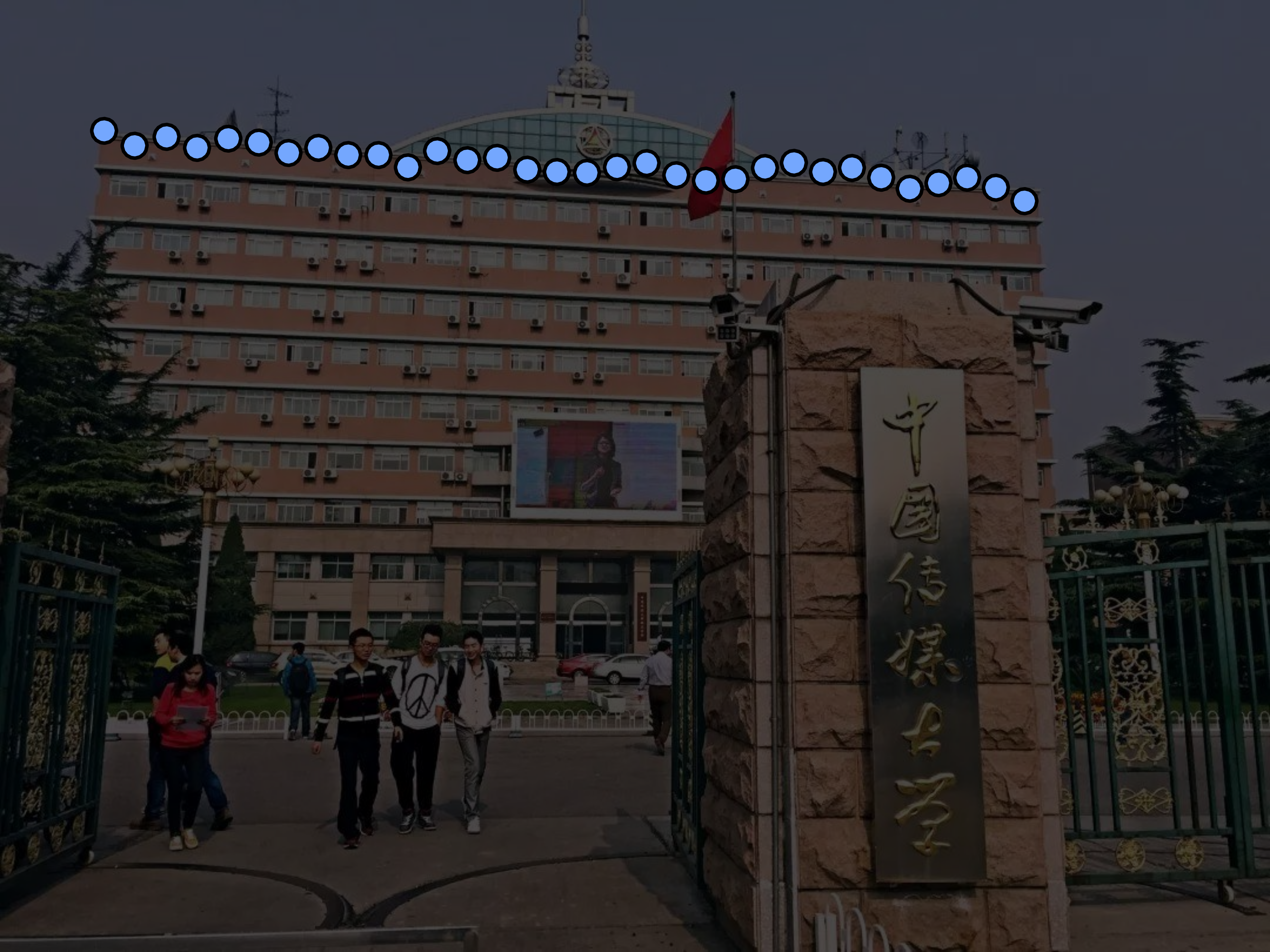
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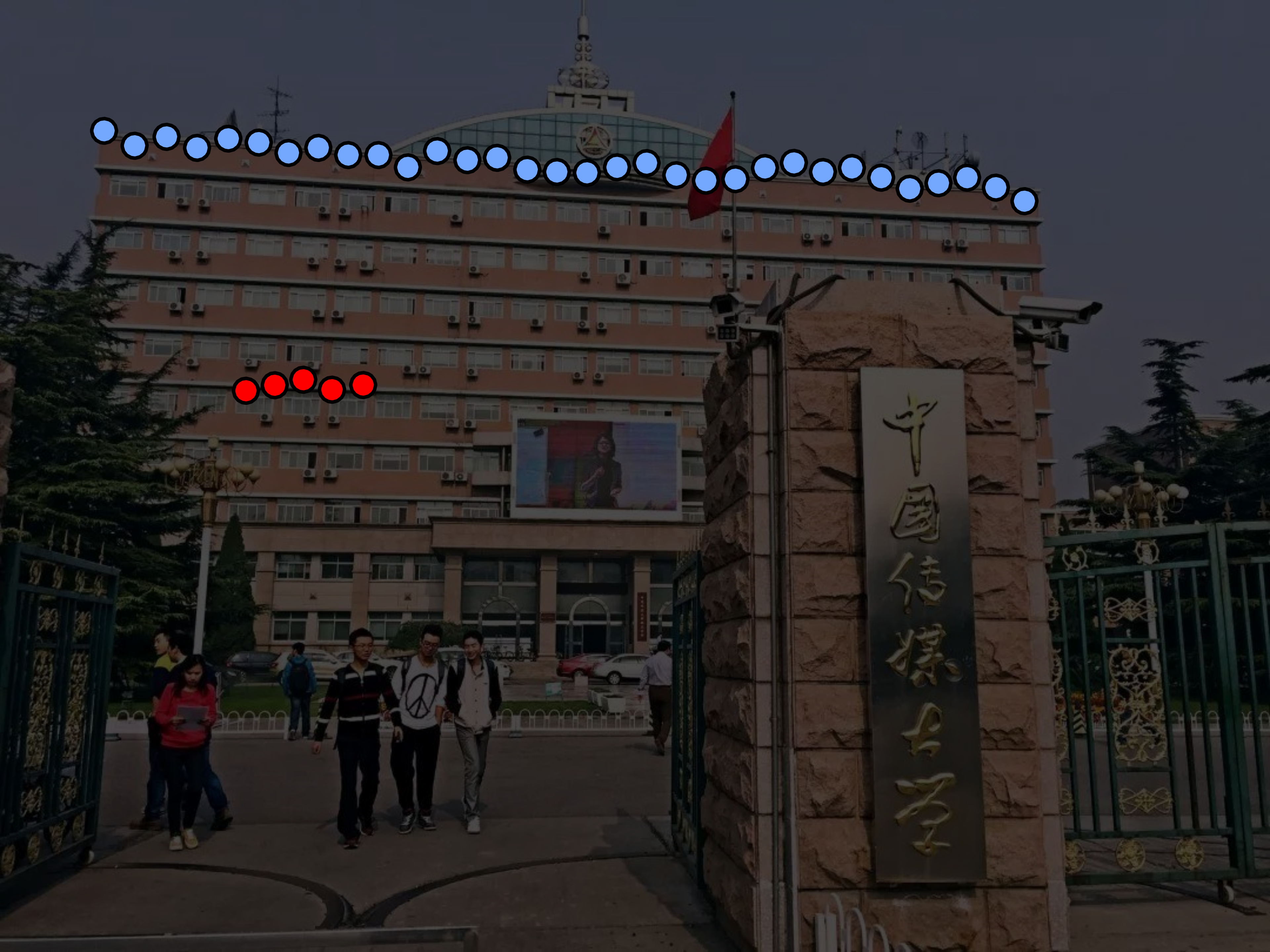
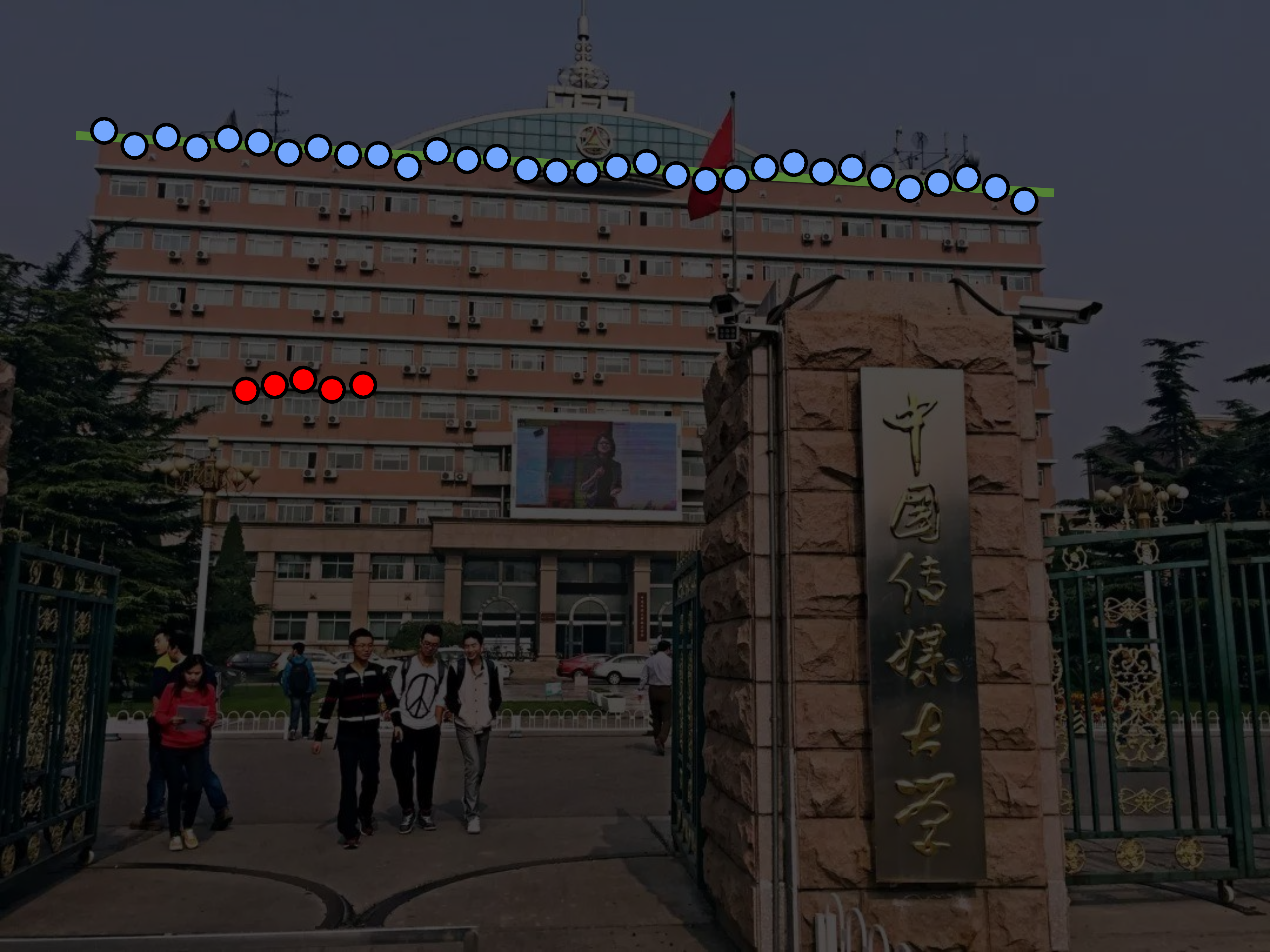
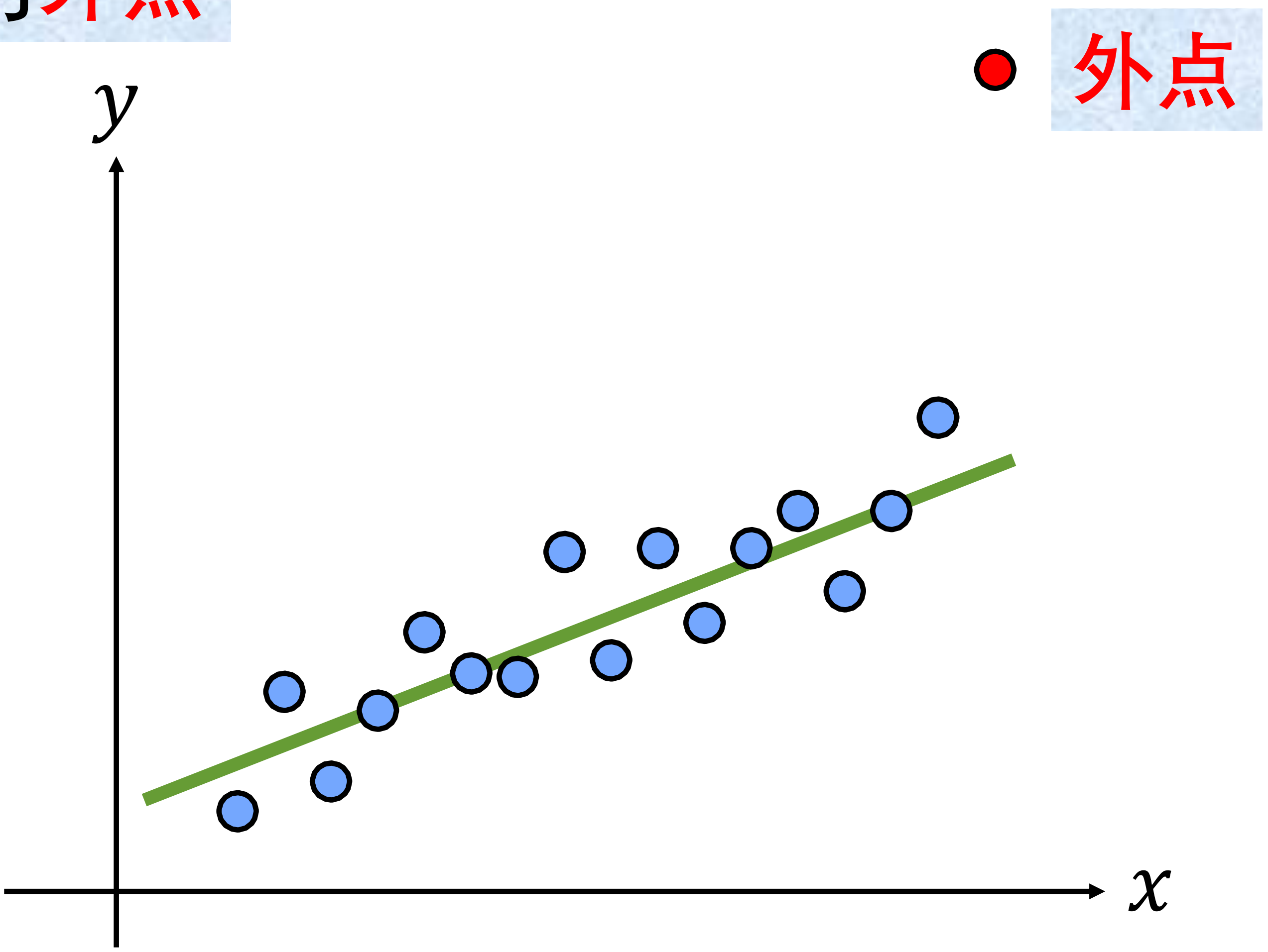
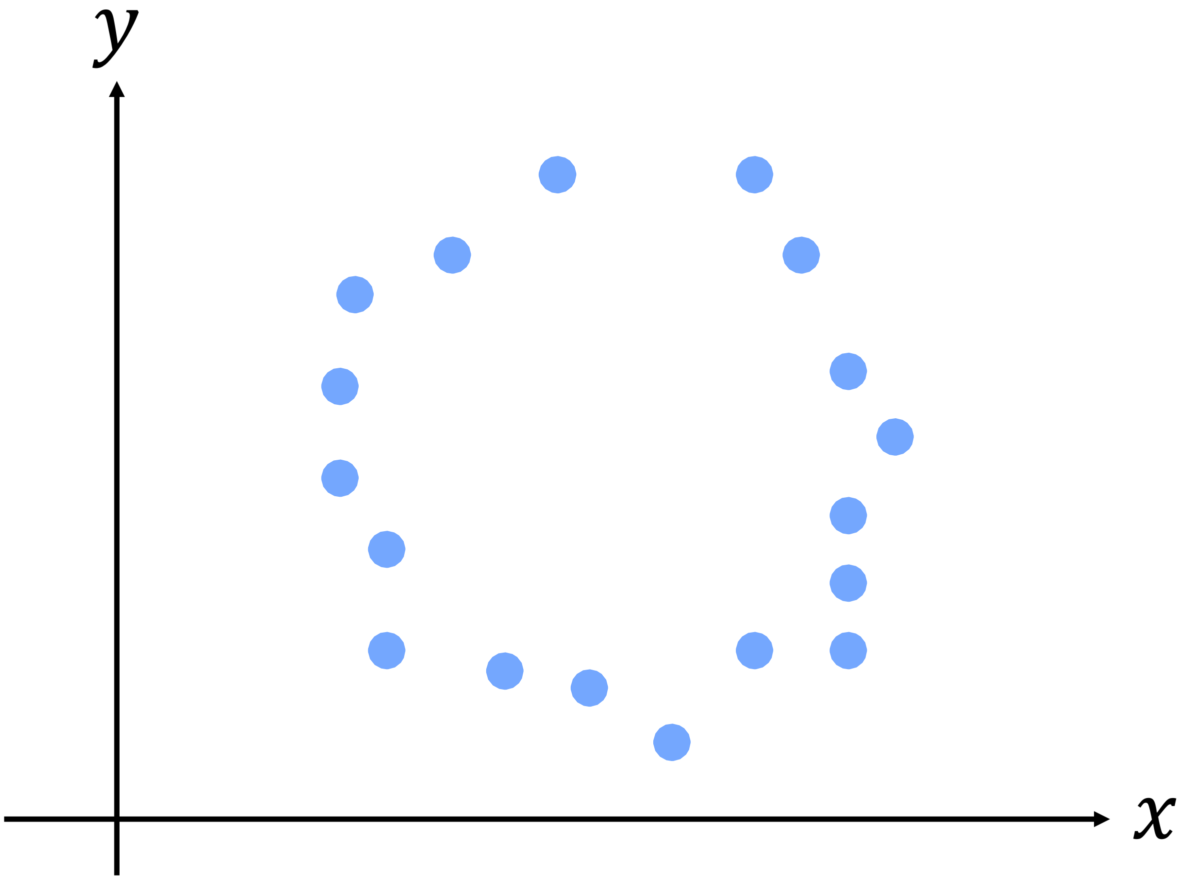

中国传媒大学


中国传媒大学

数据中的外点



缺失数据



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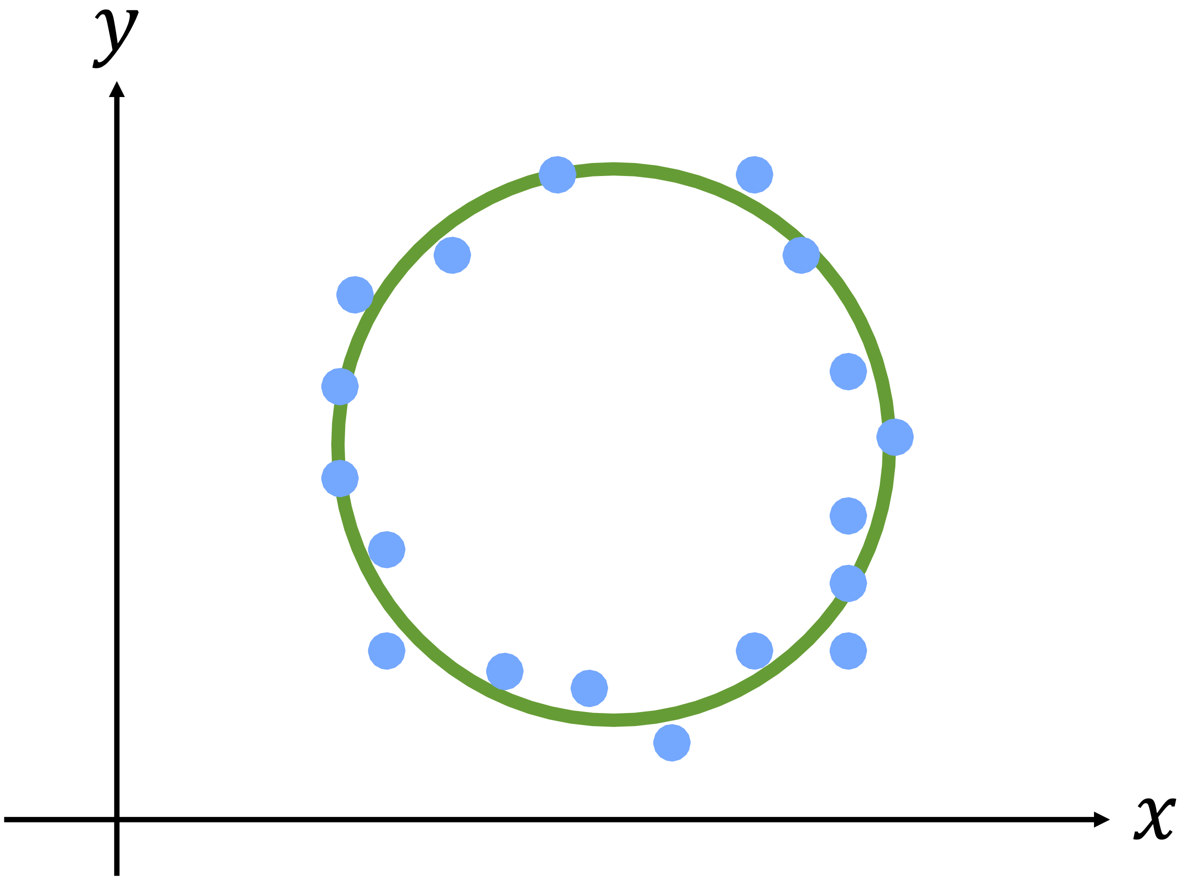

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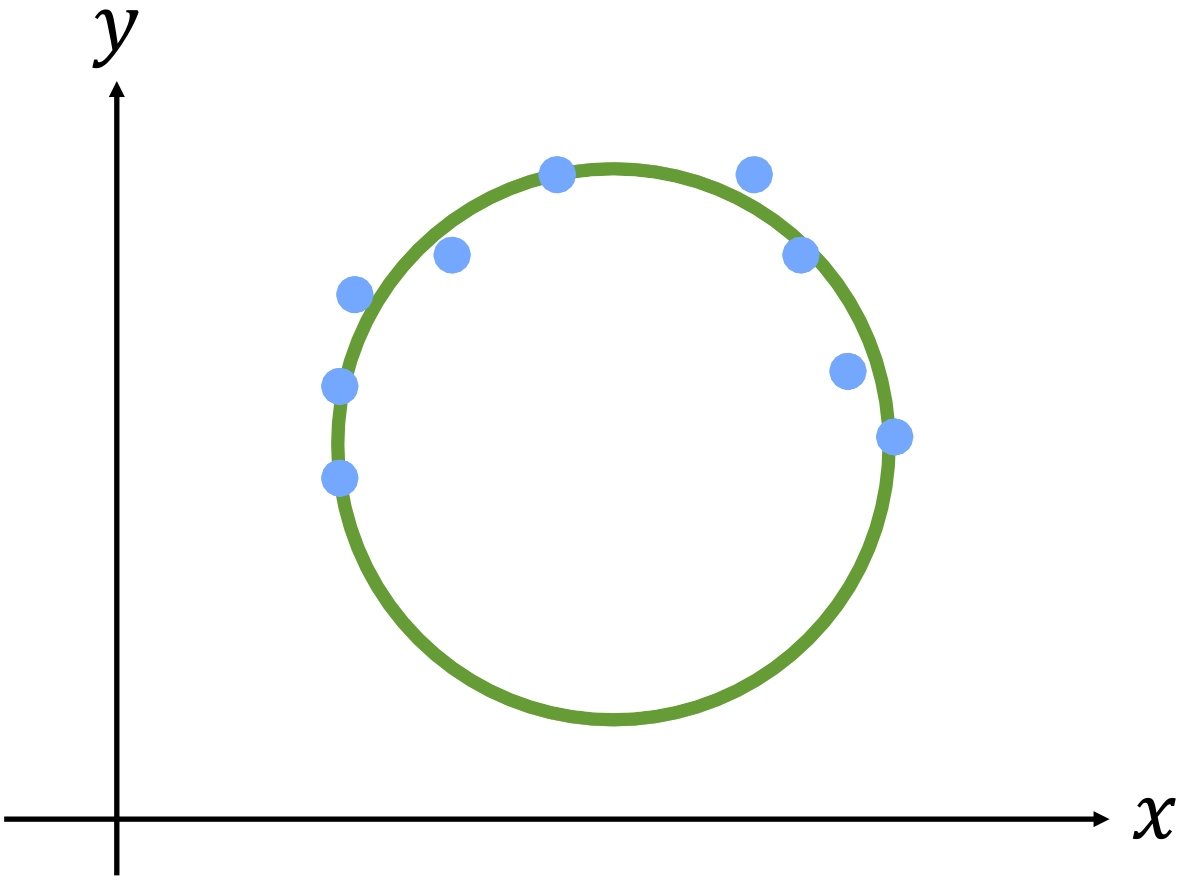

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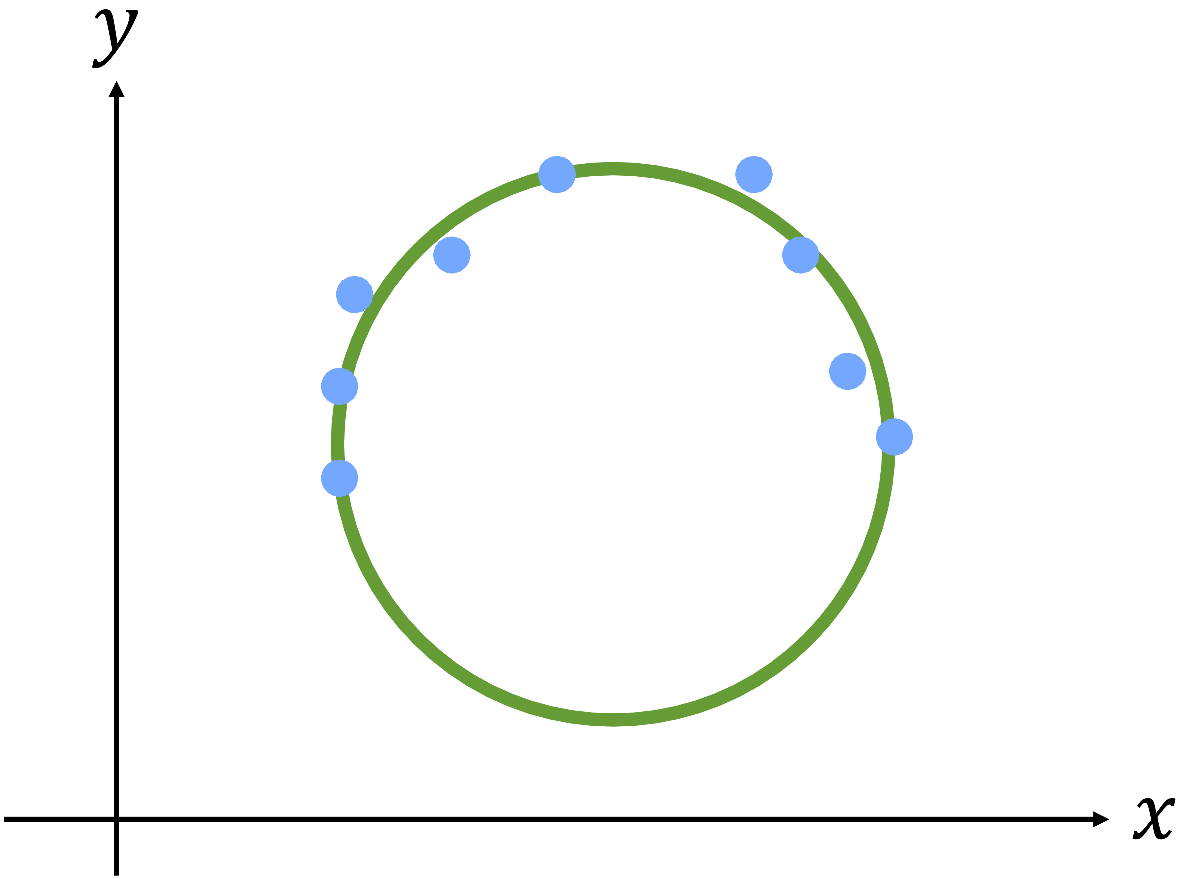
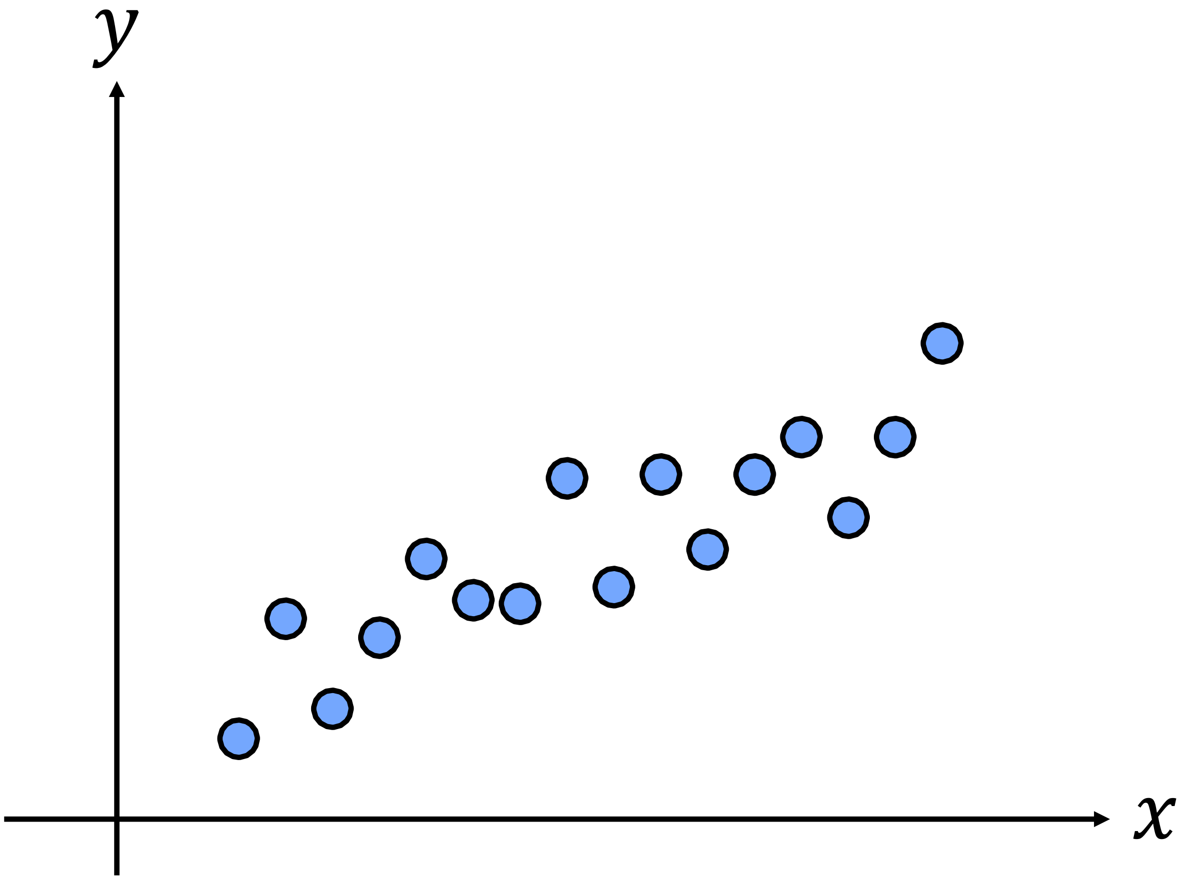
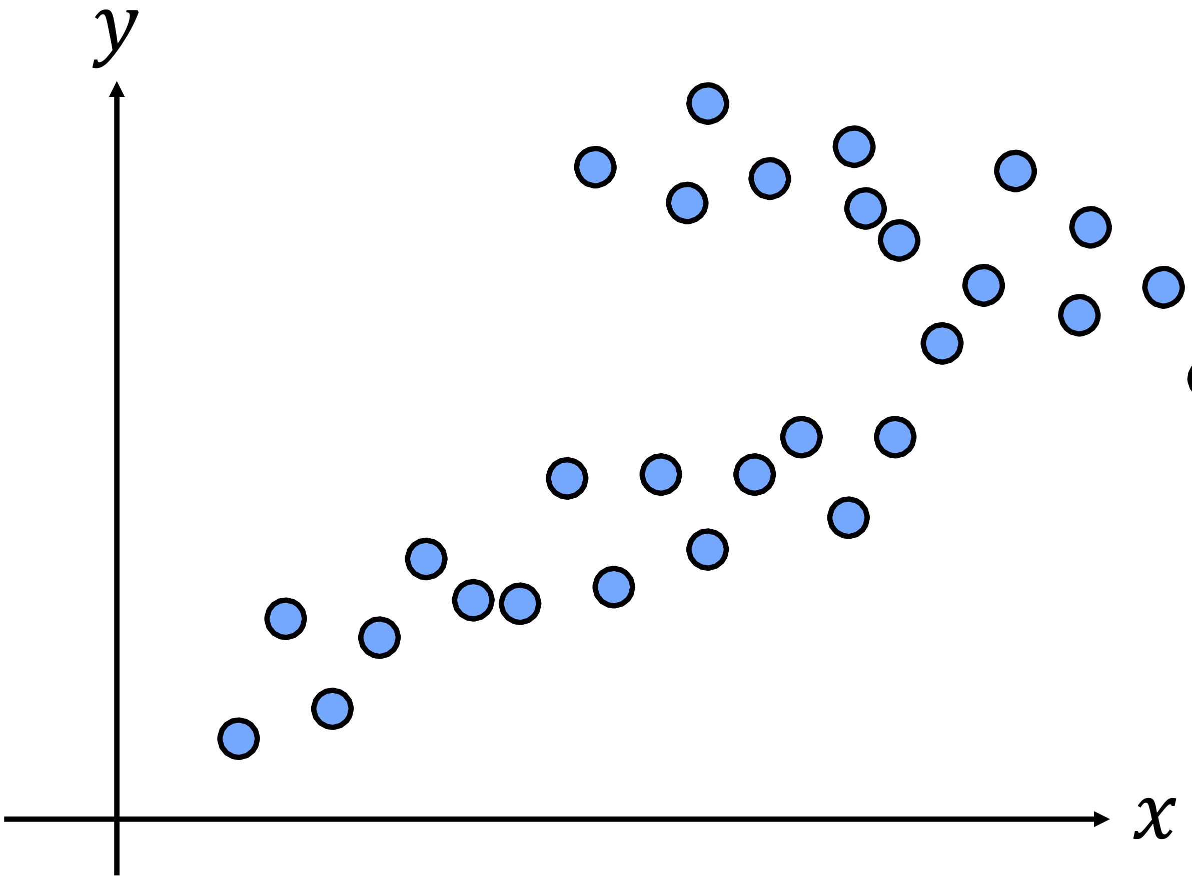


特征捆绑

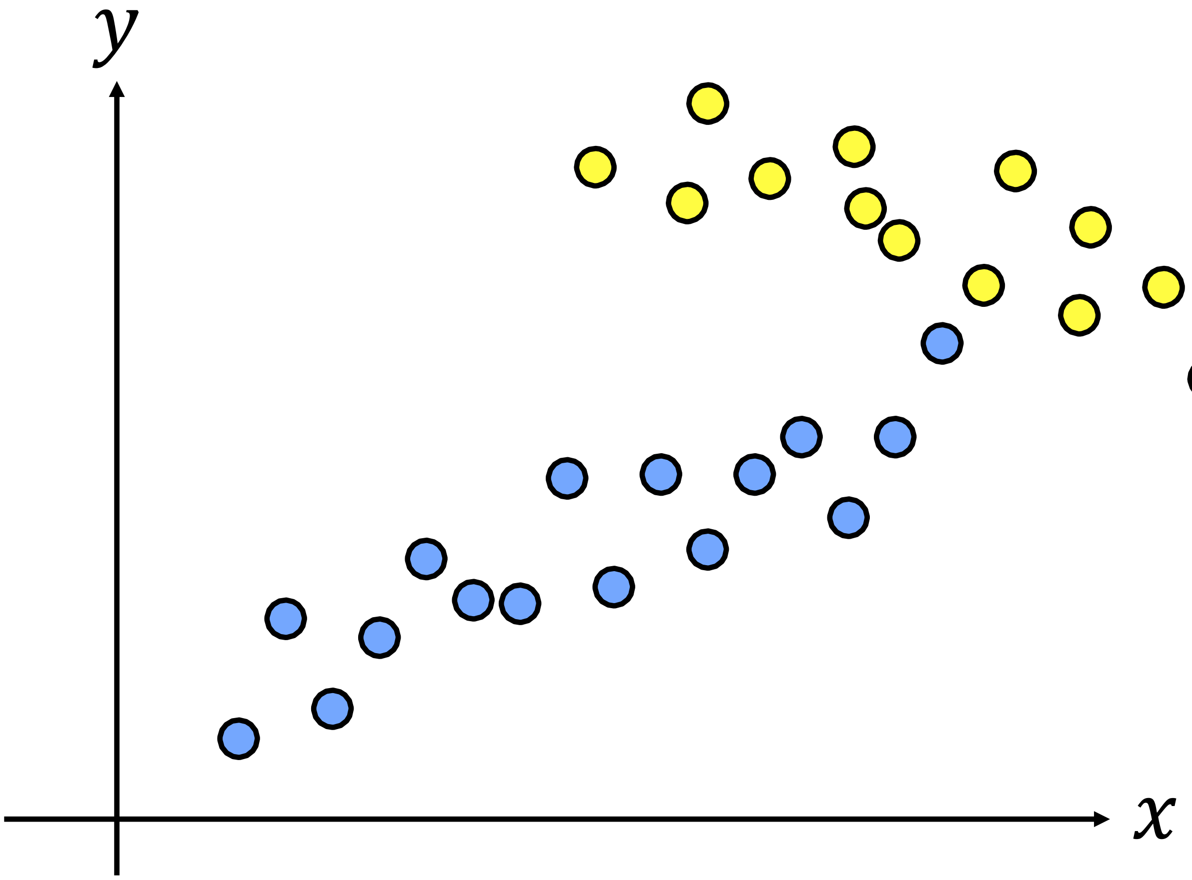
特征捆绑



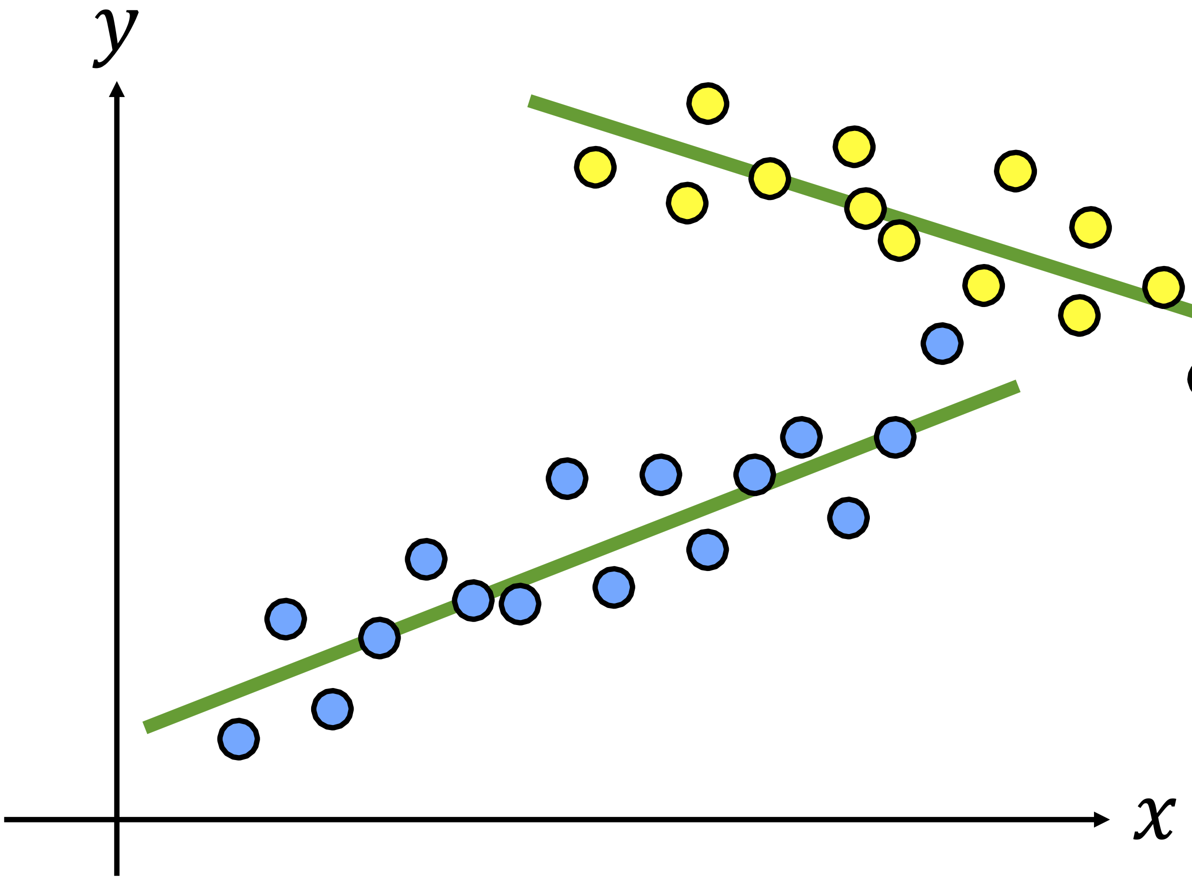
特征捆绑



特征捆绑



特征捆绑



最小二乘法

拟合

卡尔·弗里德里希·高斯

卡尔·弗里德里希·高斯

阿德里安-马里·勒让德

卡尔·弗里德里希·高斯

~~阿德里安-马里·勒让德~~

路易斯·勒让德

卡尔·弗里德里希·高斯

阿德里安-马里·勒让德

Least-squares estimation: from Gauss to Kalman

The Gaussian concept of estimation by least squares, originally stimulated by astronomical studies, has provided the basis for a number of estimation theories and techniques during the ensuing 170 years—probably none as useful in terms of today's requirements as the Kalman filter

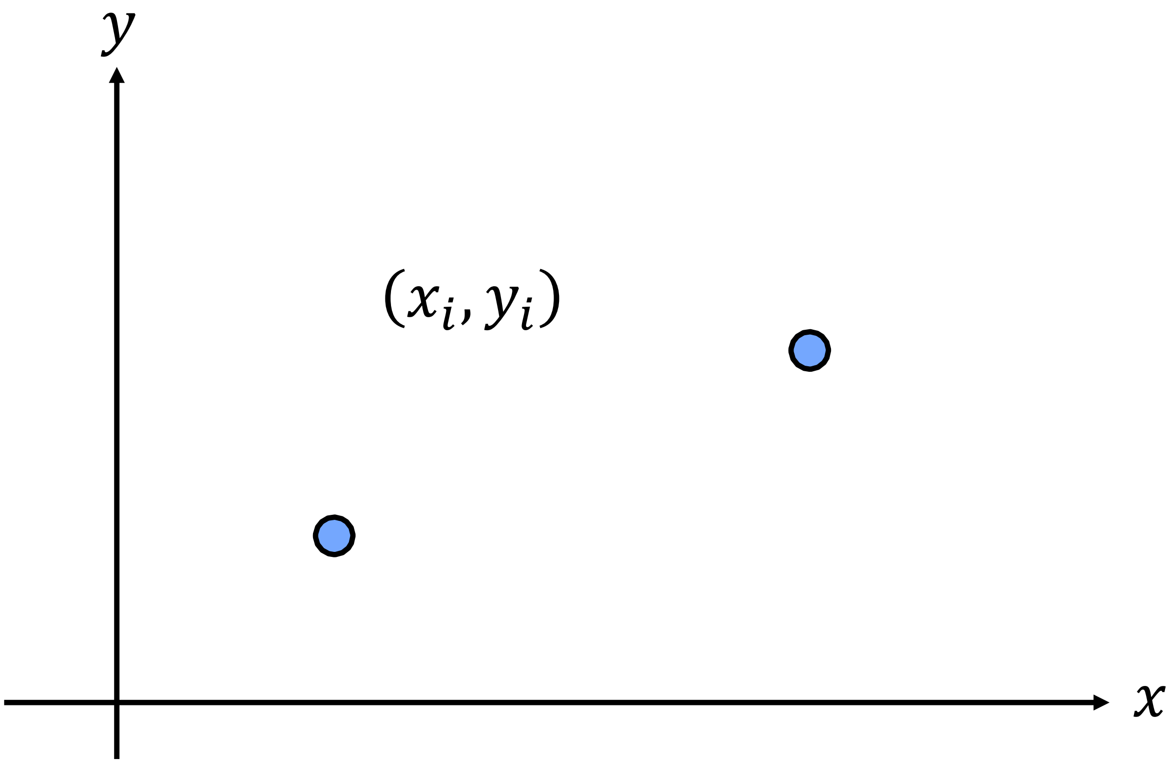
H. W. Sorenson University of California, San Diego

This discussion is directed to least-squares estimation theory, from its inception by Gauss[1] to its modern form, as developed by Kalman.[2] To aid in furnishing the desired perspective, the contributions and insights provided by Gauss are described and related to developments that have appeared more recently (that is, in the 20th century). In the author's opinion, it is enlightening to consider just how far (or how little) we have advanced since the initial developments and to recognize the truth in the saying that we "stand on the

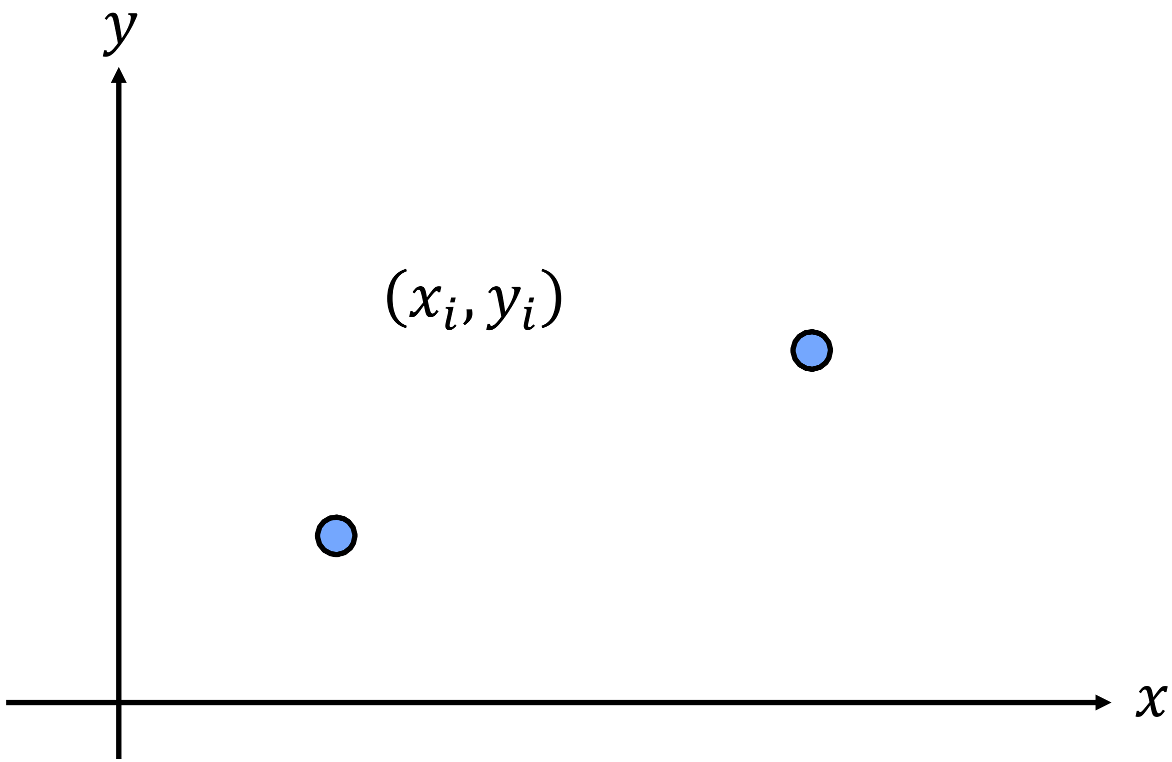
have made use of since the year 1795, has lately been published by Legendre in the work *Nouvelles méthodes pour la determination des orbites des cometes, Paris, 1806*, where several other properties of this principle have been explained which, for the sake of brevity, we here omit." This reference angered Legendre who, with great indignation, wrote to Gauss and complained[3] that "Gauss, who was already so rich in discoveries, might have had the decency not to appropriate the method of least-squares." It is interesting to note that Gauss, who is now regarded

IEEE Spectrum, 1970

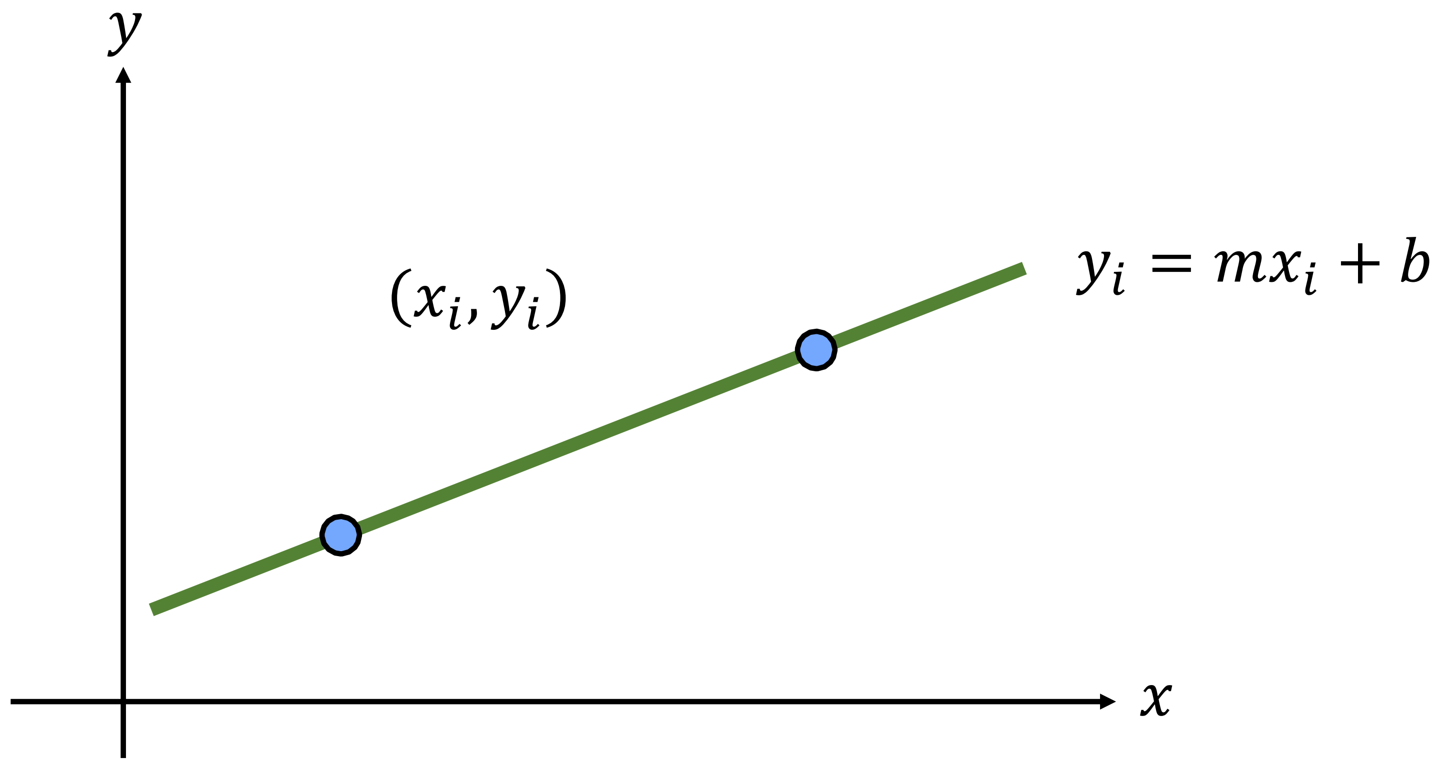

y
x



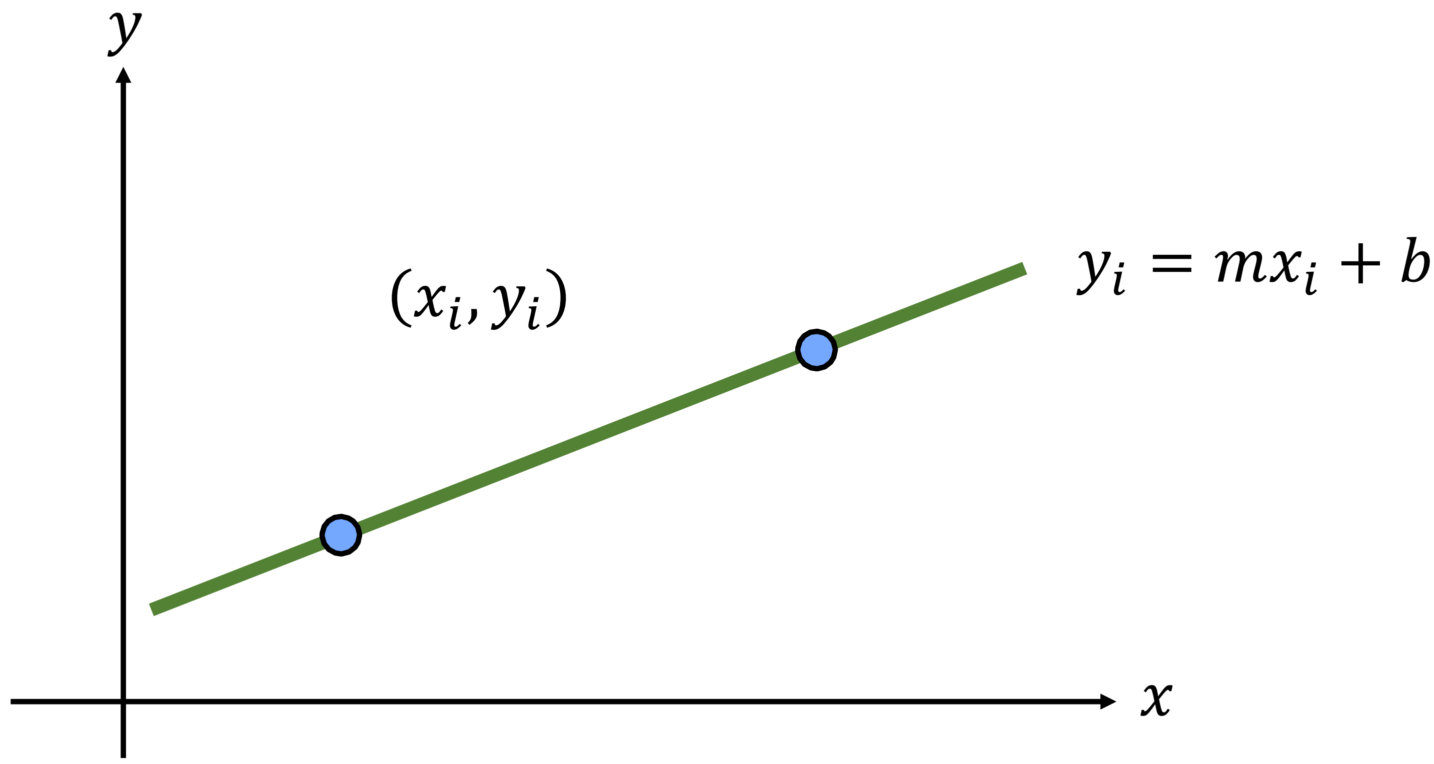
y
(x_i, y_i)
x



如何找到这条线?

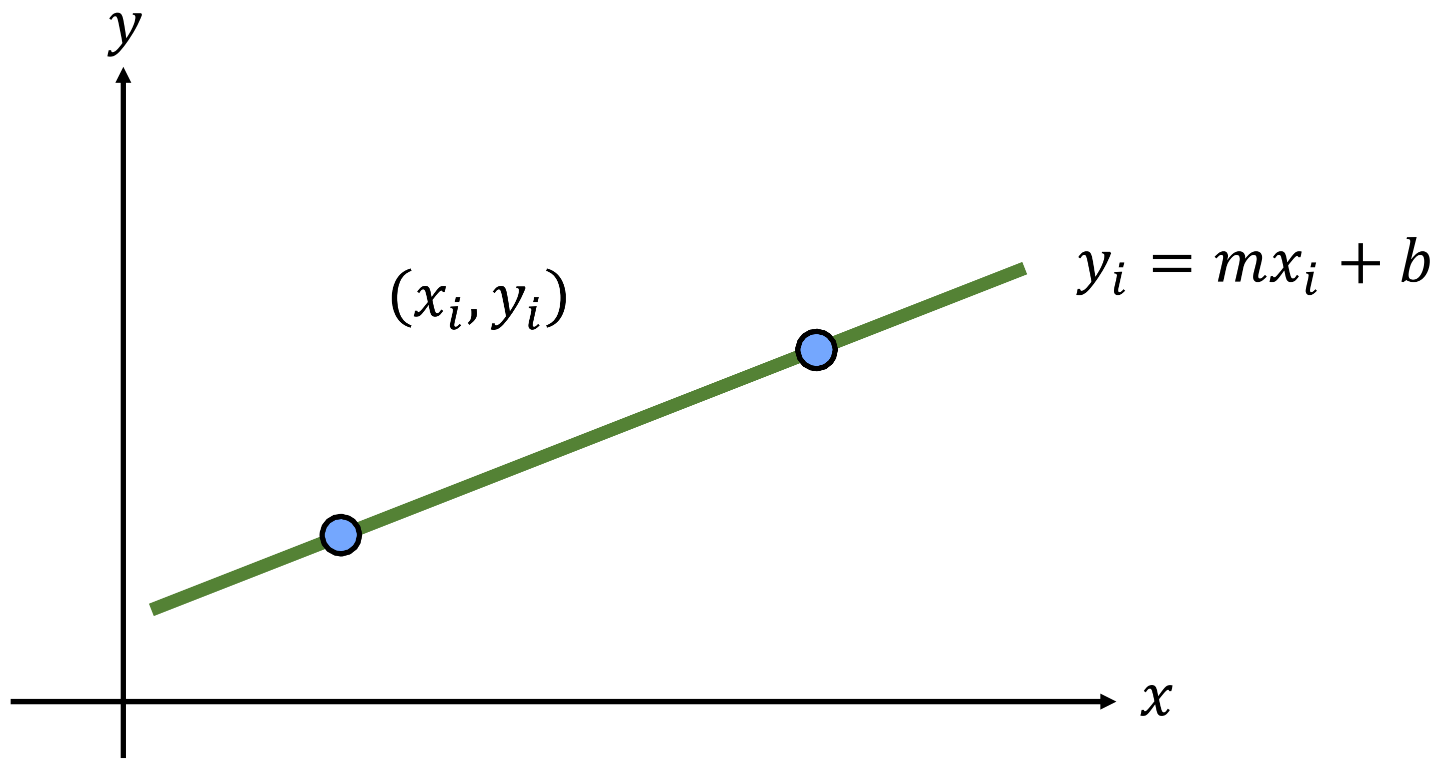


如何找到这条线?



如何找到这条线?

$$y_0 = mx_0 + b$$
$$y_1 = mx_1 + b$$



如何找到这条线?

$$\begin{pmatrix} x_0 & 1 \\ x_1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

如何找到这条线?

$$\begin{pmatrix} x_0 & 1 \\ x_1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

如何找到这条线?

$$\begin{pmatrix} x_0 & 1 \\ x_1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

$\mathbf{A}$

如何找到这条线?

$$\begin{pmatrix} x_0 & 1 \\ x_1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

$\mathbf{A}$ $\mathbf{p}$

如何找到这条线?

$$\begin{pmatrix} x_0 & 1 \\ x_1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

$\mathbf{A}$ $\mathbf{p}$ $\mathbf{b}$

如何找到这条线?

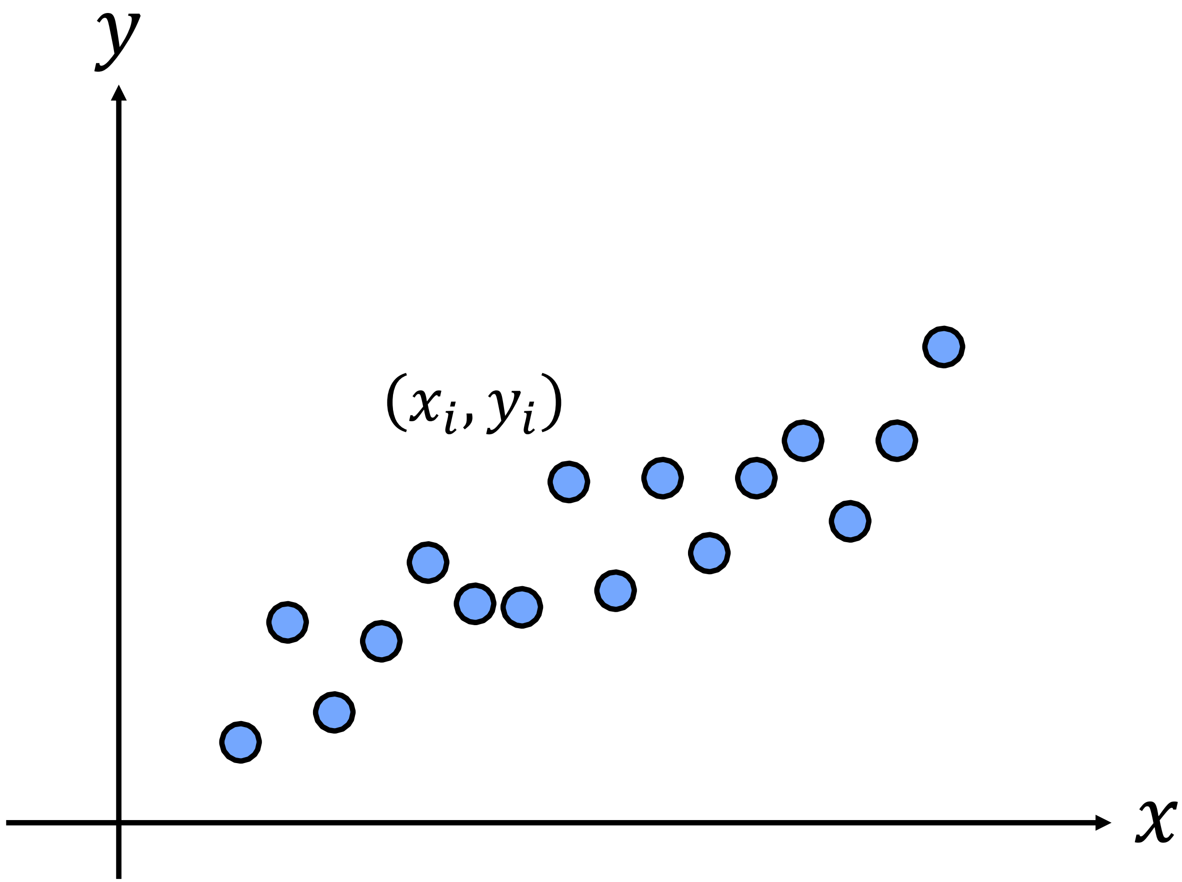
$$\begin{pmatrix} x_0 & 1 \\ x_1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$$

$\mathbf{A}$ $\mathbf{p}$ $\mathbf{b}$

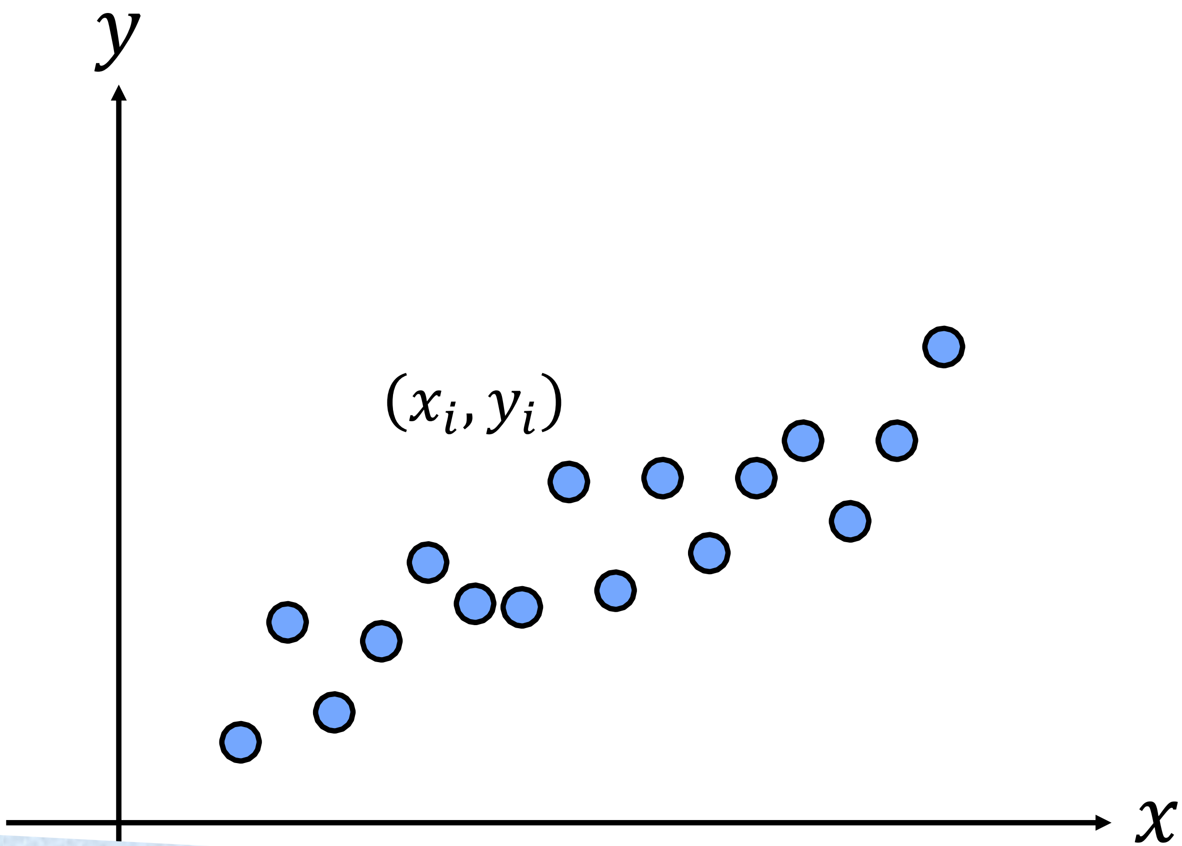
如何求解直线参数?

$$\mathbf{p} = \mathbf{A}^{-1}\mathbf{b}$$

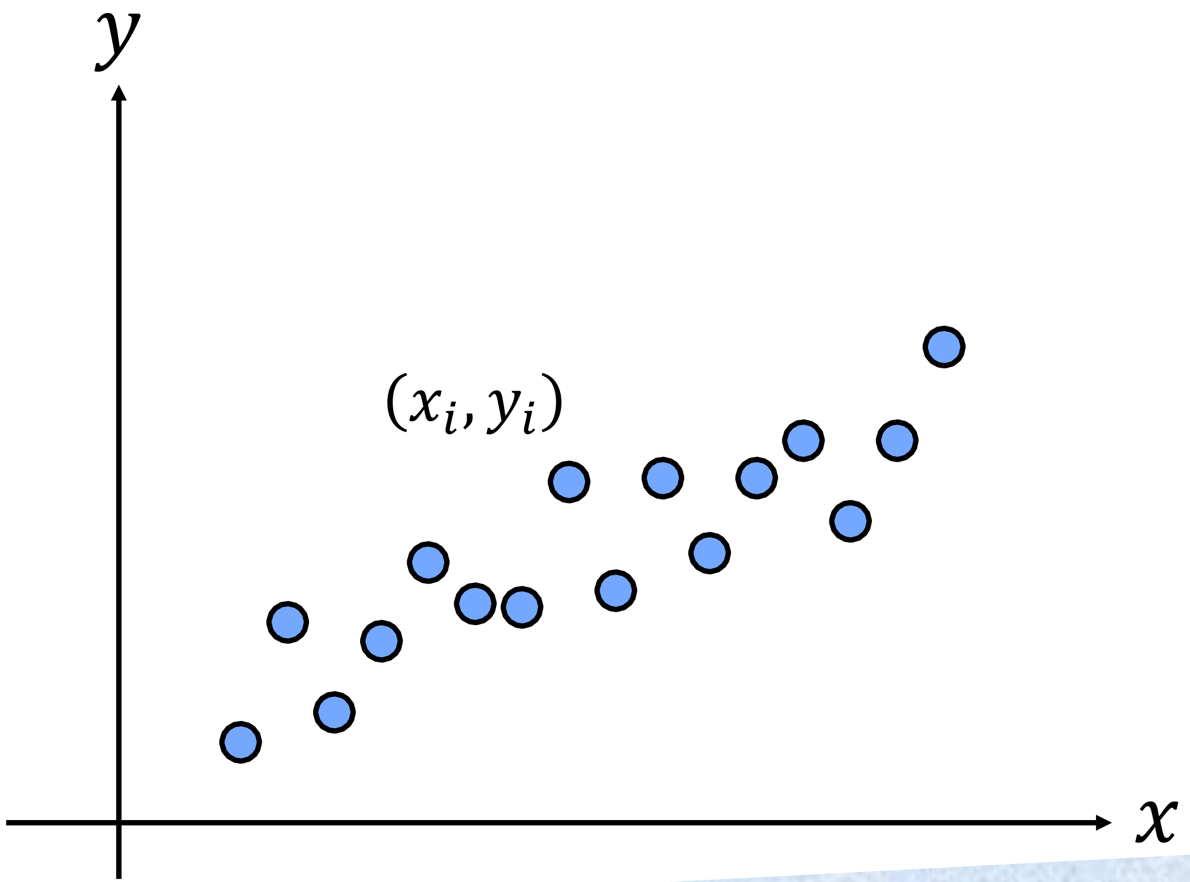

y
x



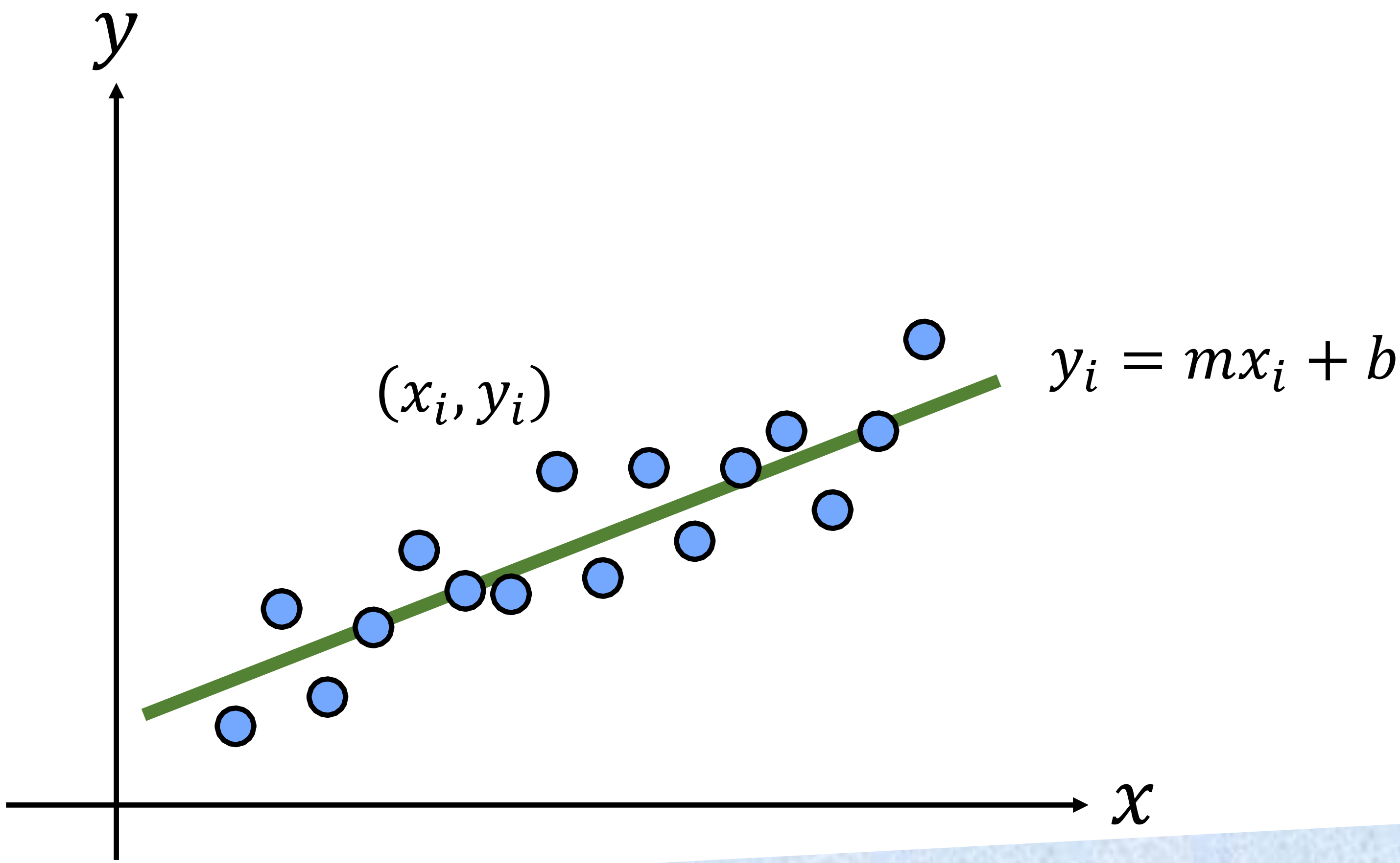
y
(x_i, y_i)
x



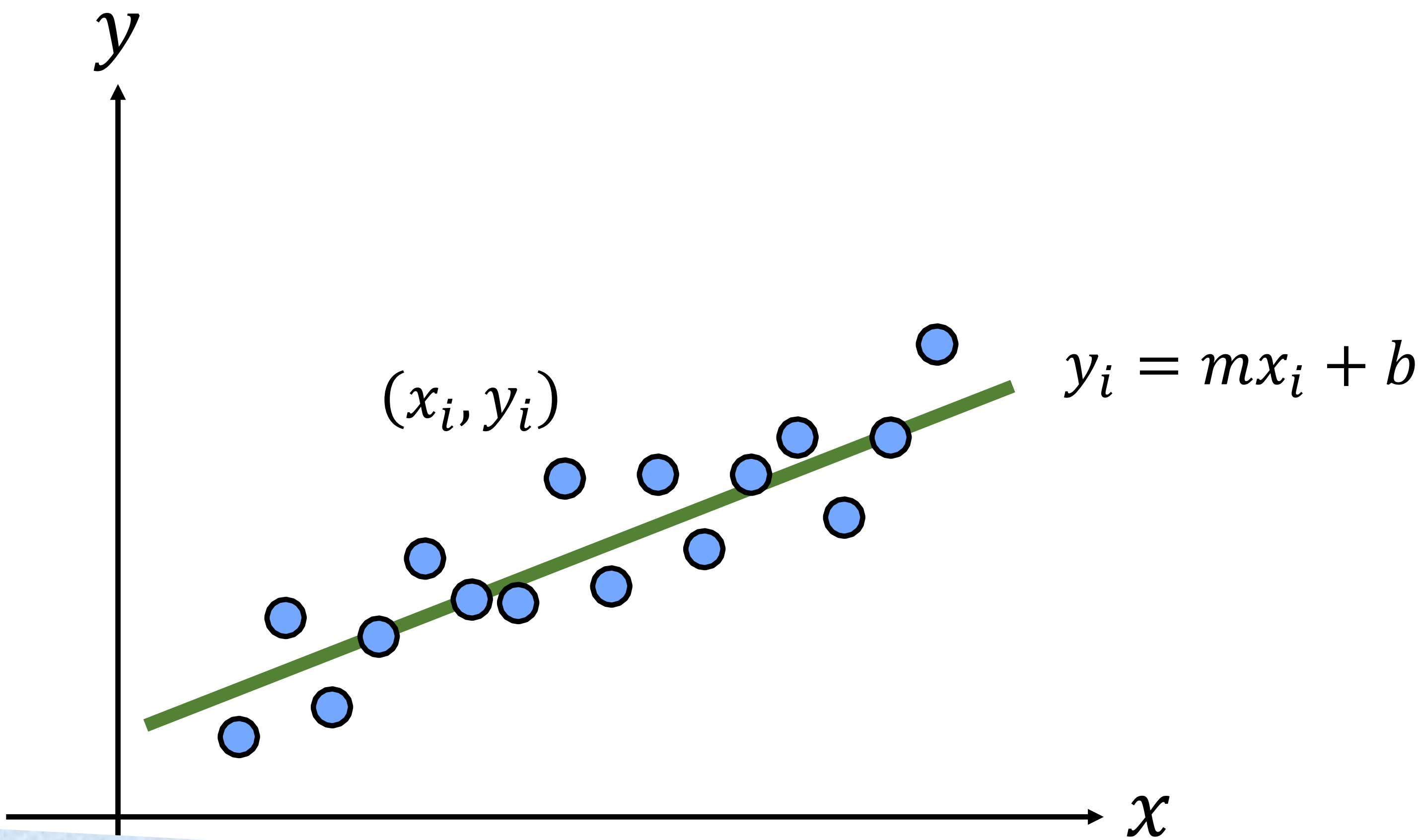
如何拟合一条直线?



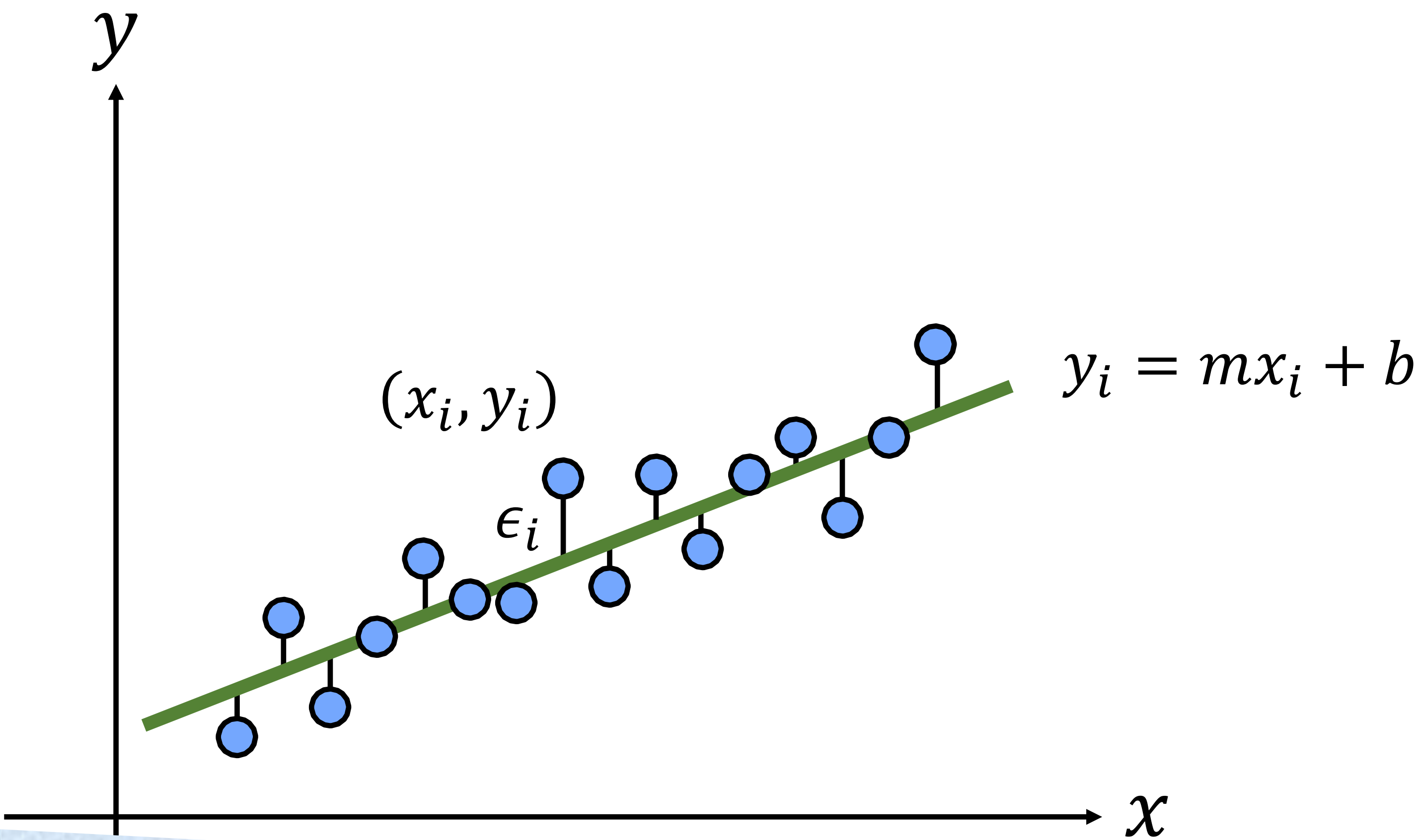
找到“最佳”拟合线，
使其到数据点的“距离”最小化



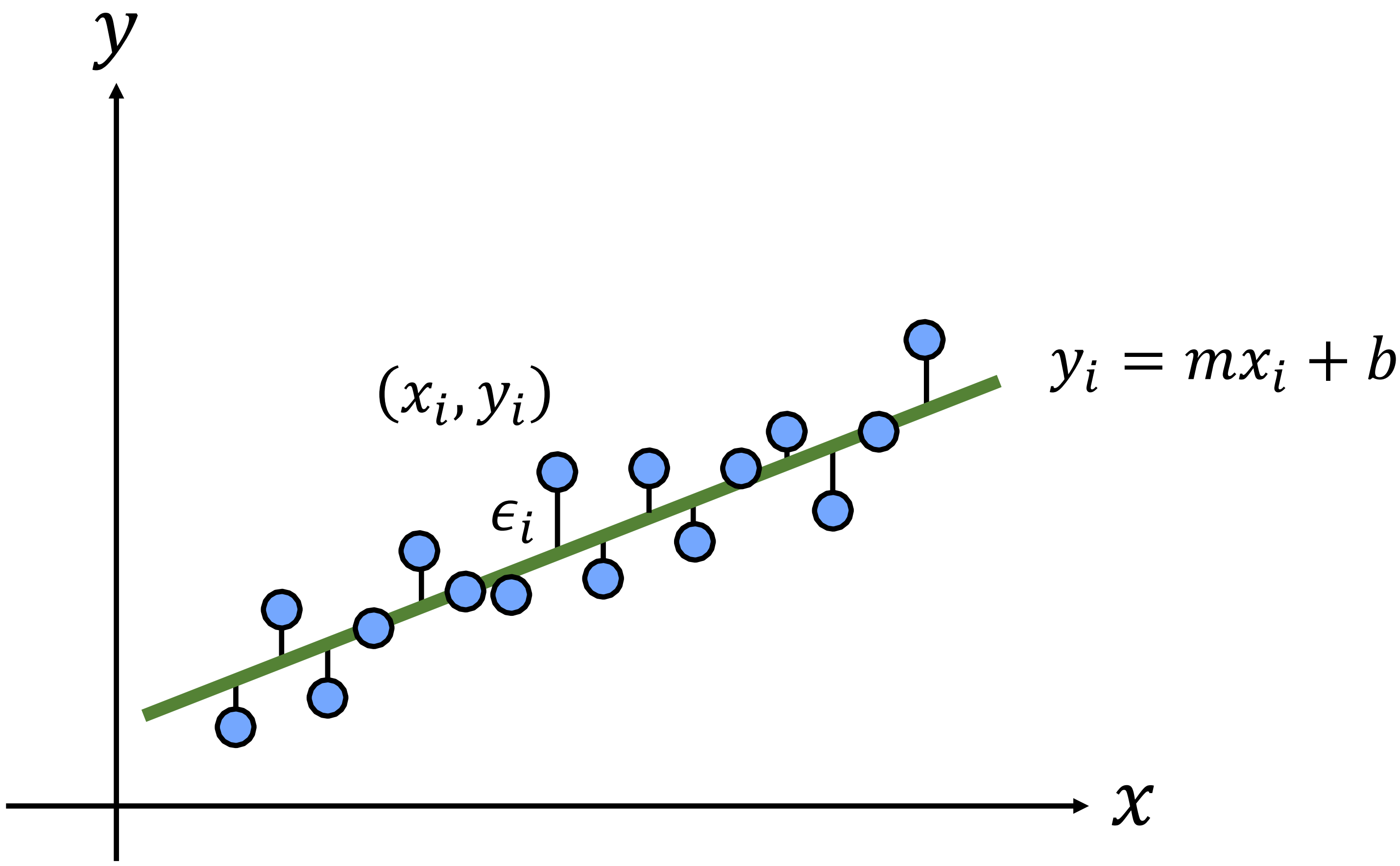
找到“最佳”拟合线，
使其到数据点的“距离”最小化



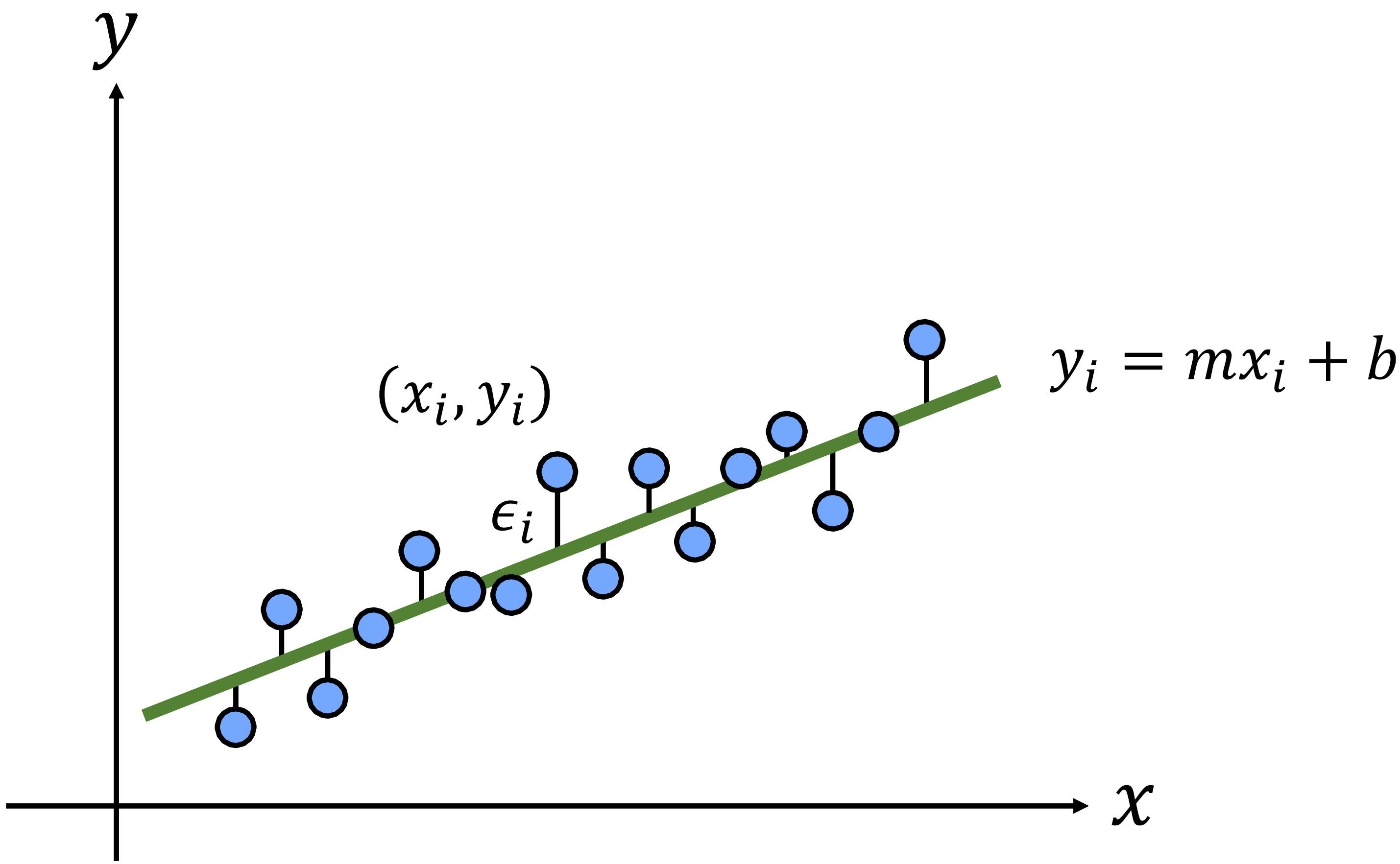
如何测量到这条线的距离?



如何测量到这条线的距离？



$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$



$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

未知的

$$\epsilon = \sum_{i=1}^{N} (y_i - m x_i - b)^2$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - \begin{pmatrix} x_i & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right)^2$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - (x_i \quad 1) \binom{m}{b} \right)^2$$

$$= \left\| \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} - \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix} \binom{m}{b} \right\|^2$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - \begin{pmatrix} x_i & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right)^2$$

$$= \left\| \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} - \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right\|^2$$

$$\mathbf{b}$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - \begin{pmatrix} x_i & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right)^2$$

$$= \left\| \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{\mathbf{b}} - \underbrace{\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}}_{\mathbf{A}} \begin{pmatrix} m \\ b \end{pmatrix} \right\|^2$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - \begin{pmatrix} x_i & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right)^2$$

$$= \left\| \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{\mathbf{b}} - \underbrace{\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} m \\ b \end{pmatrix}}_{\mathbf{x}} \right\|^2$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - \begin{pmatrix} x_i & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right)^2$$

$$= \left\| \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{\mathbf{b}} - \underbrace{\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} m \\ b \end{pmatrix}}_{\mathbf{x}} \right\|^2$$

$$= \|\mathbf{b} - \mathbf{Ax}\|^2$$

$$\epsilon = \sum_{i=1}^{N} (y_i - mx_i - b)^2$$

$$= \sum_{i=1}^{N} \left(y_i - (x_i \quad 1) \begin{pmatrix} m \\ b \end{pmatrix} \right)^2$$

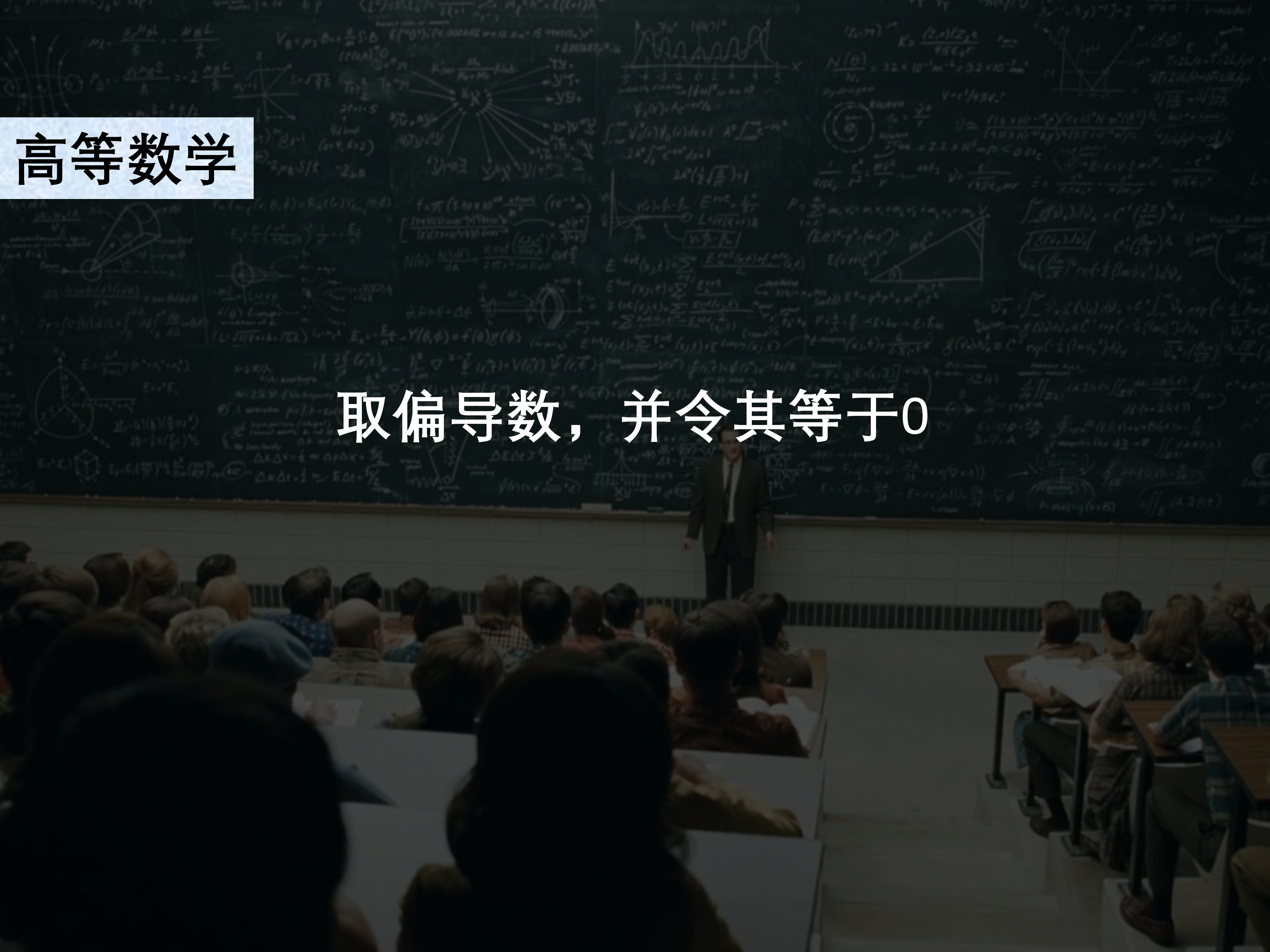
$$= \left\| \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} - \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} \right\|^2$$

$$\mathbf{b} \qquad \qquad \mathbf{A} \qquad \qquad \mathbf{x}$$

$$= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2$$

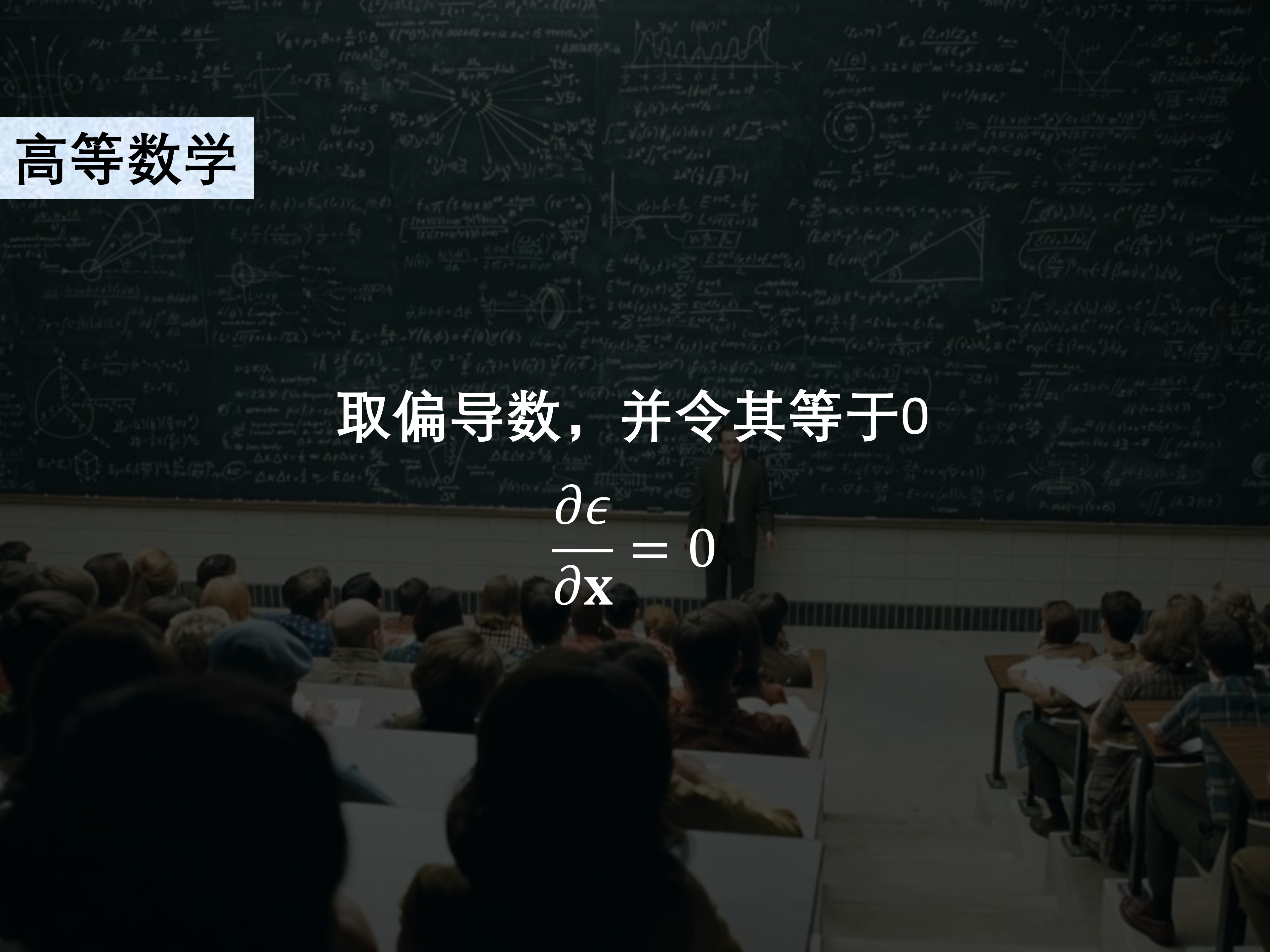
如何最小化ϵ?

高等数学



高等数学

取偏导数，并令其等于0

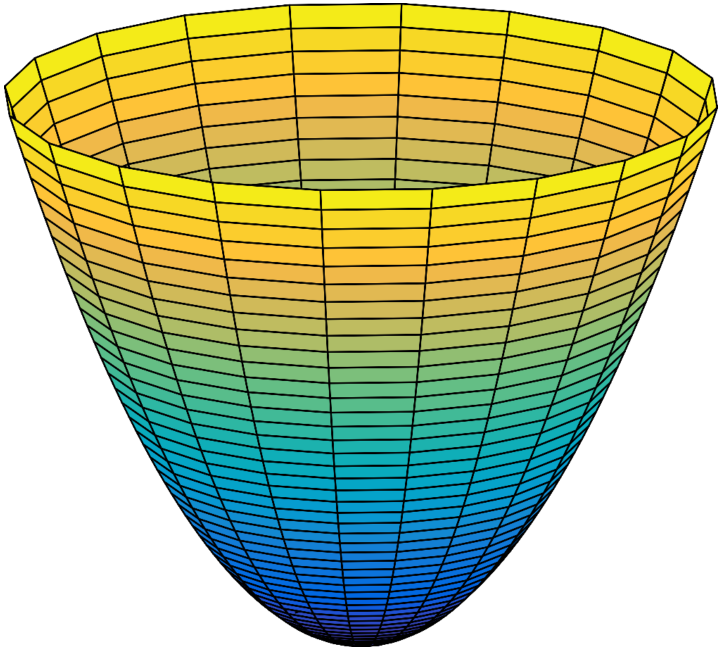


高等数学

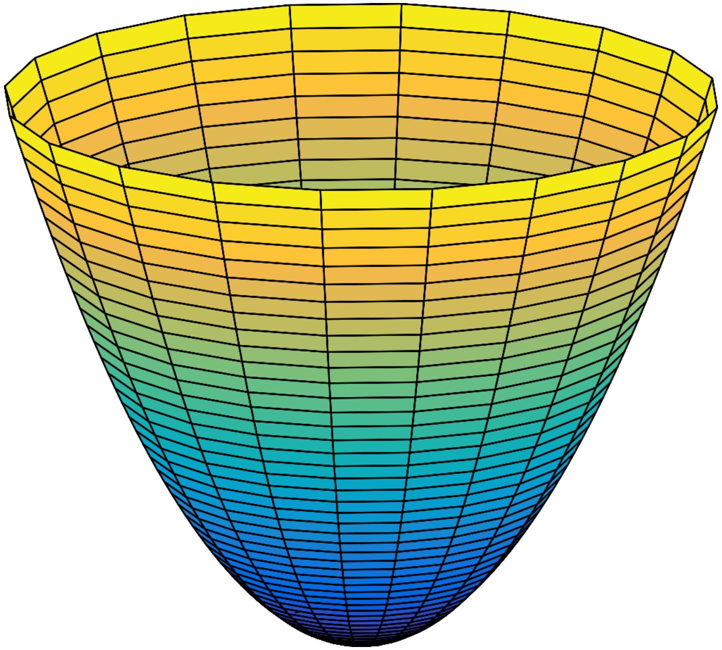
取偏导数，并令其等于0

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

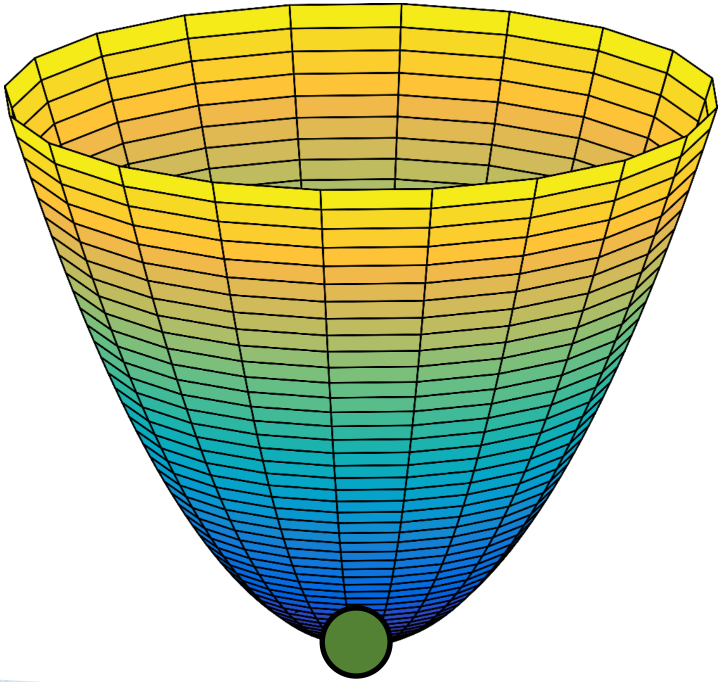


$$\text{解}\quad \frac{\partial \epsilon}{\partial \mathbf{x}} = 0$$



梯度在极值处等于0

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$



梯度在极值处等于0

解 $$\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$$

解 $$\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$$

$$\epsilon = \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned} \epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\ &= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \end{aligned}$$

解 $\dfrac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$
\begin{aligned}
\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\
&= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x})
\end{aligned}
$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned}
\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\
&= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}
\end{aligned}$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned}
\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\
&= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}
\end{aligned}$$

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{b}\right) - \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right) - \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b}\right) + \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$
\begin{aligned}
\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\
&= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}
\end{aligned}
$$

$$
\frac{\partial \epsilon}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{b}\right) - \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right) - \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b}\right) + \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)
$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned}\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\ &= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\ &= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x}) \\ &= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\end{aligned}$$

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{b}\right) - \underbrace{\frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)}_{\mathbf{A}^{\mathrm{T}}\mathbf{b}} - \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b}\right) + \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\epsilon = \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2$$
$$= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x})$$
$$= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x})$$
$$= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}$$

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{b}\right) - \underbrace{\frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)}_{\mathbf{A}^{\mathrm{T}}\mathbf{b}} - \underbrace{\frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b}\right)}_{\mathbf{A}^{\mathrm{T}}\mathbf{b}} + \frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)$$

解 $\dfrac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\epsilon = \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2$$

$$= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x})$$

$$= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x})$$

$$= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}$$

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = \cancel{\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{b}\right)} - \underbrace{\frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)}_{\mathbf{A}^{\mathrm{T}}\mathbf{b}} - \underbrace{\frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b}\right)}_{\mathbf{A}^{\mathrm{T}}\mathbf{b}} + \underbrace{\frac{\partial \epsilon}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\right)}_{2\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}}$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned}
\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\
&= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x}) \\
&= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}
\end{aligned}$$

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = -2\mathbf{A}^{\mathrm{T}}\mathbf{b} + 2\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned}\epsilon &= \|\mathbf{b} - \mathbf{Ax}\|^2 \\ &= (\mathbf{b} - \mathbf{Ax})^{\mathrm{T}}(\mathbf{b} - \mathbf{Ax}) \\ &= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{Ax}) \\ &= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{Ax} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{Ax}\end{aligned}$$

$$\frac{\partial \epsilon}{\partial \mathbf{x}} = -2\mathbf{A}^{\mathrm{T}}\mathbf{b} + 2\mathbf{A}^{\mathrm{T}}\mathbf{Ax} = \mathbf{0}$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\begin{aligned}\epsilon &= \|\mathbf{b} - \mathbf{A}\mathbf{x}\|^2 \\ &= (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathrm{T}}(\mathbf{b} - \mathbf{A}\mathbf{x}) \\ &= \left(\mathbf{b}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\right)(\mathbf{b} - \mathbf{A}\mathbf{x}) \\ &= \mathbf{b}^{\mathrm{T}}\mathbf{b} - \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{x} - \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}\end{aligned}$$

$$-2\mathbf{A}^{\mathrm{T}}\mathbf{b} + 2\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{0}$$

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

解 $\dfrac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$ 正规方程

解 $$\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

正规方程

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$ 正规方程

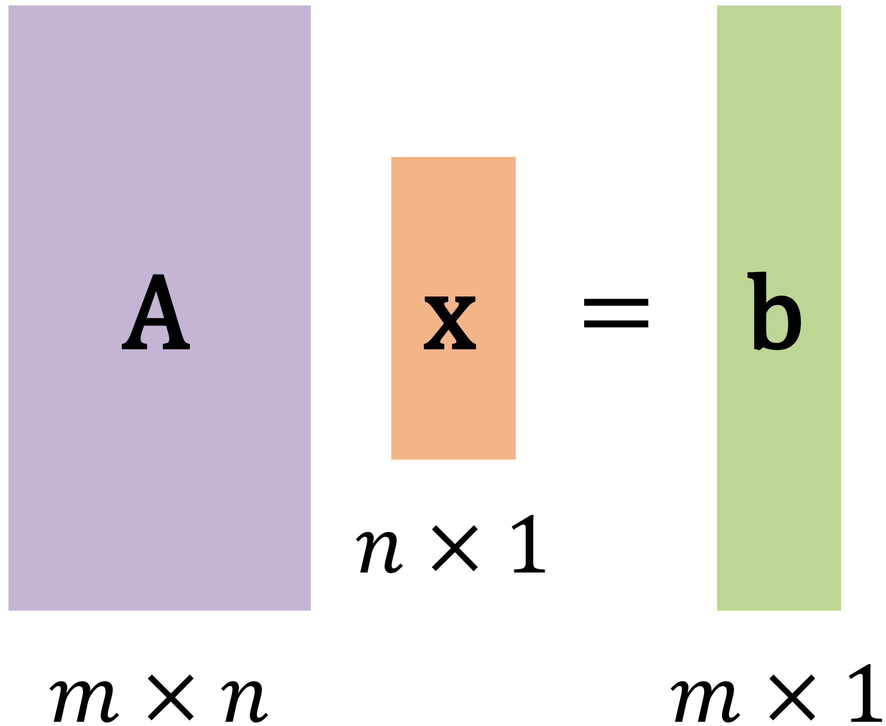
最小二乘解 $\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$

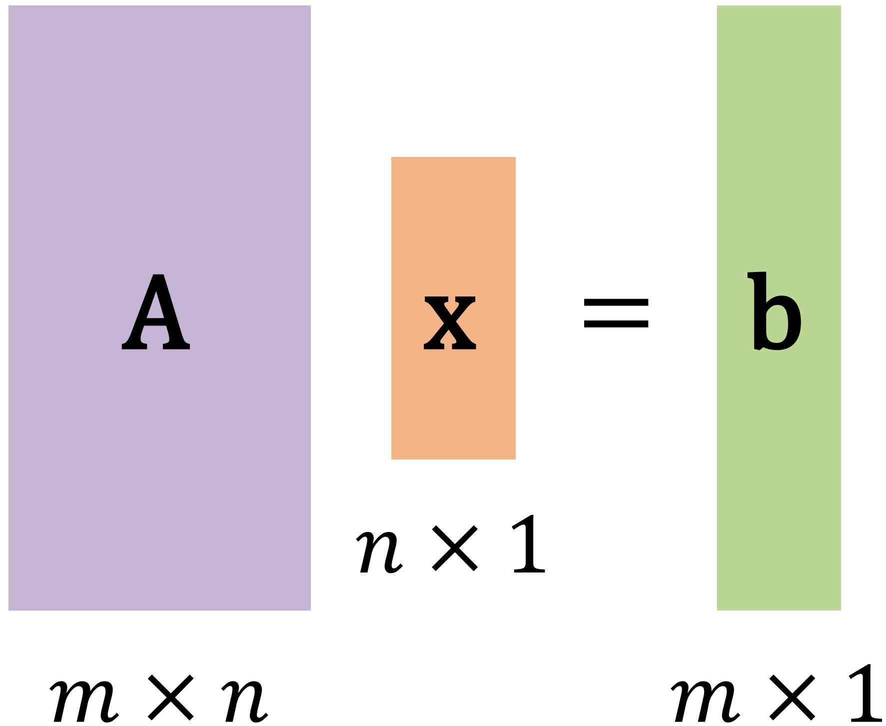
解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$ 正规方程

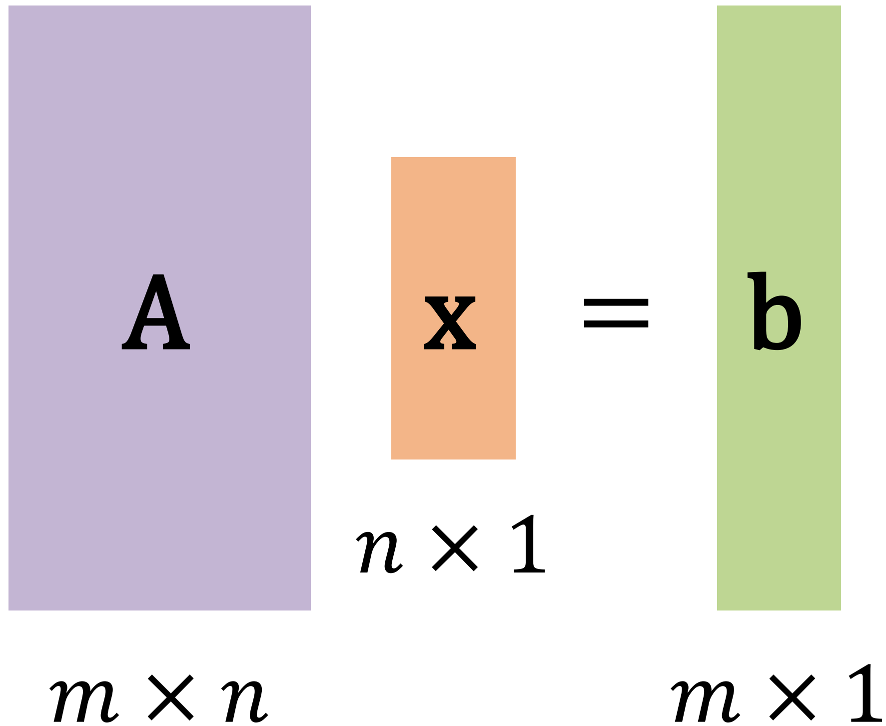
最小二乘解 $\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$

A的伪逆

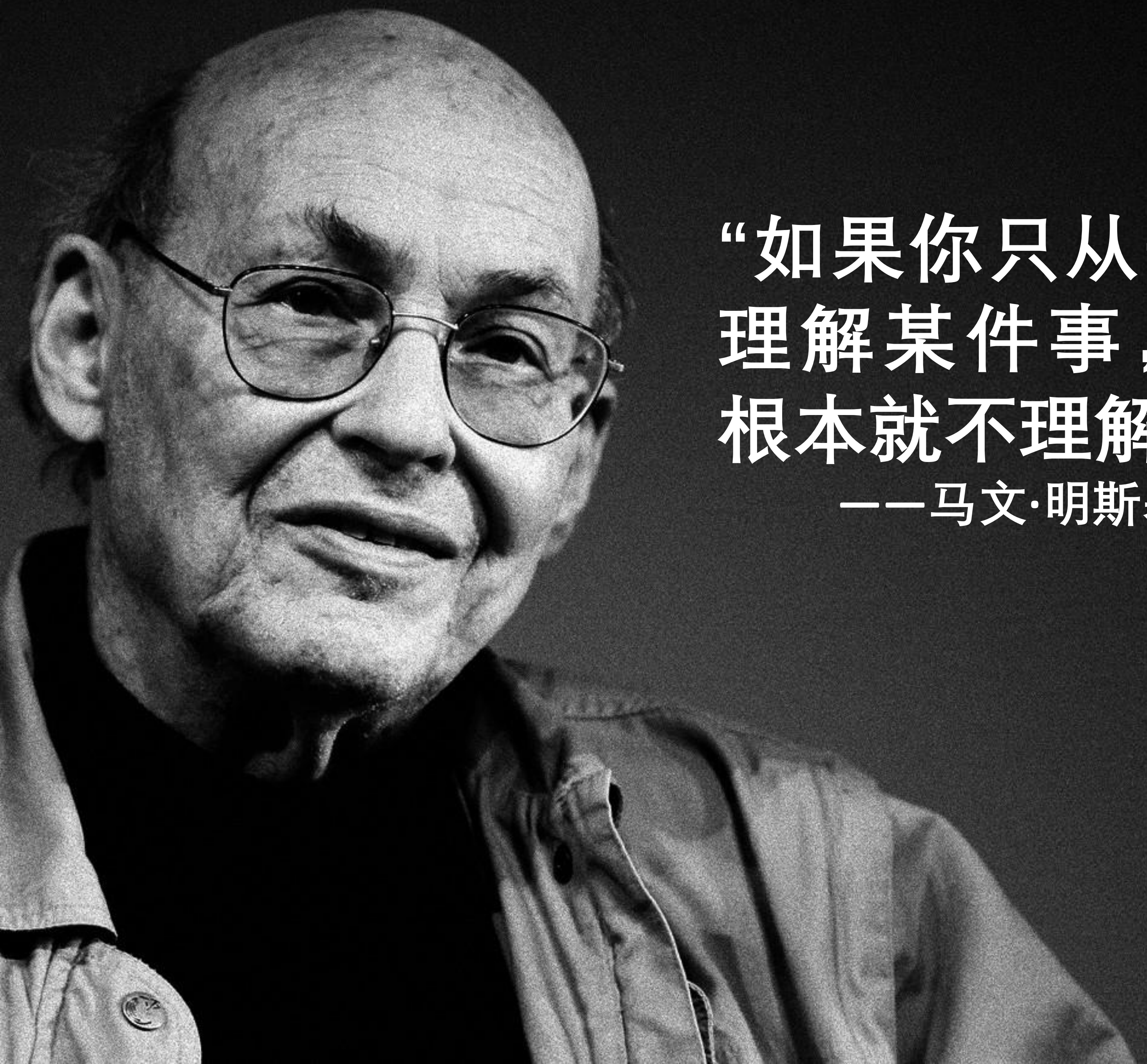

A
x
=
b
n × 1
m × n
m × 1



$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2$$

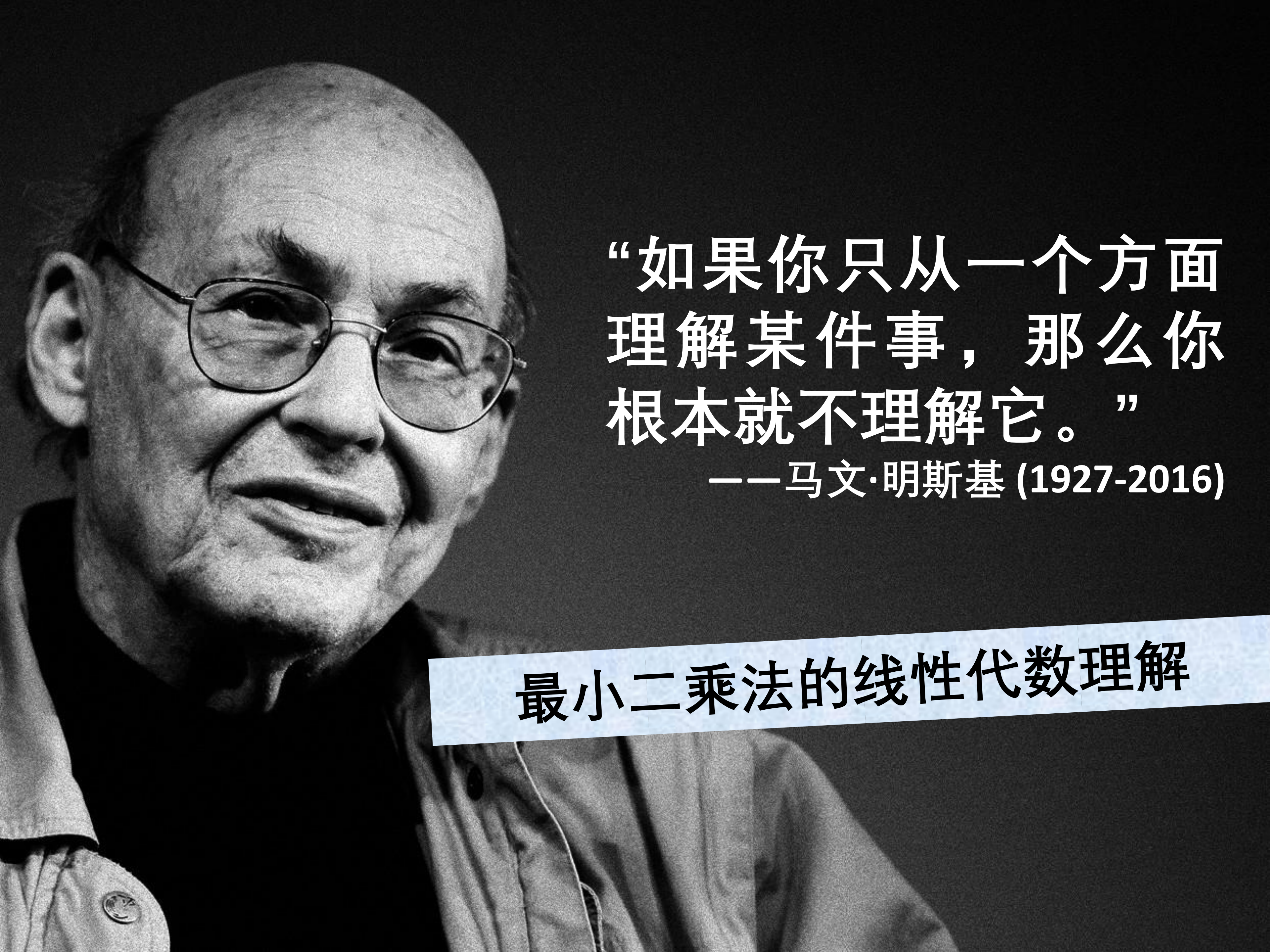


$$\mathbf{x}^* = \arg\min_{\mathbf{x}} ||\mathbf{A}\mathbf{x} - \mathbf{b}||^2 = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$



“如果你只从一个方面理解某件事，那么你根本就不理解它。”

——马文·明斯基 (1927-2016)


“如果你只从一个方面
理解某件事，那么你
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最小二乘法的线性代数理解

目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

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$$\mathbf{A}\mathbf{x}=\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}$$

目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

$$\mathbf{A}\mathbf{x}=\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}$$

$$\underbrace{}_{\mathbf{a}_1}$$

目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

$$\mathbf{A}\mathbf{x} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix}$$

$$\underline{}\ \ \underline{}$$

$$\mathbf{a}_1 \quad \mathbf{a}_2$$

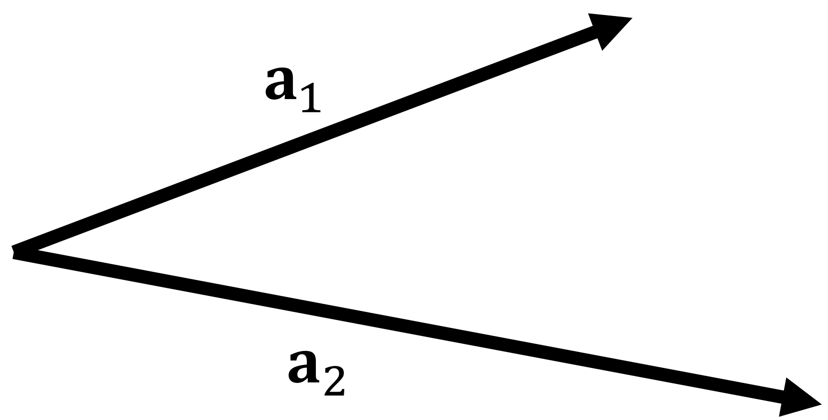
目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

$$\mathbf{A}\mathbf{x}=\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}=(\mathbf{a}_1 \quad \mathbf{a}_2)\begin{pmatrix} m \\ b \end{pmatrix}$$

$\mathbf{a}_1$ $\mathbf{a}_2$

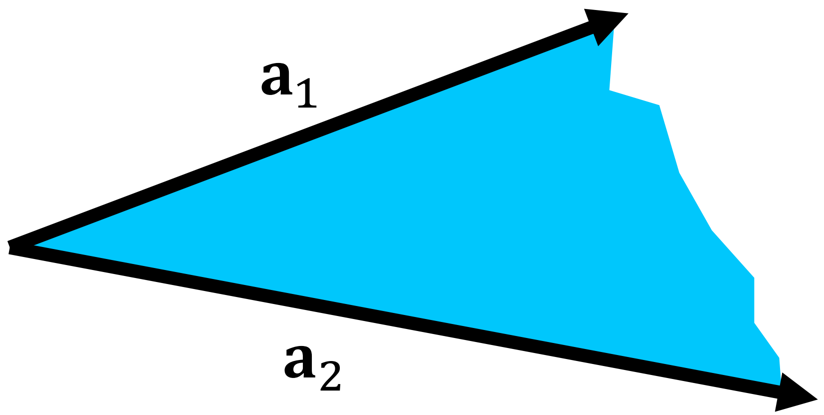
目标： 使$\|\mathbf{b}-\mathbf{Ax}\|$尽可能小

$$\mathbf{Ax}=\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}=(\mathbf{a}_1 \quad \mathbf{a}_2)\begin{pmatrix} m \\ b \end{pmatrix}$$



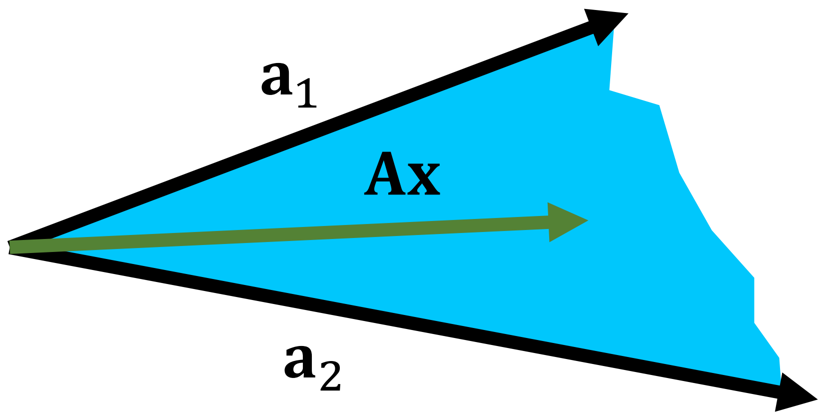
目标： 使$\|\mathbf{b}-\mathbf{Ax}\|$尽可能小

$$\mathbf{Ax}=\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}=\begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}$$

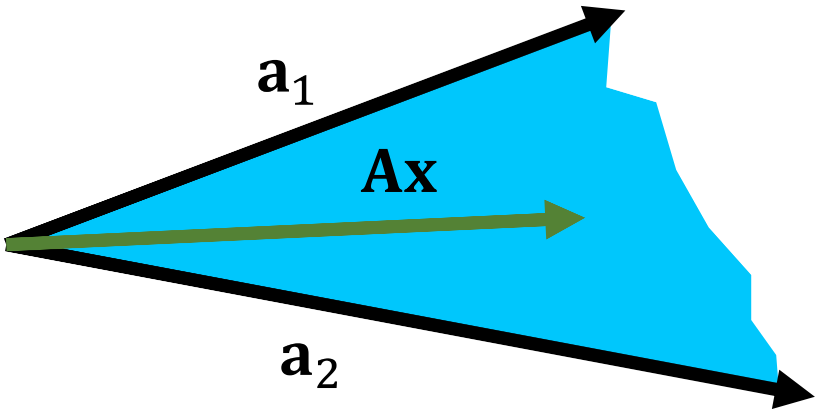


目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

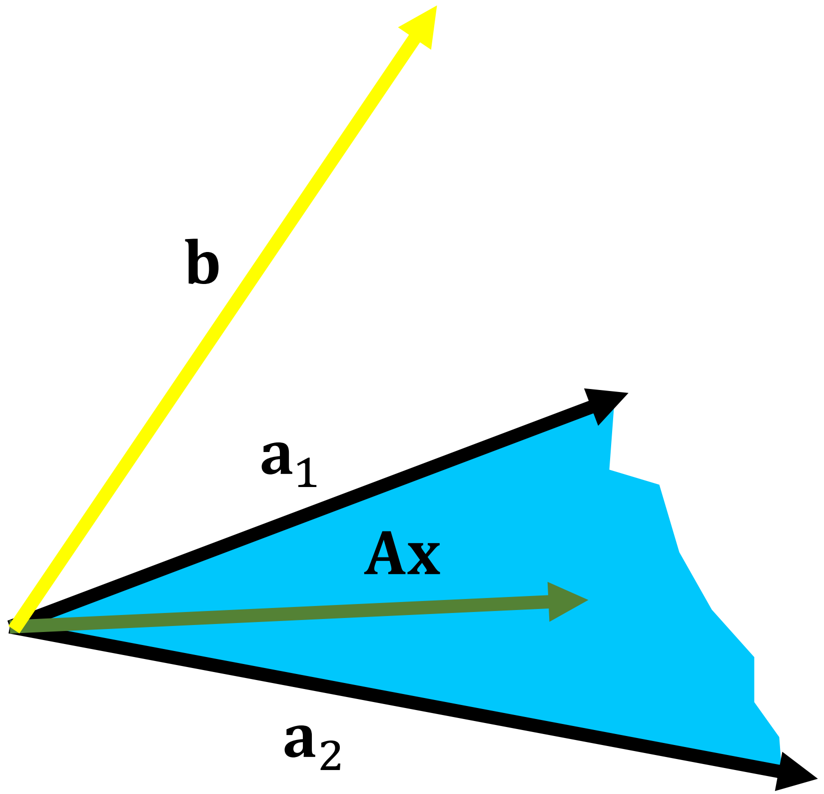
$$\mathbf{A}\mathbf{x}=\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}=\begin{pmatrix}\mathbf{a}_1 & \mathbf{a}_2\end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix}$$



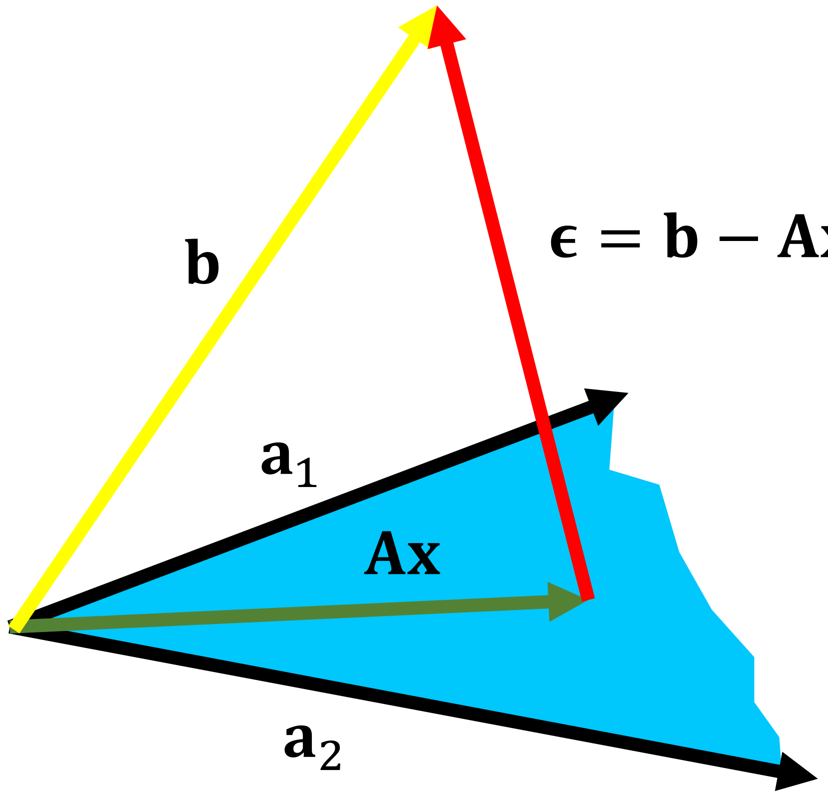
目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



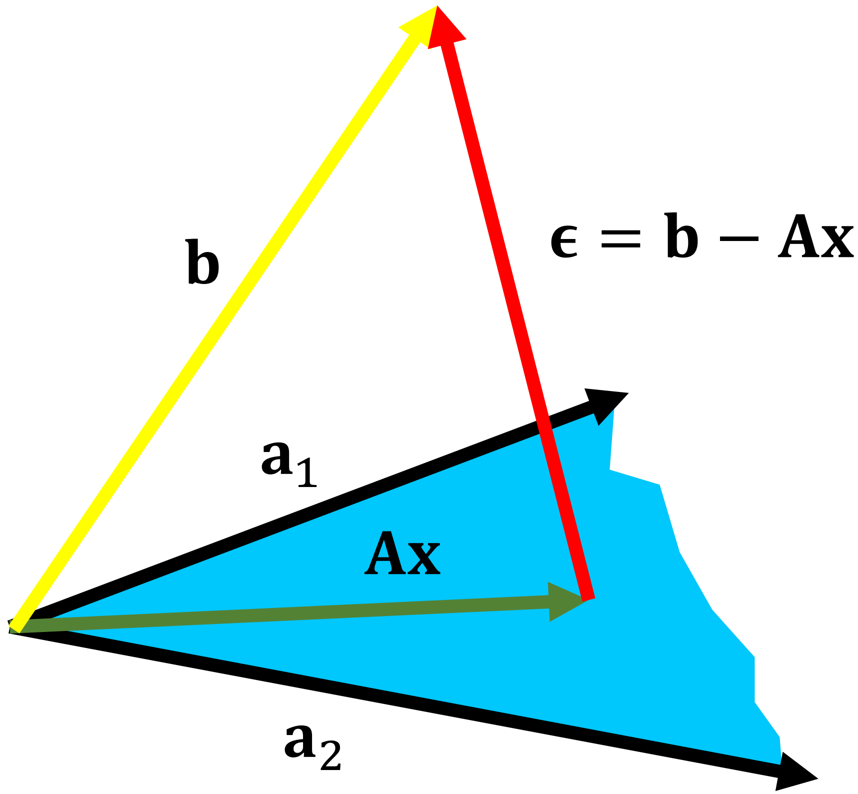
目标：使$\|\mathbf{b}-\mathbf{Ax}\|$尽可能小



目标：使$\|\mathbf{b}-\mathbf{Ax}\|$尽可能小

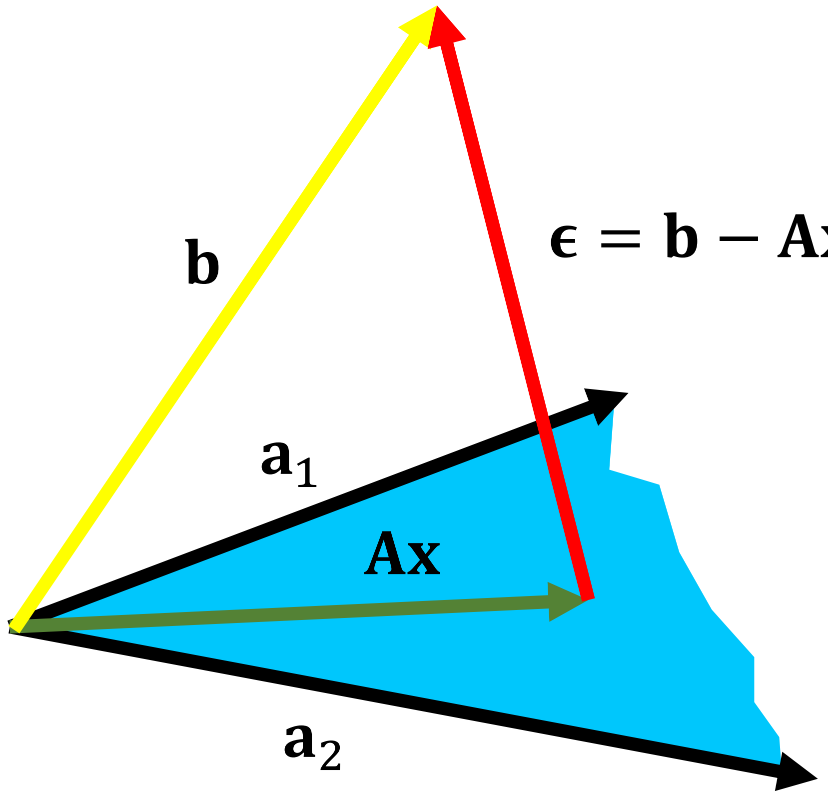


目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



怎样使ϵ的长度最小？

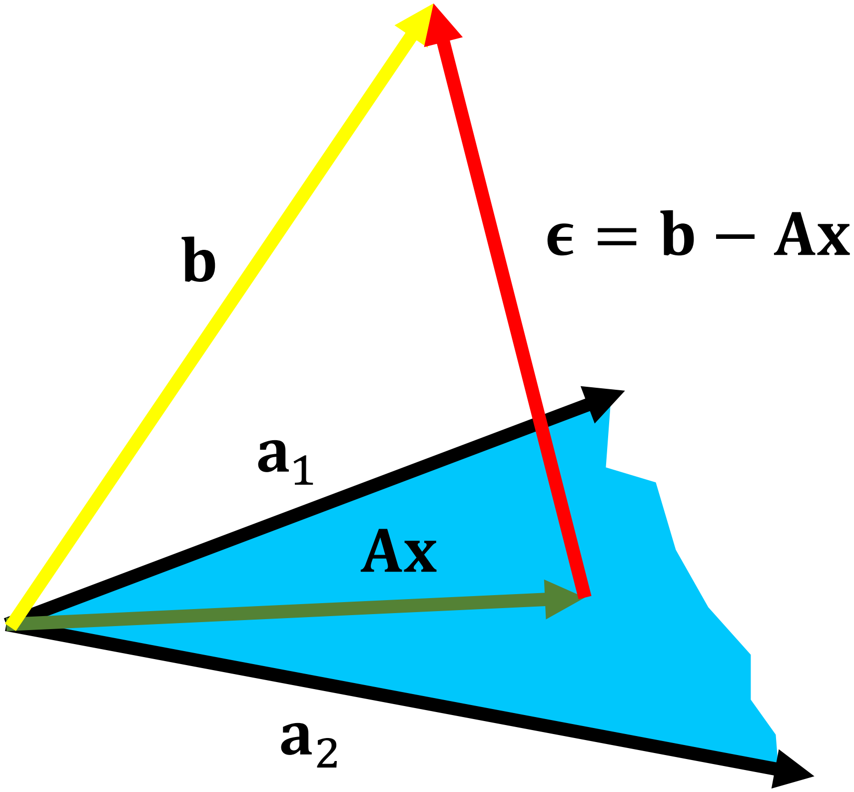
目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



怎样使ϵ的长度最小？

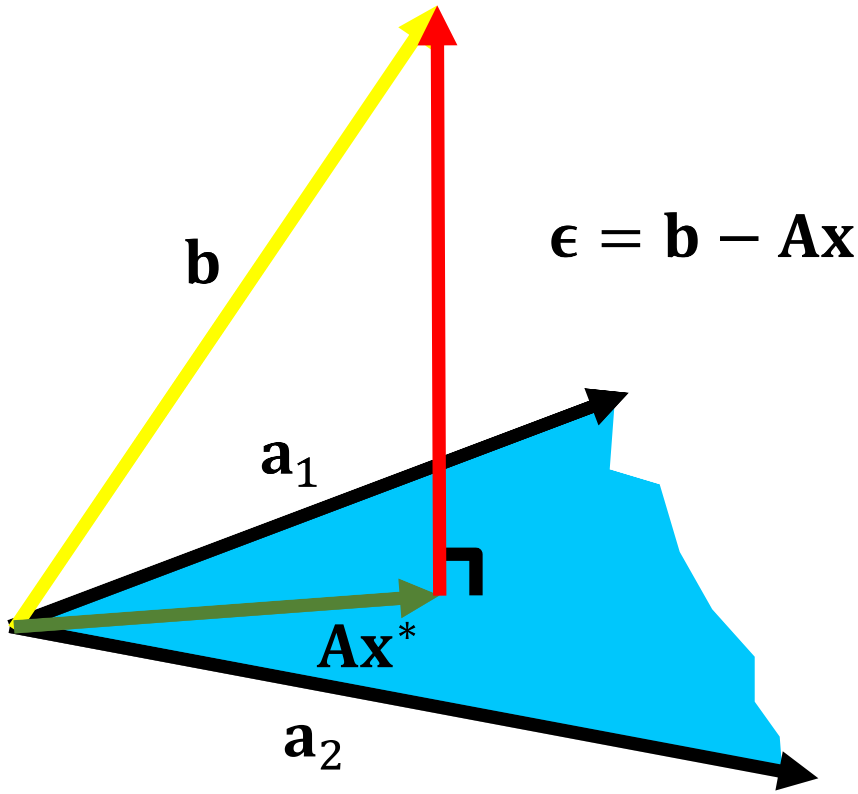
选择平面中与b最近的点

目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



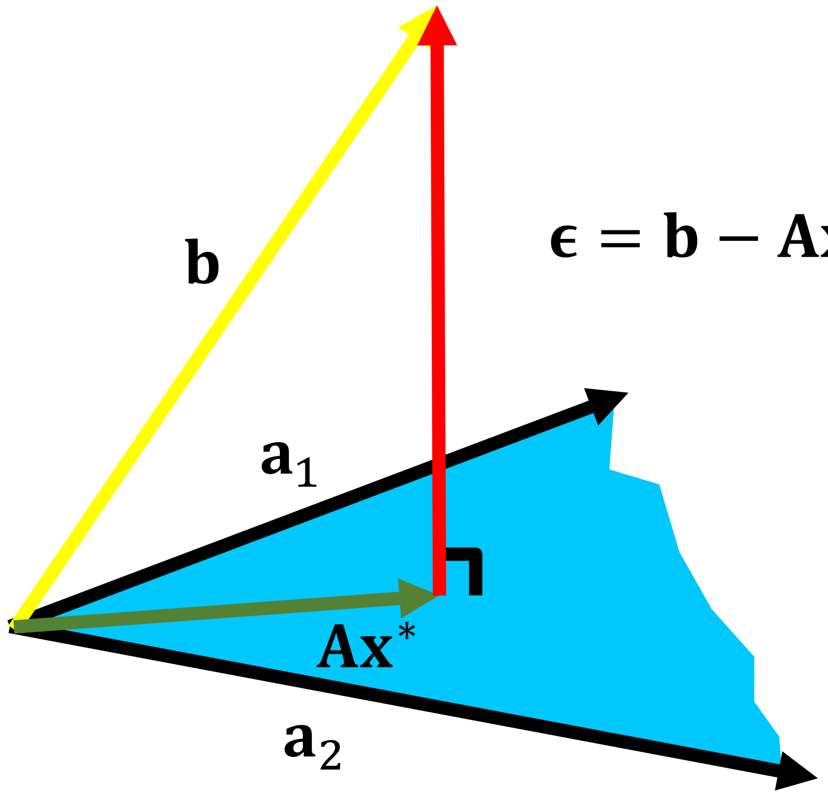
$\epsilon_{\min}$垂直于平面

目标：使$\|\mathbf{b}-\mathbf{Ax}\|$尽可能小



$\epsilon_{\min}$垂直于平面

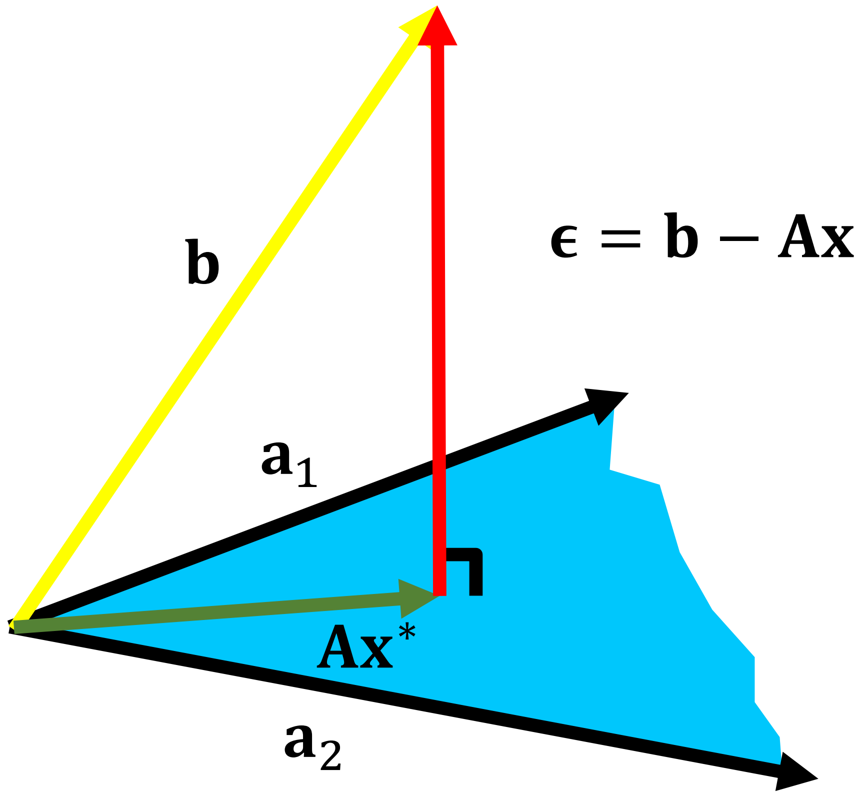
目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



$\epsilon_{\min}$垂直于平面

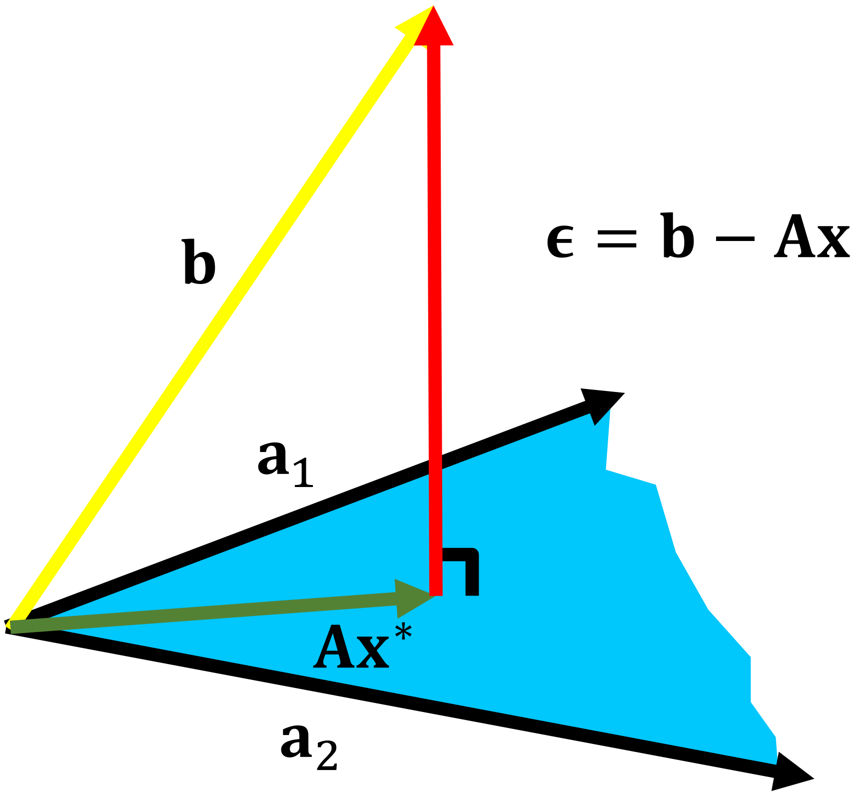
$$\mathbf{A}^{\mathrm{T}}(\mathbf{b}-\mathbf{A}\mathbf{x})=0$$

目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



$$\mathbf{A}^{\mathrm{T}}(\mathbf{b}-\mathbf{A}\mathbf{x})=0$$

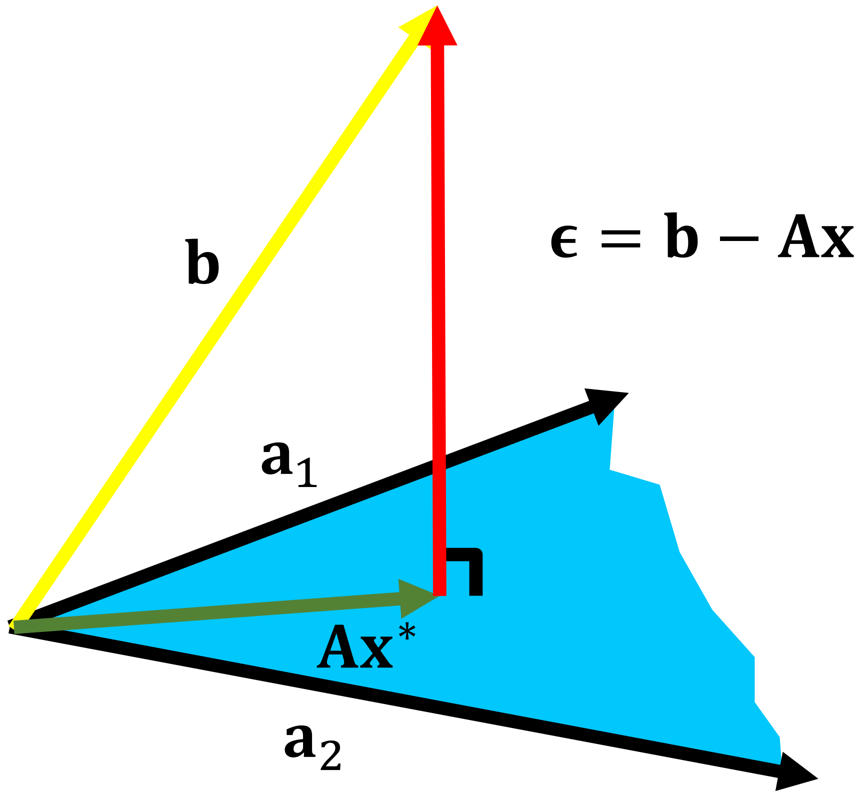
目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



$$\mathbf{A}^{\mathrm{T}}(\mathbf{b}-\mathbf{A}\mathbf{x})=0$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{b}-\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}=0$$

目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

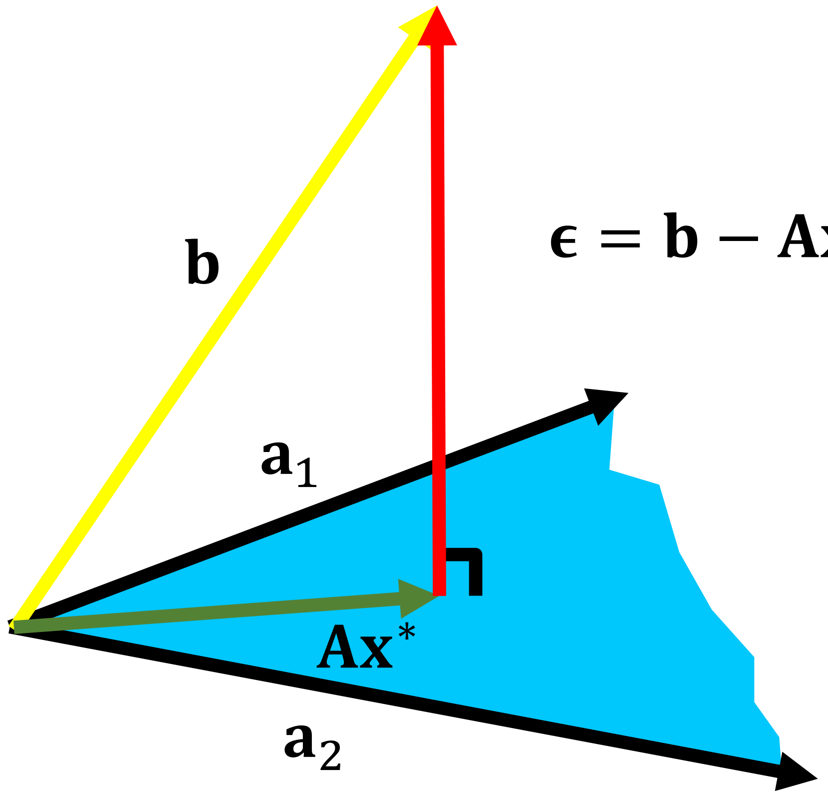


$$\mathbf{A}^{\mathrm{T}}(\mathbf{b}-\mathbf{A}\mathbf{x})=0$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{b}-\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}=0$$

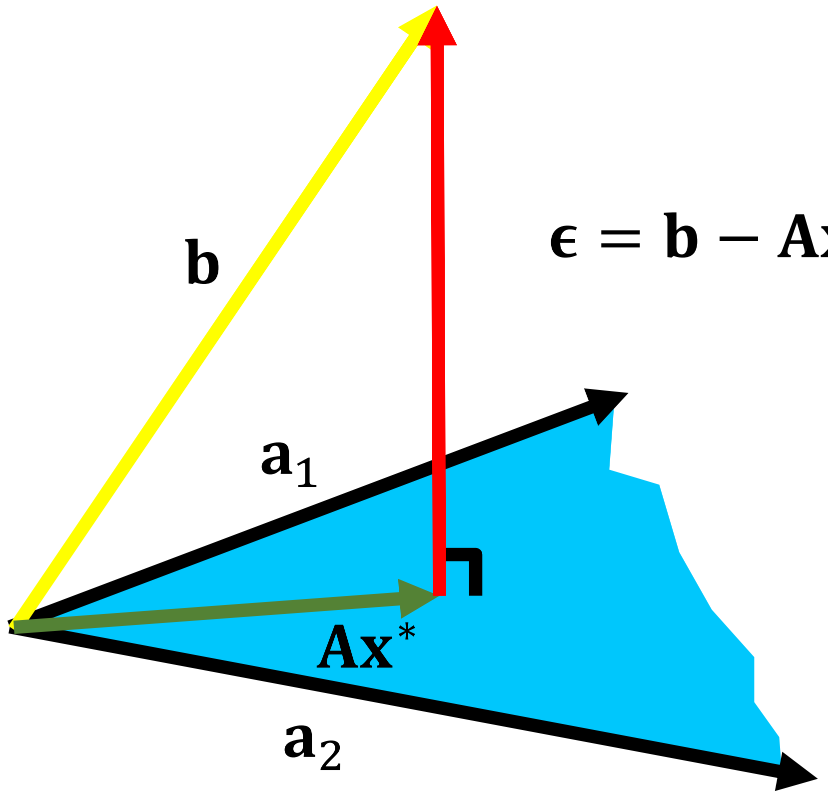
$$\mathbf{x}=\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

目标： 使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



$$\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



$$\mathbf{x}=\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

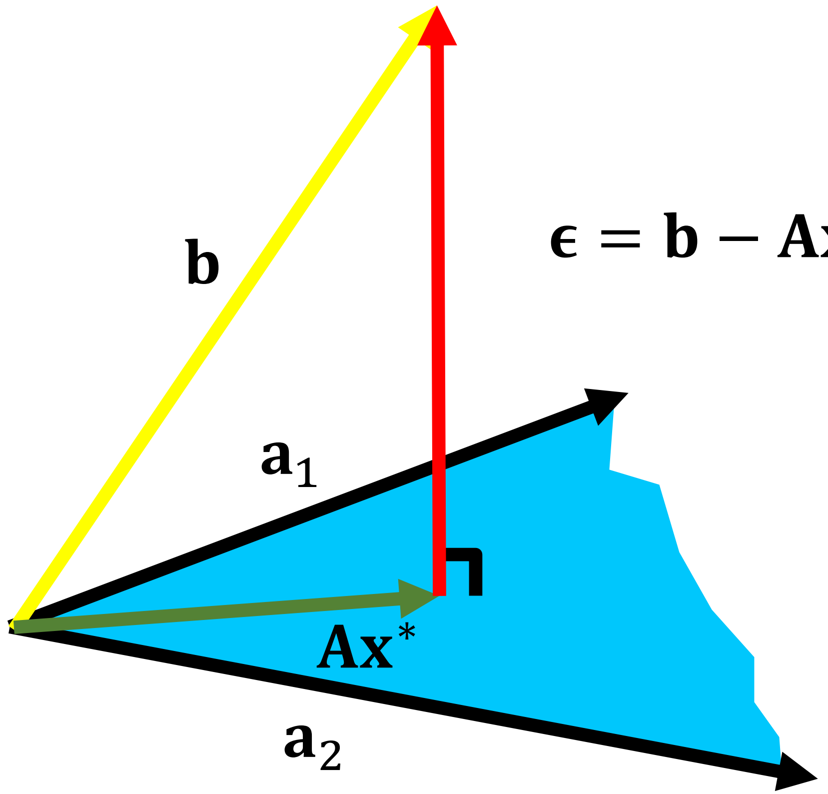
看起来眼熟吗？

解 $\frac{\partial \epsilon}{\partial \mathbf{x}} = 0$

$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$ 正规方程

最小二乘解 $\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$

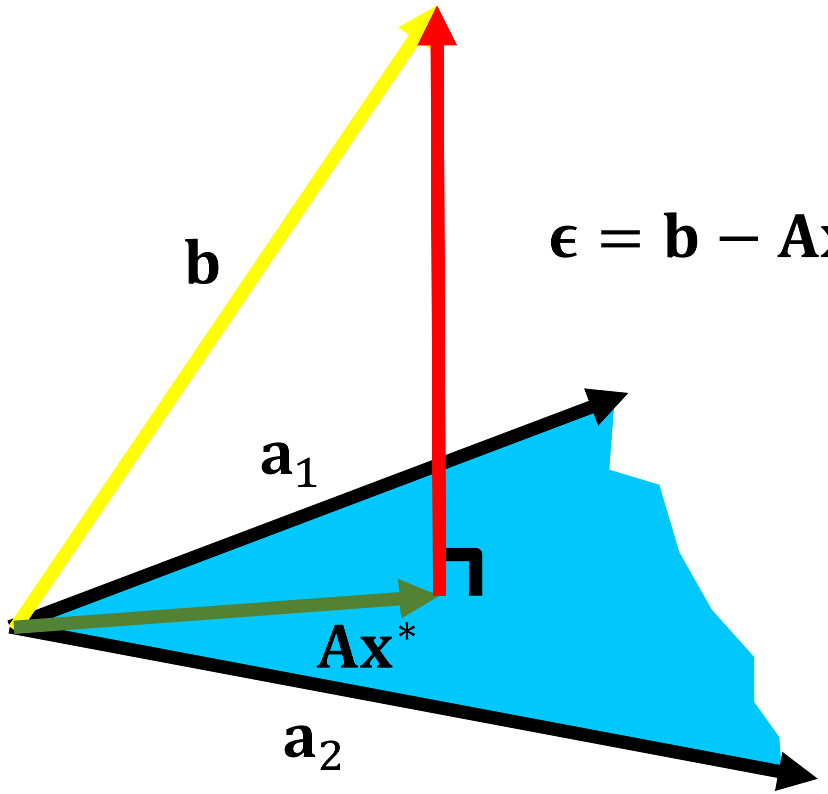
目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小



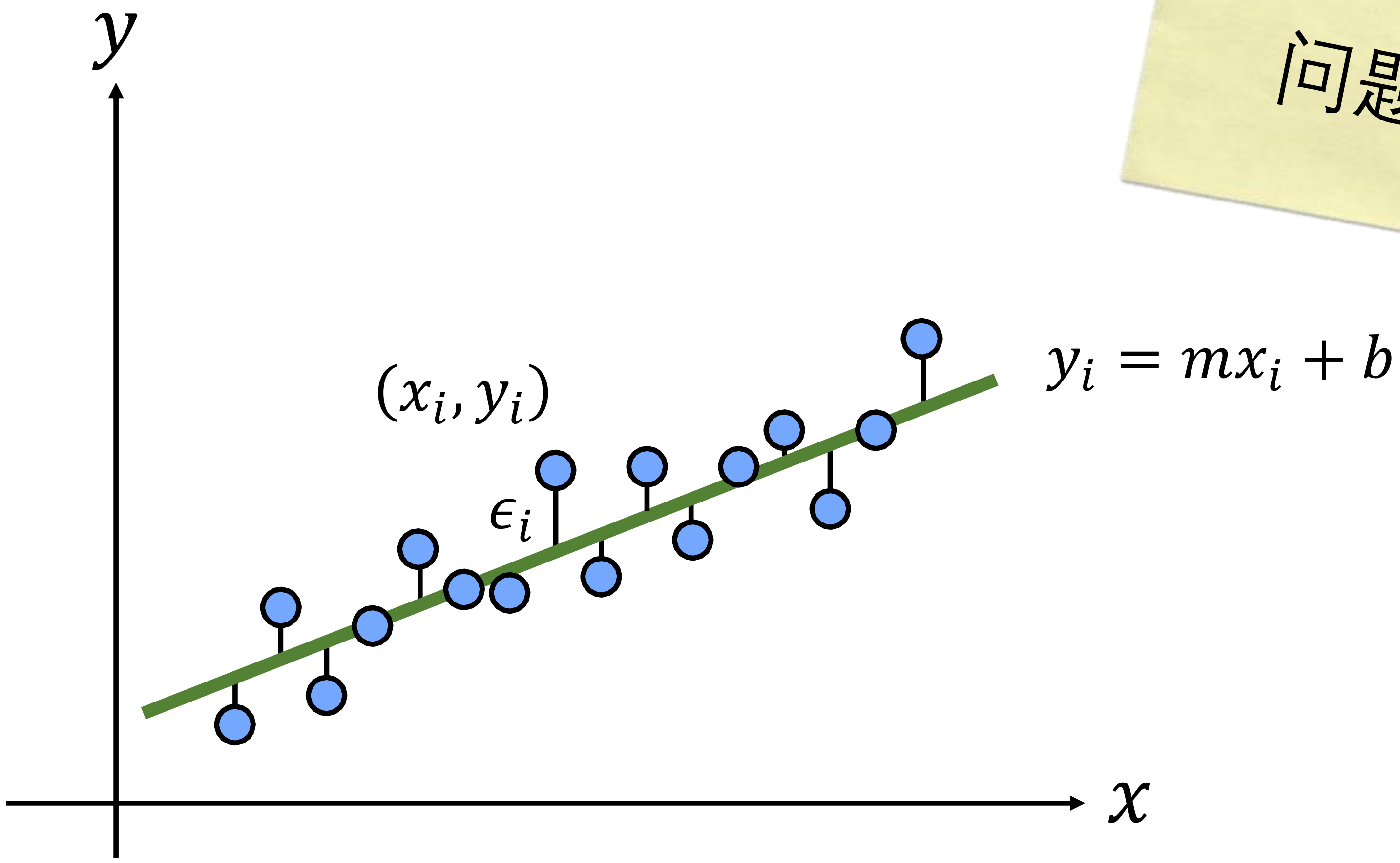
$$\mathbf{x}=\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

看起来眼熟吗？

目标：使$\|\mathbf{b}-\mathbf{A}\mathbf{x}\|$尽可能小

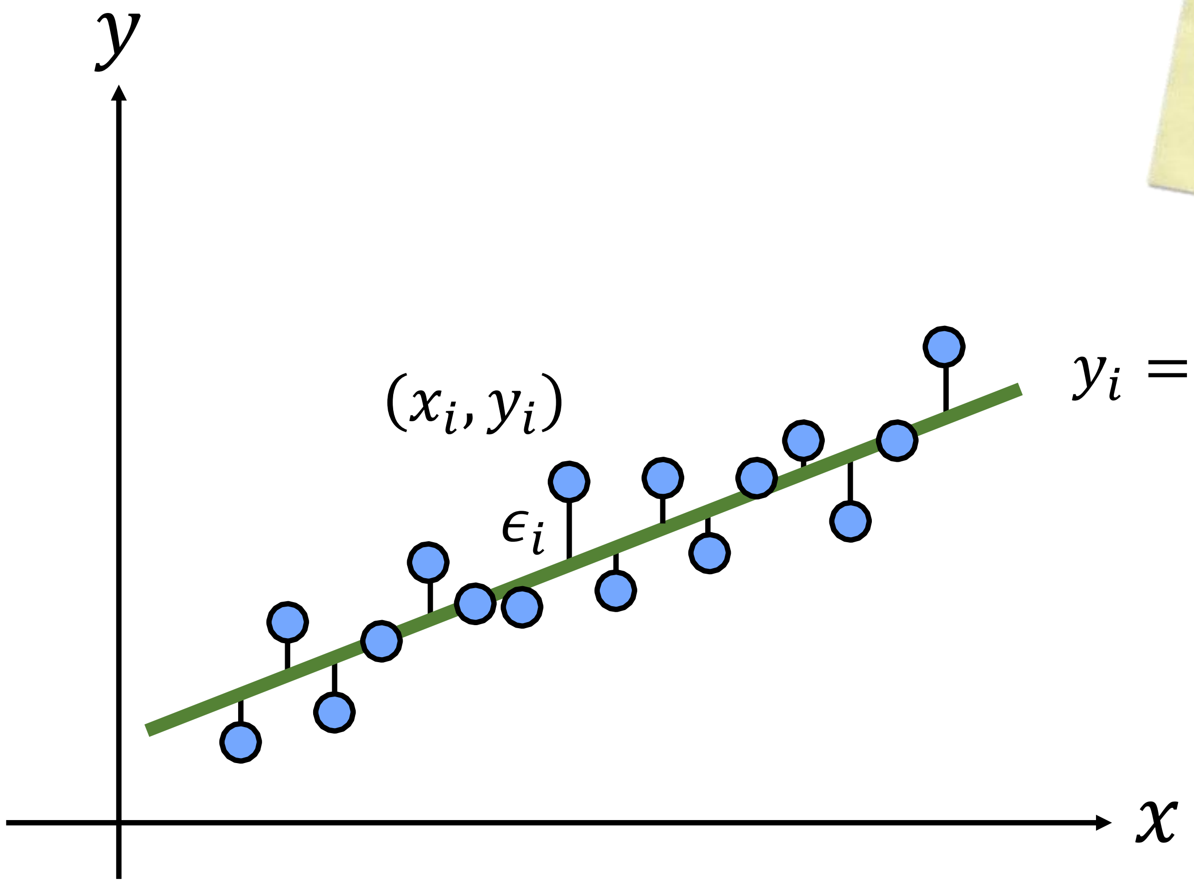


最小二乘解 $\mathbf{x}=\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$



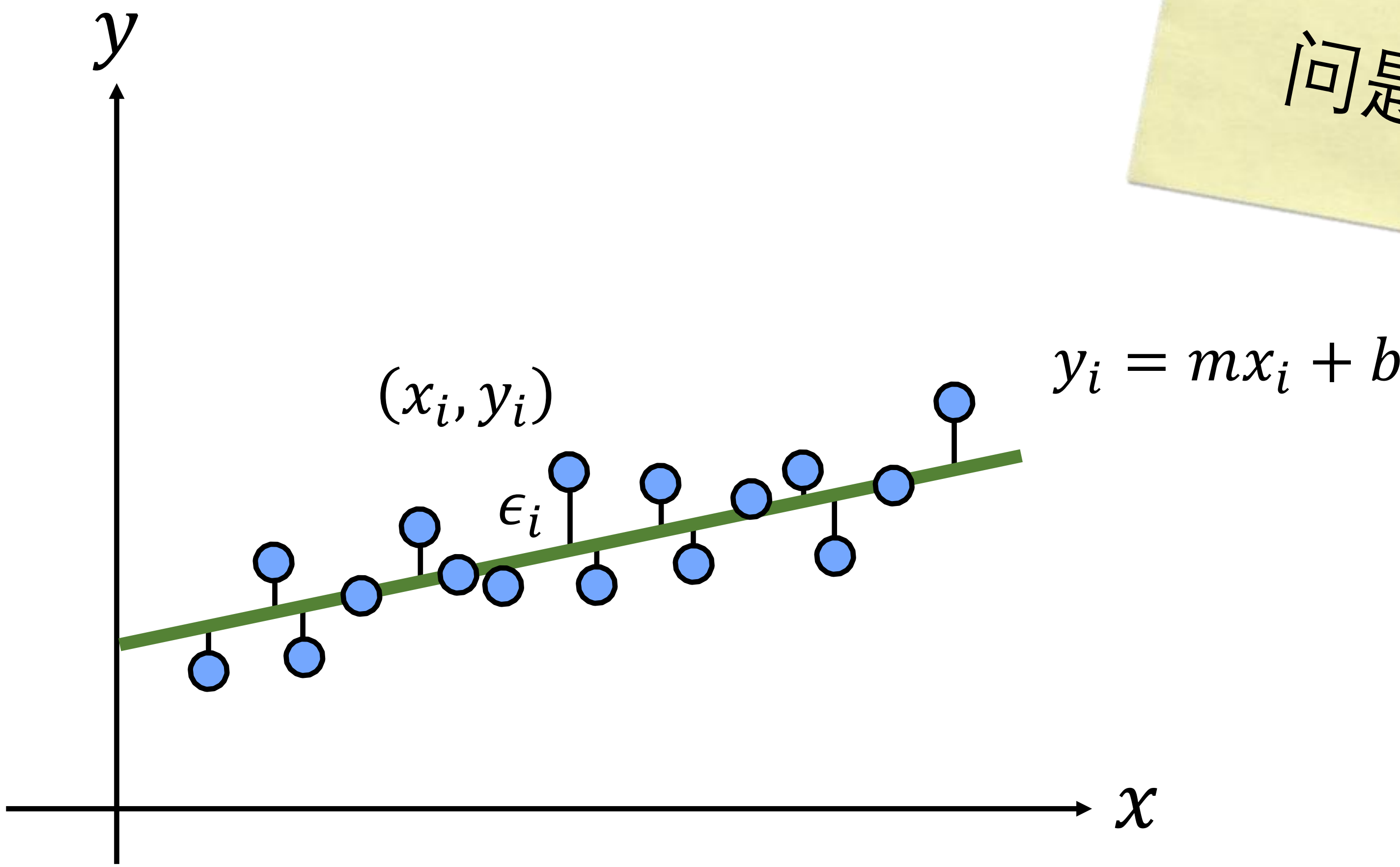
问题

问题



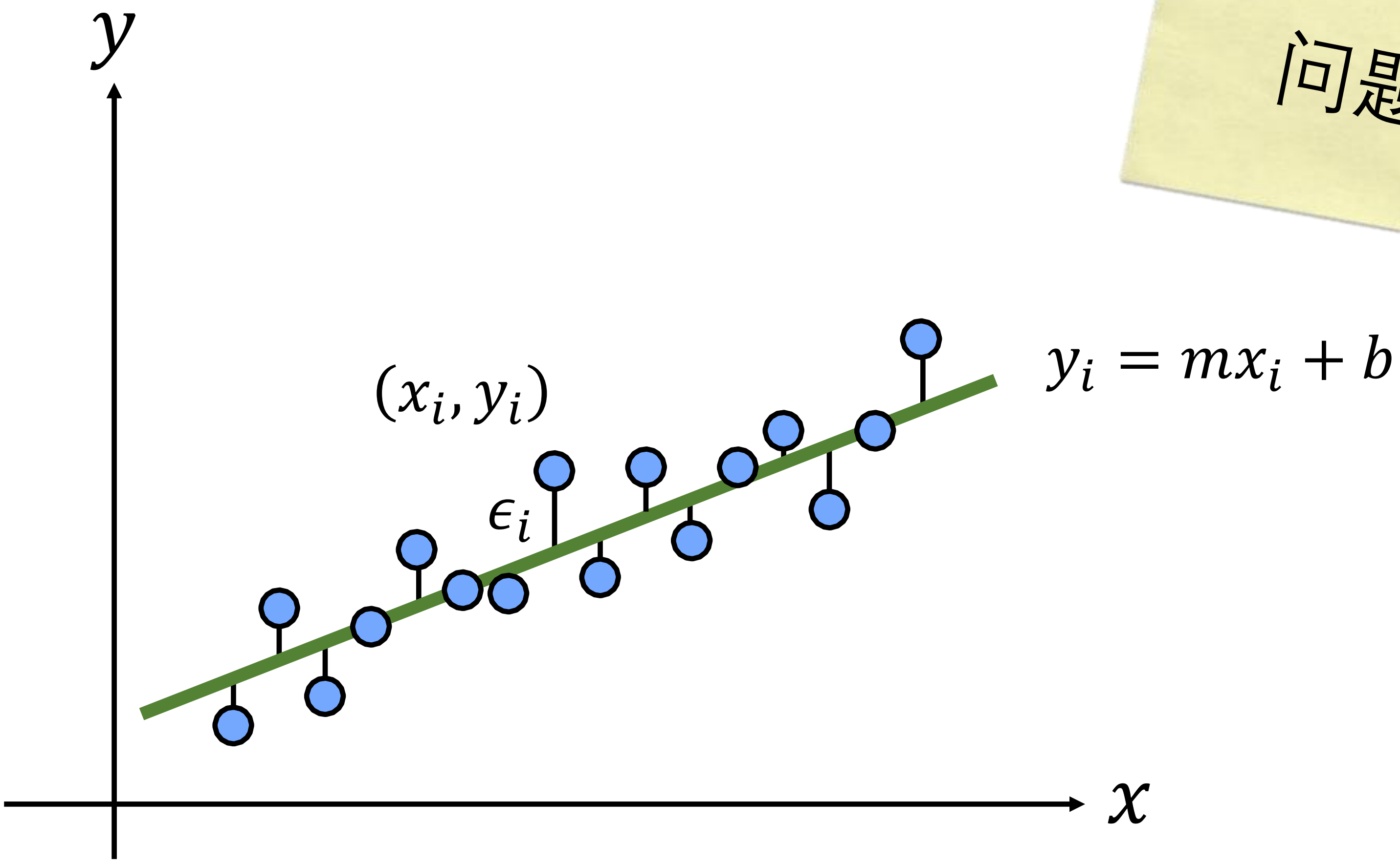
非旋转不变

问题



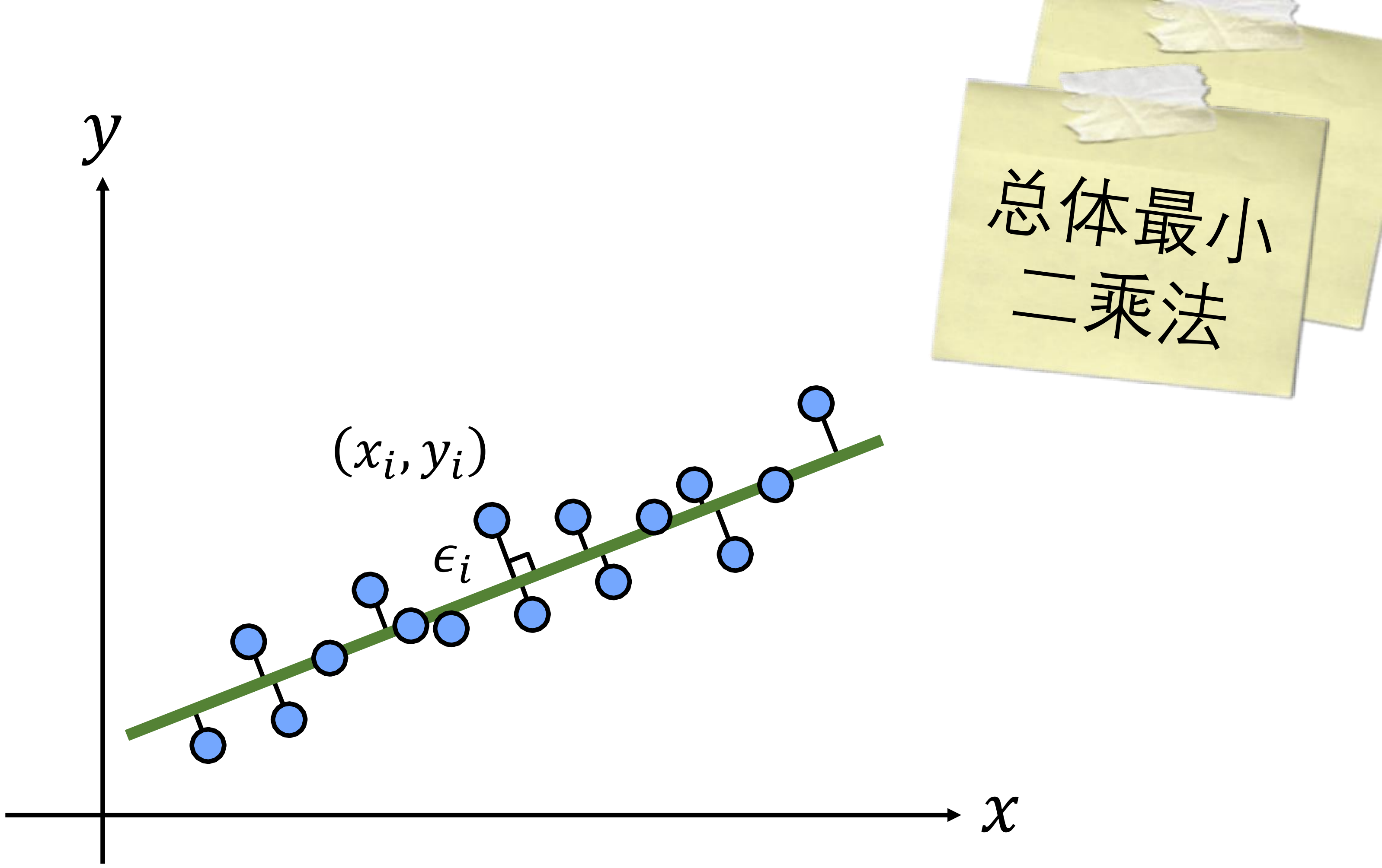
非旋转不变

问题

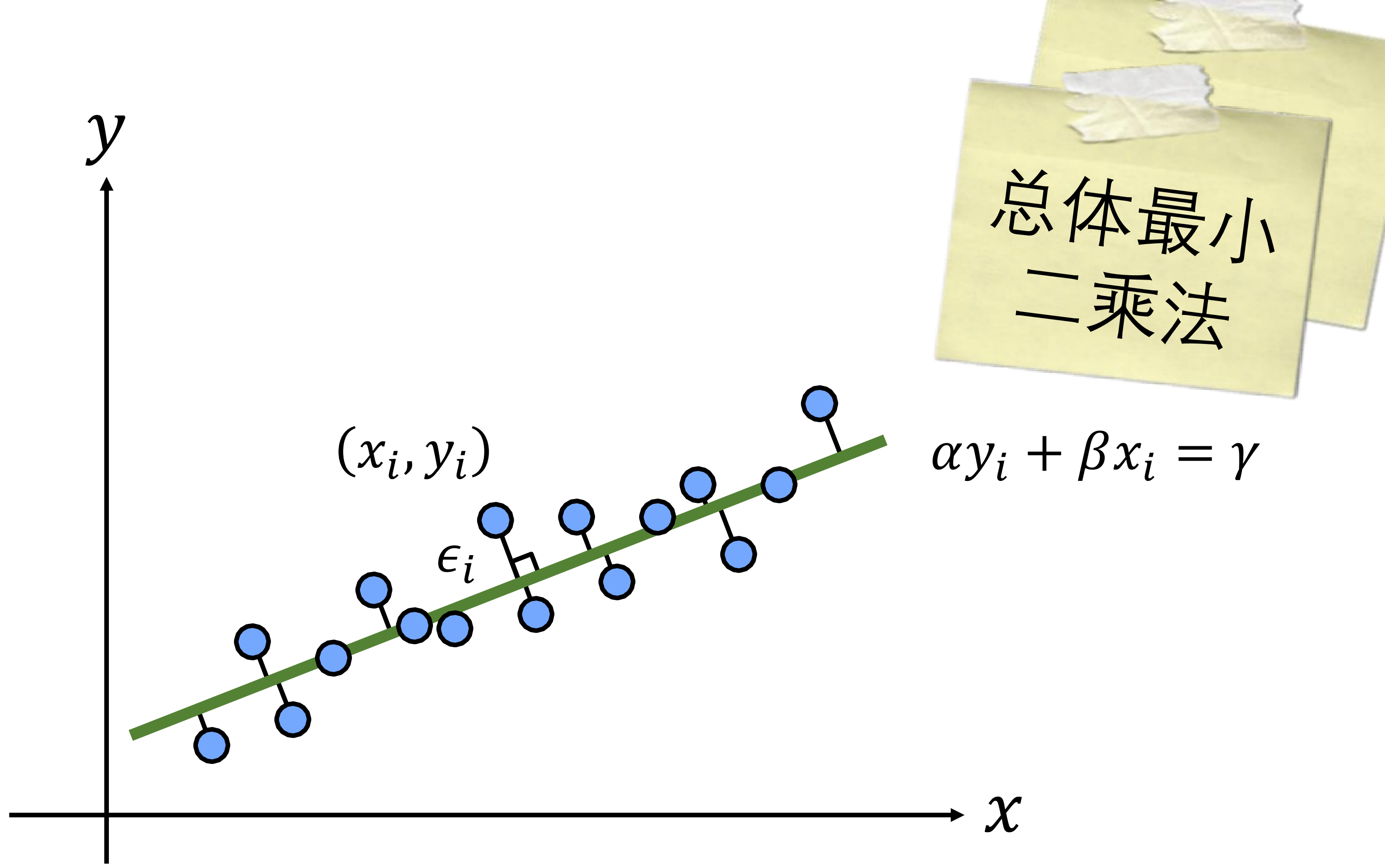


非旋转不变

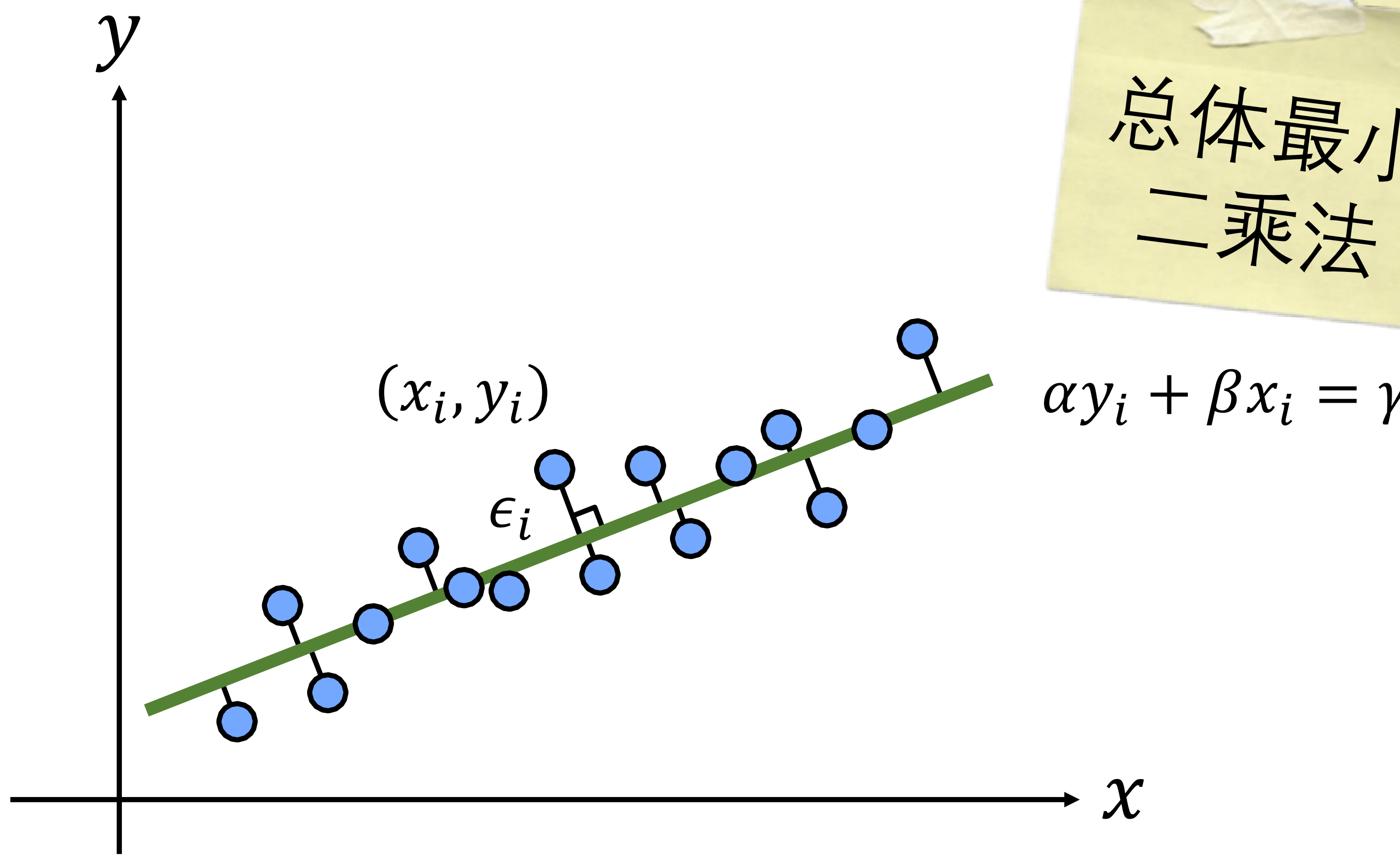
无法应对垂直直线



总体最小
二乘法
y
(x_i, y_i)
ϵ_i
x


总体最小
二乘法
y
(x_i, y_i)
$\alpha y_i + \beta x_i = \gamma$
ϵ_i
x

总体最小二乘法



$$\underset{(\alpha,\beta,\gamma)}{\arg\min} \sum_{i=1}^{N} (\alpha y_i + \beta x_i - \gamma)^2 \quad \textbf{subject to} \quad \left\| (\alpha, \beta)^{\mathrm{T}} \right\| = 1$$

Python时间

最小二乘直线拟合

```
# --- least-squares line fitting ---
x = np.arange(0, 20)
m = 2  # slope
b = 5  # y-intercept

y = m*x + b
y = y + np.random.randn(len(x))
A = np.stack([x, np.ones(x.shape)], axis=1)

mb_estimate = np.linalg.inv(A.T @ A) @ A.T @ y

# --- or equivalently ---
mb_estimate = np.linalg.pinv(A) @ y

# --- or equivalently ---
mb_estimate, _, _, _ = np.linalg.lstsq(A, y)
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向点的垂直分量添加噪声

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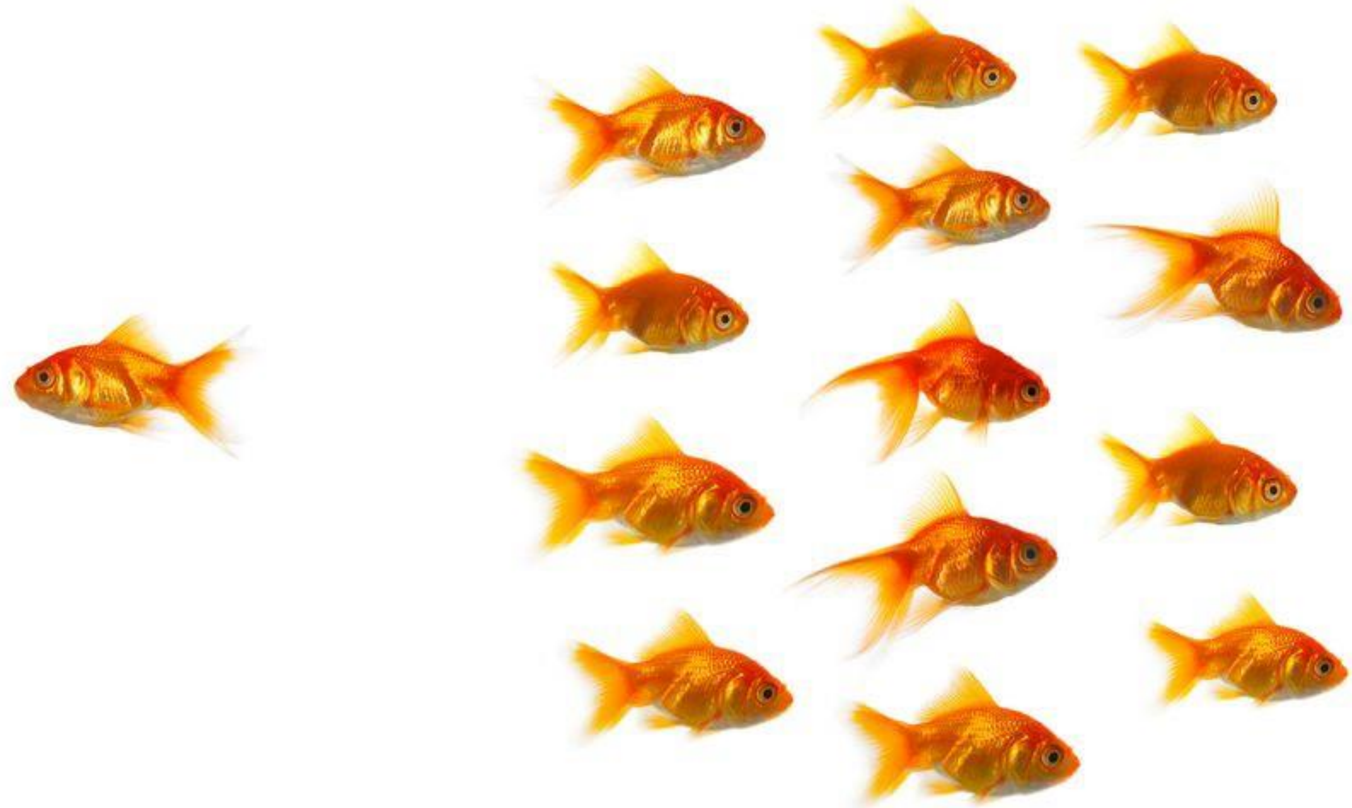
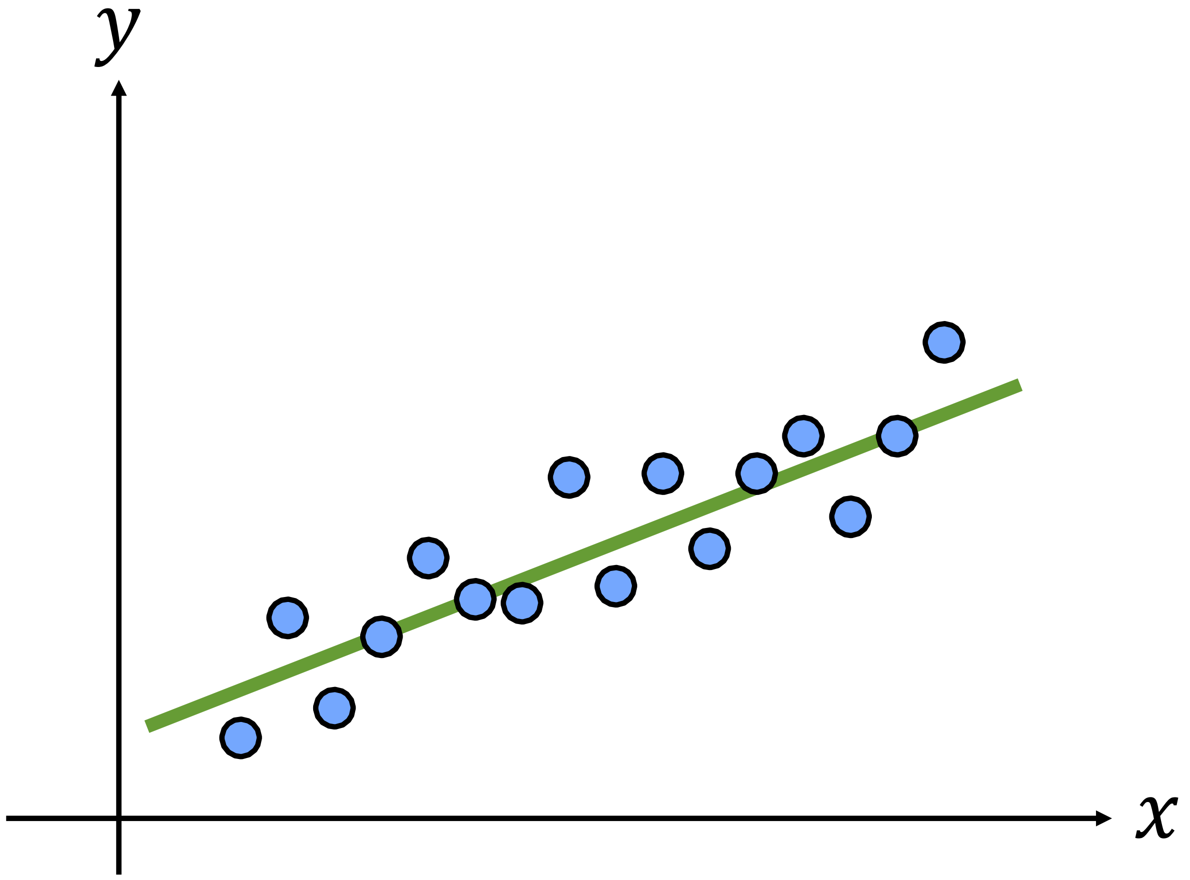
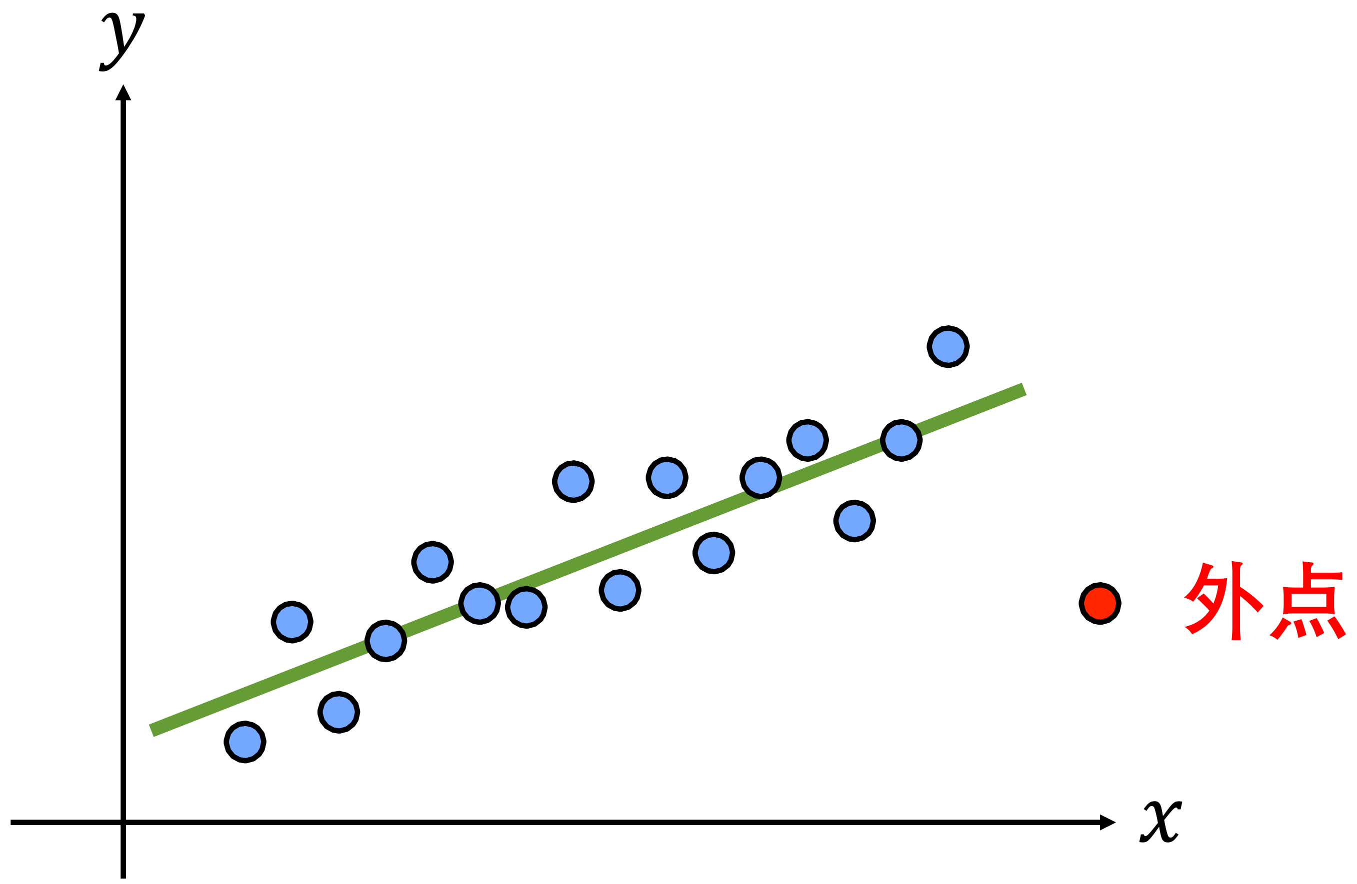
Python时间

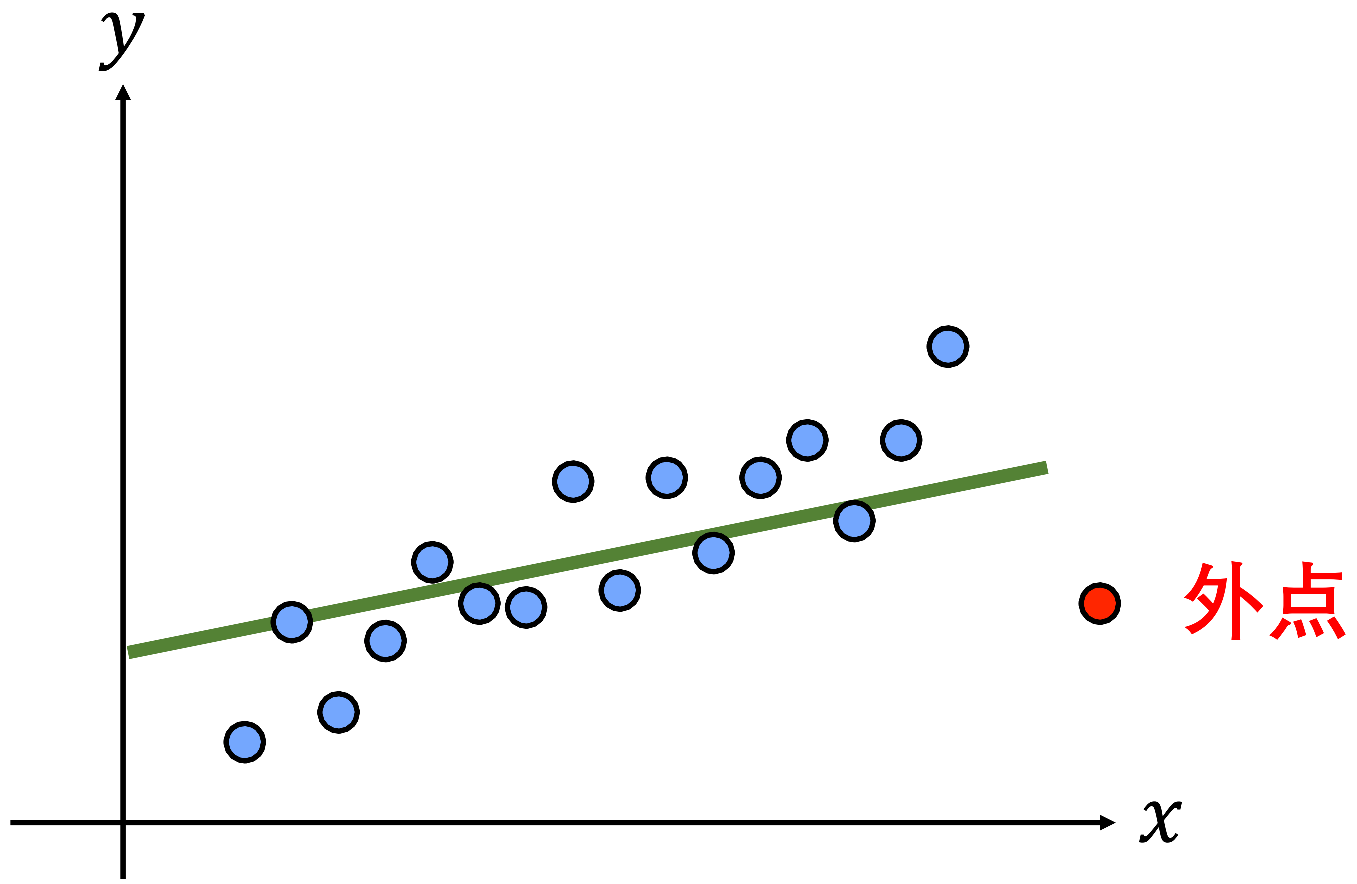
外点



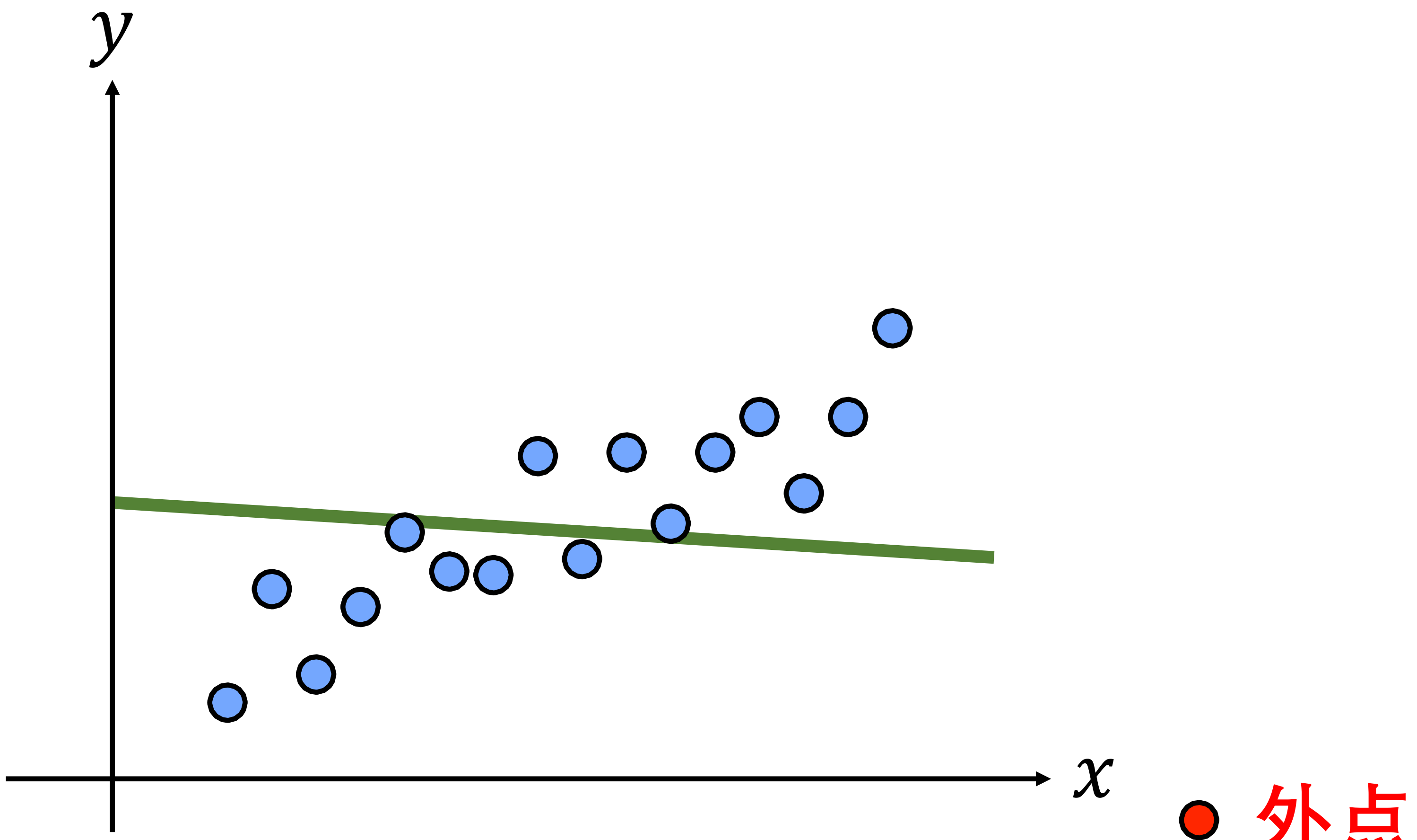

y
x

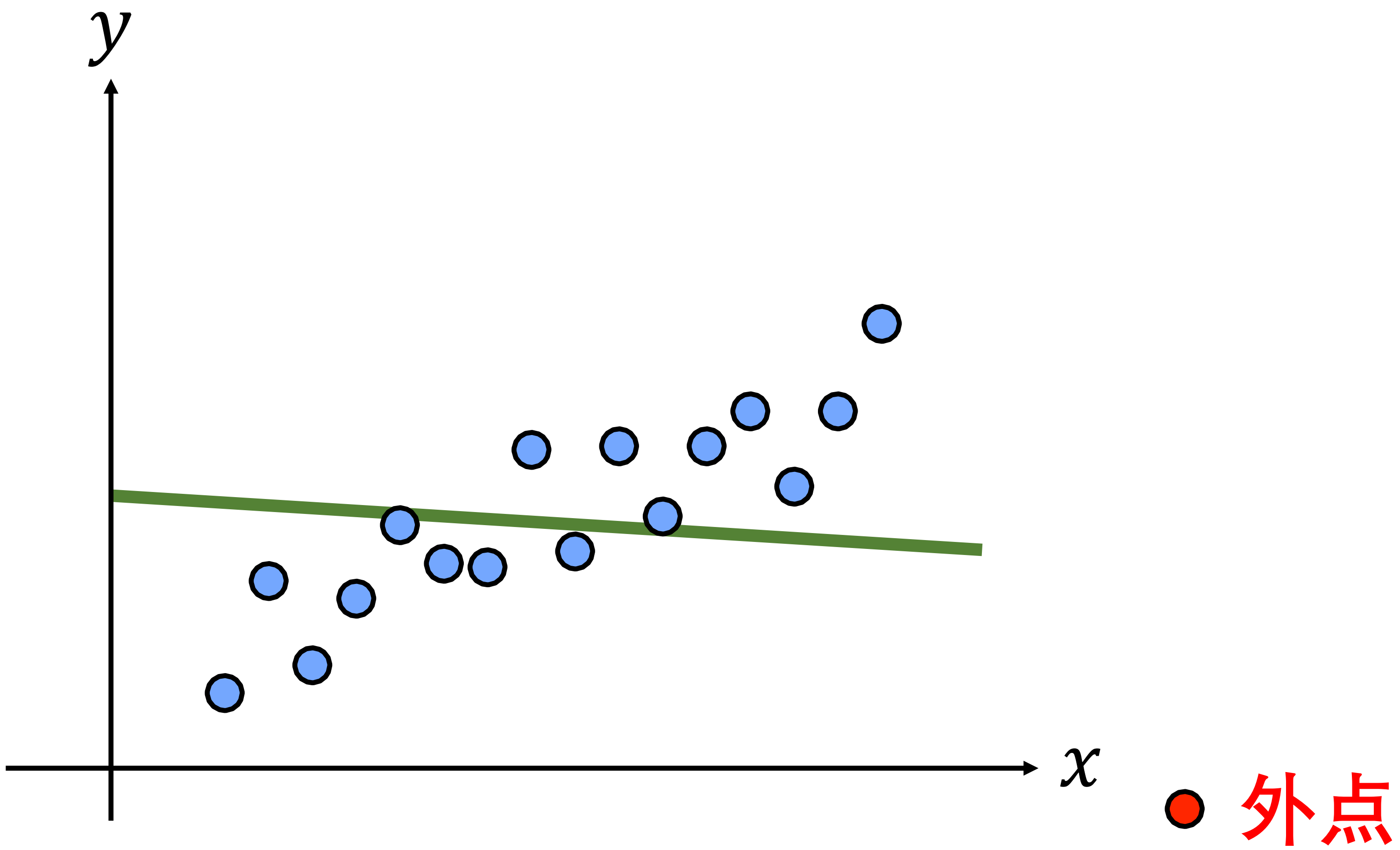


y
x
外点



y
外点
x


y
x
外点



最小二乘法对外点不具有鲁棒性

Hough

变换

MACHINE ANALYSIS OF BUBBLE CHAMBER PICTURES

P. V. C. Hough

The University of Michigan, Ann Arbor, Mich.

1. AREA ELEMENTS VERSUS LINE SEGMENTS IN PICTURE ANALYSIS

Many people have suggested that a modern digital computer should be able to recognize a fairly complex pattern of tracks in a bubble chamber photograph such as that shown in Fig. 1a [(*)]. Concrete schemes for such recognition generally assume that information is available about the presence or absence of bubbles in area elements covering the pictures and of a size appropriate to the resolution of the chamber. However, rough investigation of the time to read such information into a computer and to conduct a search for linear correlations among bubbles has so far led important reason, the slope of each line segment provides the computer with a good prediction of the location of the adjoining line segment and so reduces enormously the search time in recognizing a track.

It will be shown below that the tracks in one picture may be recognized in a time of the order of 1-2 s, and therefore a stereo pair of pictures may be analyzed in less than 5 s. It seems that an analysis time of this order of magnitude is a reasonable goal since it matches the cycle time of large accelerators.

United States Patent Office

3,069,654
Patented Dec. 18, 1962

1

3,069,654
METHOD AND MEANS FOR RECOGNIZING COMPLEX PATTERNS
Paul V. C. Hough, Ann Arbor, Mich., assignor to the United States of America as represented by the United States Atomic Energy Commission
Filed Mar. 25, 1960, Ser. No. 17,715
6 Claims. (Cl. 340—146.3)

2

of the point on the line segment from the horizontal midline **109** of the framelet **108**.

(3) Each line in the transformed plane is made to have an intercept with the horizontal midline **101** of the picture **100** equal to the horizontal coordinate of its respective point on the line segment in framelet **108**.

Thus, for a given reference point **110** on line segment **102** a line **110A** is drawn in the plane transform **102A**. The reference point **110** is approximately midway between

Dec. 18, 1962 P. V. C. HOUGH 3,069,654

METHOD AND MEANS FOR RECOGNIZING COMPLEX PATTERNS

Filed March 25, 1960 2 Sheets-Sheet 1

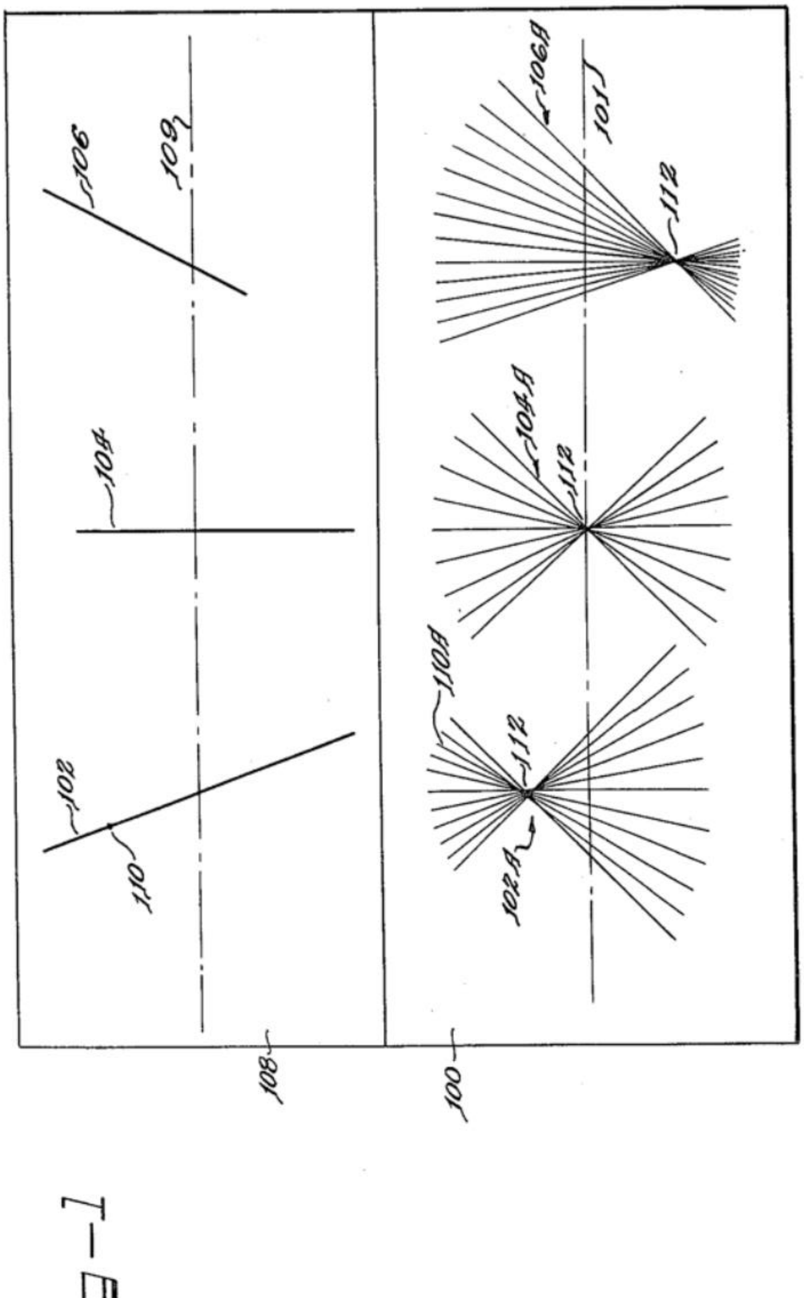


Fig-1

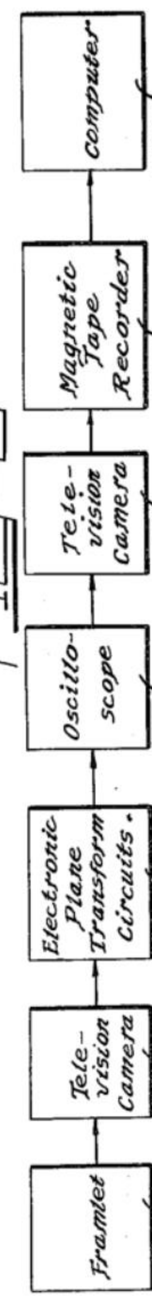


Fig-2

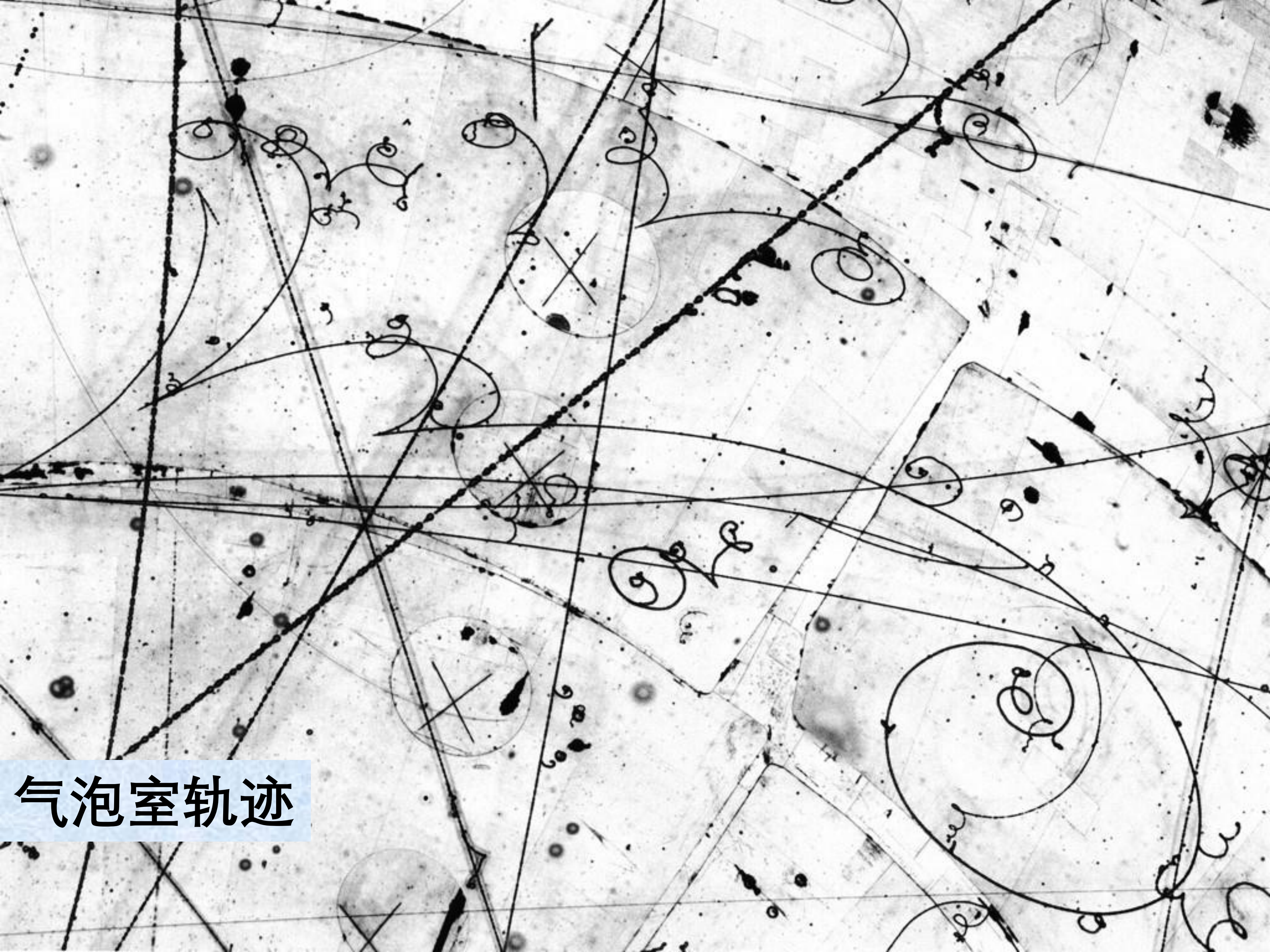
INVENTOR.

Paul V.C. Hough

BY

Roland G. [signature]

Attorney



气泡室轨迹

How the Hough Transform Was Invented

EDITOR'S INTRODUCTION

Our guest in this column is Dr. Peter Hart. Dr. Hart was born on 17 February 1941 in Brooklyn, New York where he attended public elementary and high schools. He received the B.E.E. degree from the Renesselaer Polytechnic Institute in 1962 and the M.S. and Ph.D. degrees from Stanford University in 1963 and 1966, respectively. He is married to educational writer Diane Hart and they have one daughter, Laura. Dr. Hart is currently the chair and founder of Ricoh Innovations, Inc. in Menlo Park, California.

If you have visited the Computer History Museum in Mountain View, California, you probably saw Shakey, the world's first mobile, intelligent robot. Dr. Hart was the head of the project within SRI International that developed Shakey. Besides Shakey and the topic of the column, which is by itself an important milestone in the computing era, Dr. Hart has had many achievements in his career, and in this limited space we will highlight just some of them.

Our guest's leadership skills and vision are well proven by the number of companies and international research centers he founded and/or led. At SRI International, where he served as the director of the Artificial Intelligence Center, Dr. Hart coinvented the A* algorithm for finding the shortest path through a graph. This is the basic algorithm used today in various Web services and GPS products to compute driving directions. Also, while at SRI International, he invented the modern form of the Hough transform and coauthored one of the most cited references in the field of computer science, *Pattern Classification and Scene Analysis*, which was in print for more than 25 years before being supplanted by a second edition. The textbook has been translated into four languages: Russian, Japanese, Chinese, and Korean.

On top of all these early-career achievements, in 1980 our guest founded the world's first corporate artificial intelligence research laboratory at Fairchild/Schlumberger. In 1983 he cofounded Syntelligence, Inc. and delivered commercial expert for creating new technology and business opportunities for the worldwide Ricoh Group. Moreover, Dr. Hart was the first non-Japanese person to serve as a corporate officer of Ricoh Company, Ltd.

All these leadership positions did not distract Dr. Hart from technical research and innovations. He holds 70 U.S. and foreign patents, the most recent of which was issued in August 2009. Dr. Hart has shaped technological advancements not only by his own contributions and research leadership positions but also through his service on numerous committees on technology strategy that advised the director of the National Science Foundation (NSF), the Undersecretary of Defense, the director of Central Intelligence, and the administrator of NASA. Dr. Hart is an IEEE Fellow, ACM Fellow, and AAAI Fellow. In 1998 he received the IEEE Information Society Golden Jubilee Award for work done with Prof. Thomas M. Cover establishing error bounds on the nearest-neighbor rule for pattern classification.

With all these achievements comes a pleasing personality and an interest in music and sports. If you look at Dr. Hart's profile on Facebook, you will see him in his biking clothes. He is humble and likes people to call him Peter. He is a source of inspiration to all those who work with him and "he challenges all of his colleagues with his intellect and by identifying issues that everybody else misses and continuously making amazing technical inventions," says Berna Erol at Ricoh Innovations Ltd. She adds, "Oftentimes we hear from our colleagues that whenever they encounter difficult situations they ask themselves 'what would Peter do'." For these coworkers, Peter is the big brother who cares deeply about his employees and guides them technically to success.

In this article, Dr. Hart shares with readers how the Hough transform was invented. You will be intrigued to learn how this standard item in the computer vision tool kit evolved almost by chance from a geometric insight to a theoretically

Peter Hart, Signal Processing Magazine, 2009

The Hough transform, used to detect geometric features

in computer vision [1]–[3]. Although nobody tabulates the frequency with

"Hough transform." This is several times larger than the number of citations


MAKE
张


每个特征都会对与其支持的所有模型投票
MAKE
张

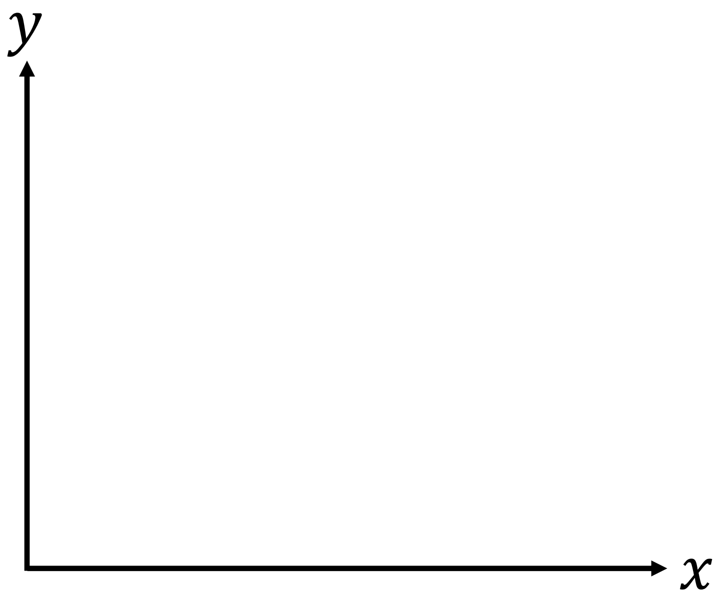

每个特征都会对与其支持的所有模型投票
获得投票最多的模型当选

每个特征都会对与其支持的所有模型投票

获得投票最多的模型当选

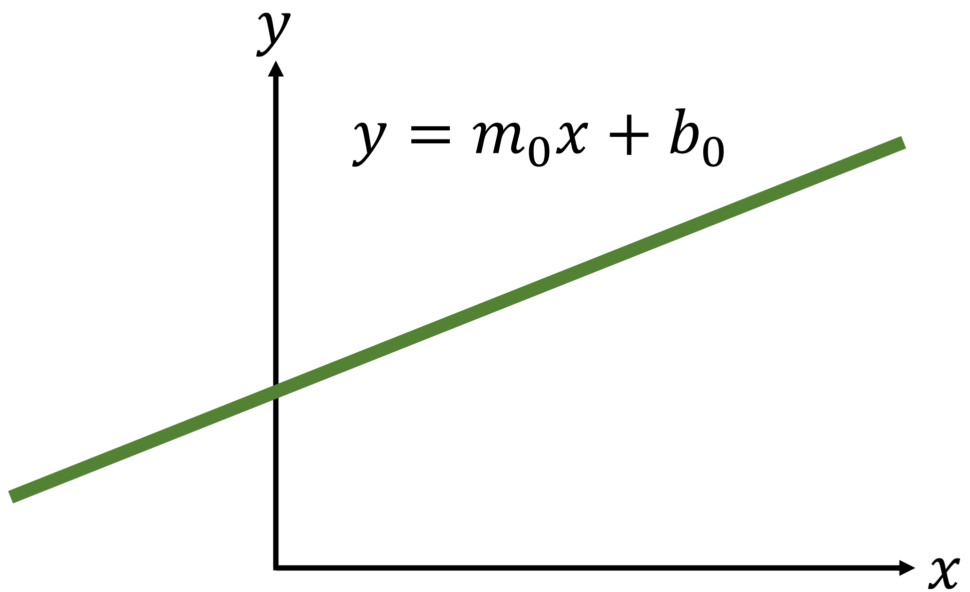
噪点和外点也投票，但通常他们的投票是不一致的

Hough
直线



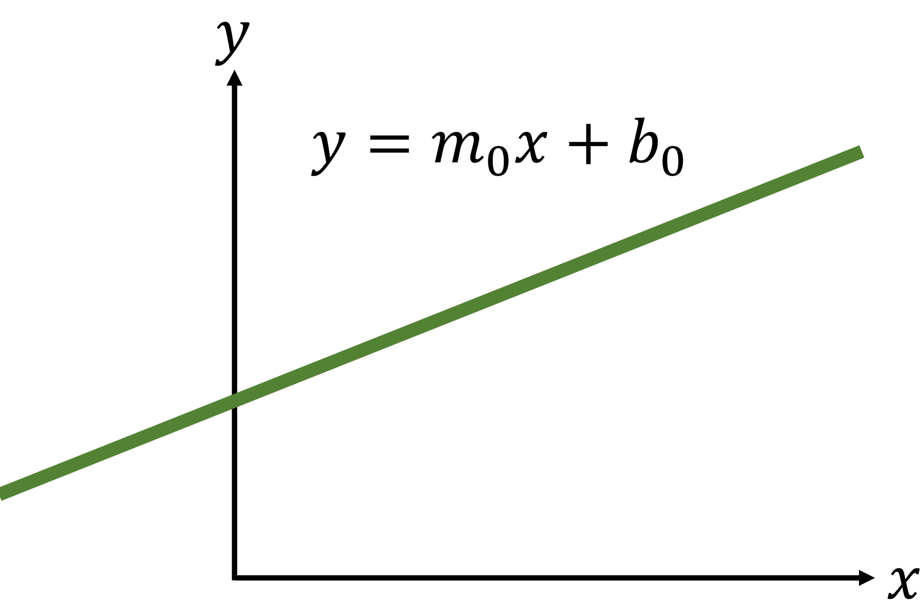
图像空间

Hough
直线



图像空间

Hough 直线



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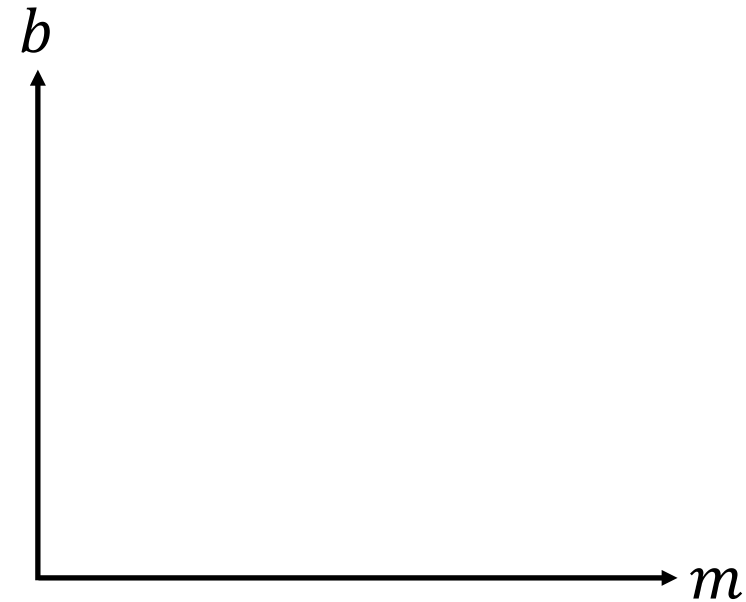
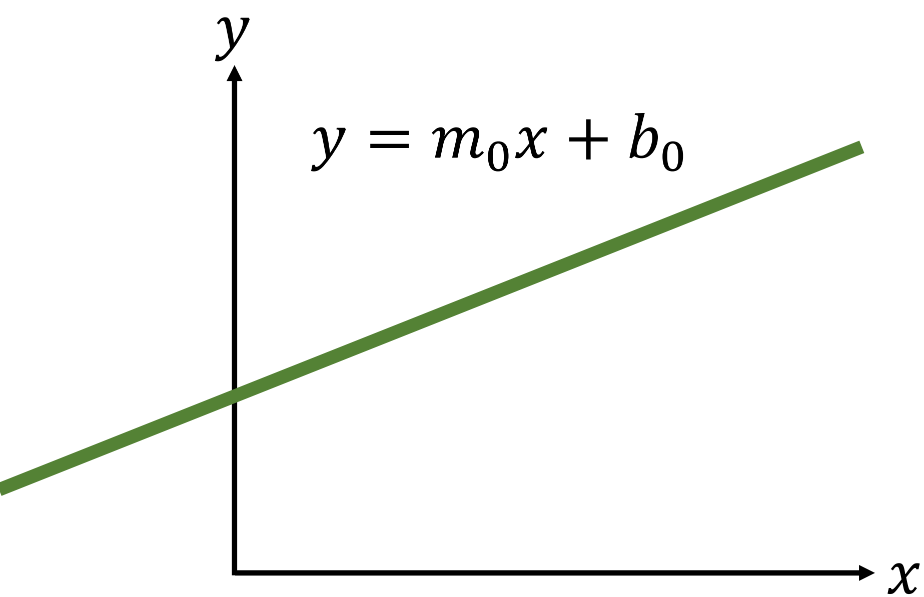


Hough (参数) 空间

Hough 直线



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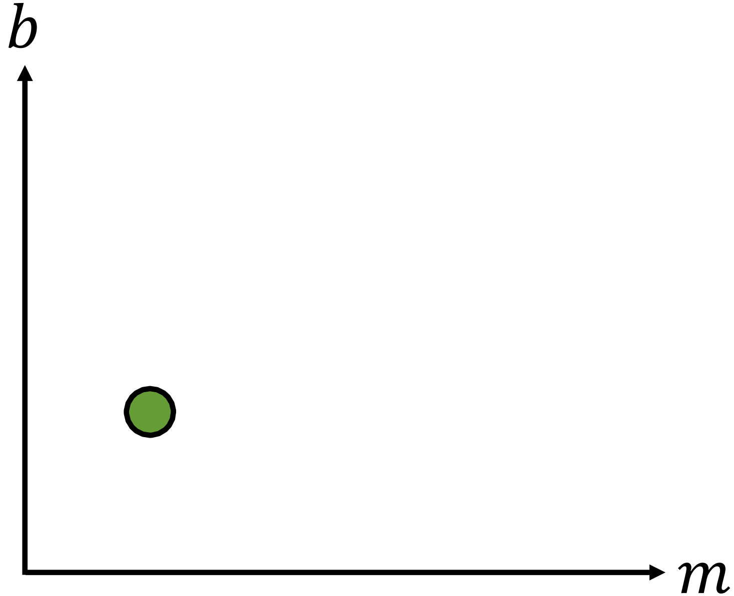
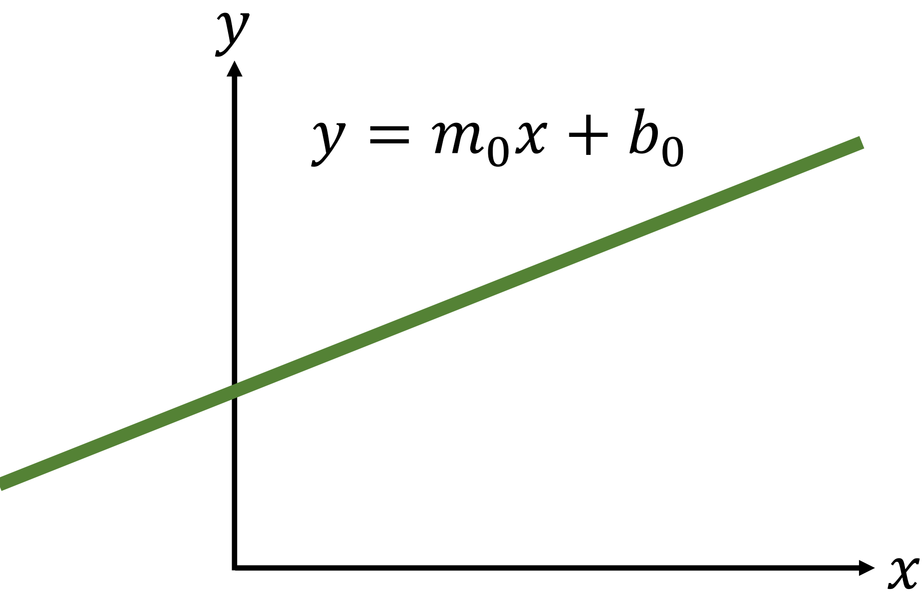


Hough (参数) 空间

Hough
直线



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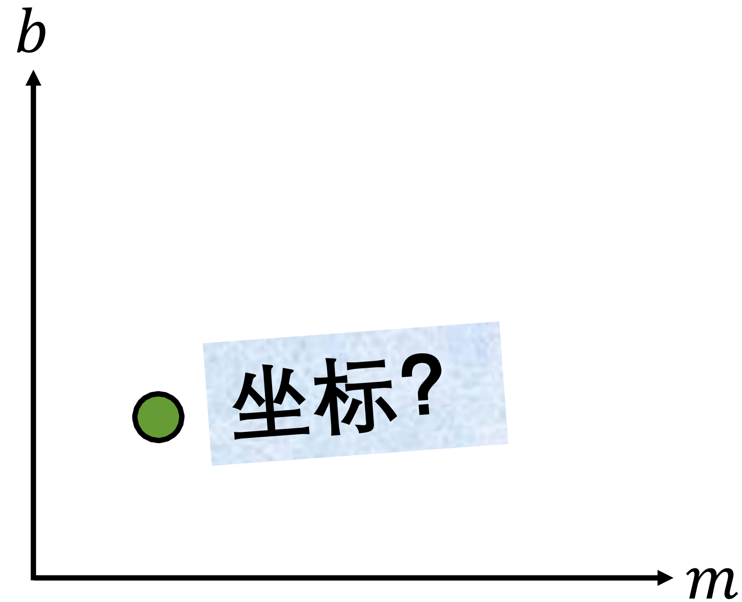
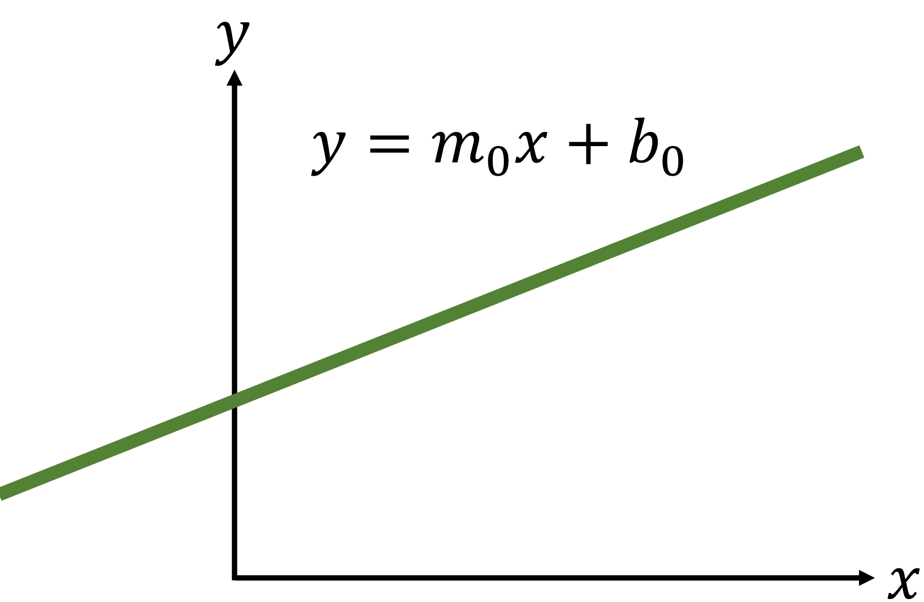


Hough (参数) 空间

Hough 直线



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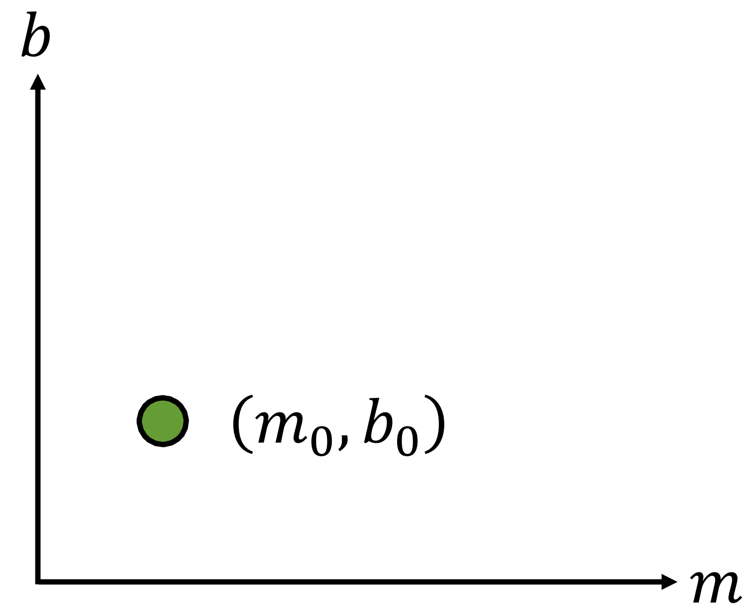
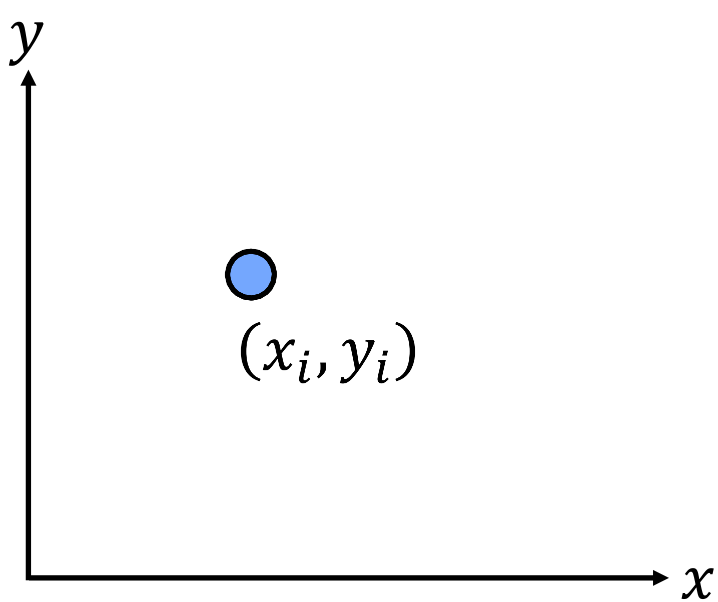


Hough (参数) 空间

点线对偶



图像空间

Hough (参数) 空间

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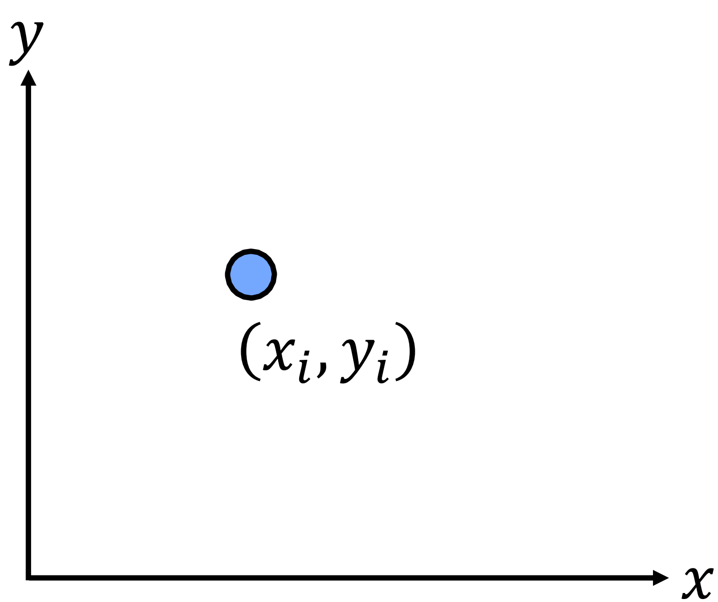

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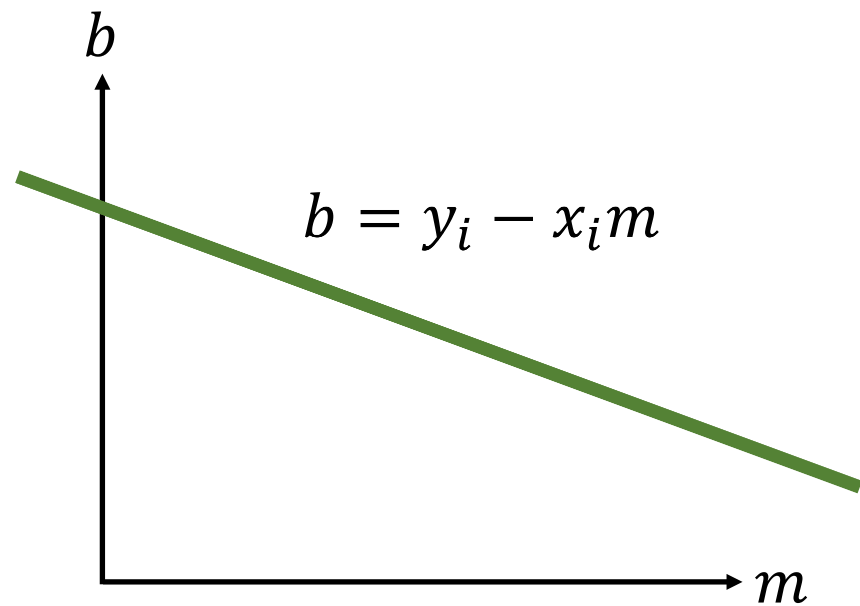
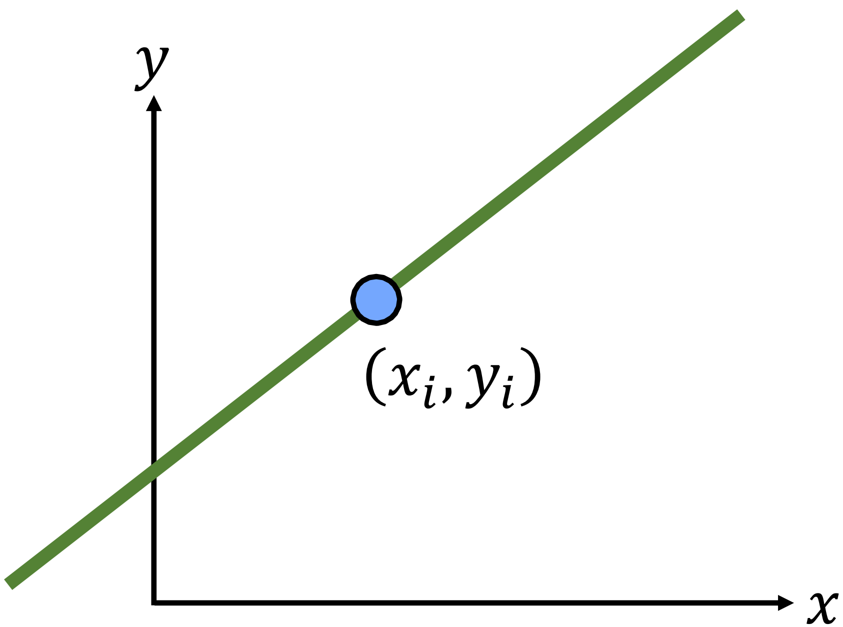


Hough (参数) 空间

Hough 直线



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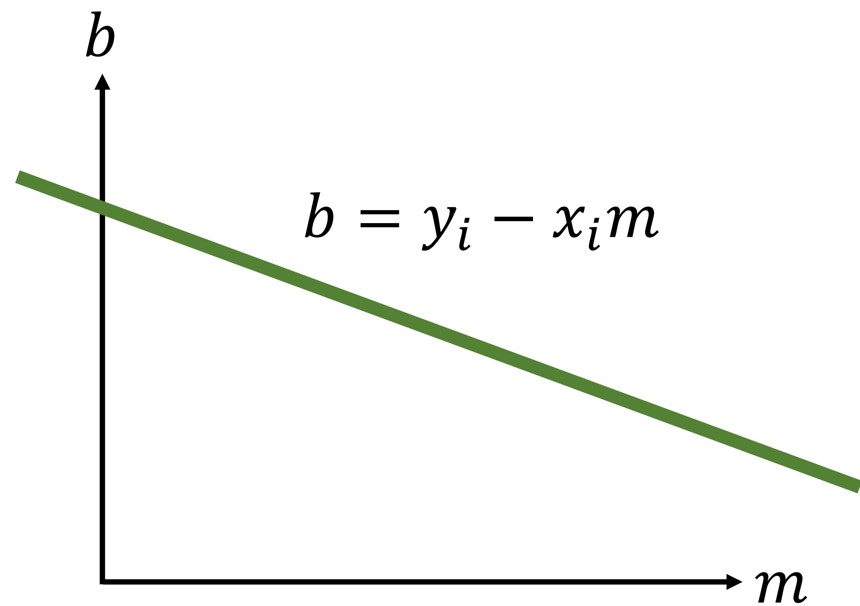
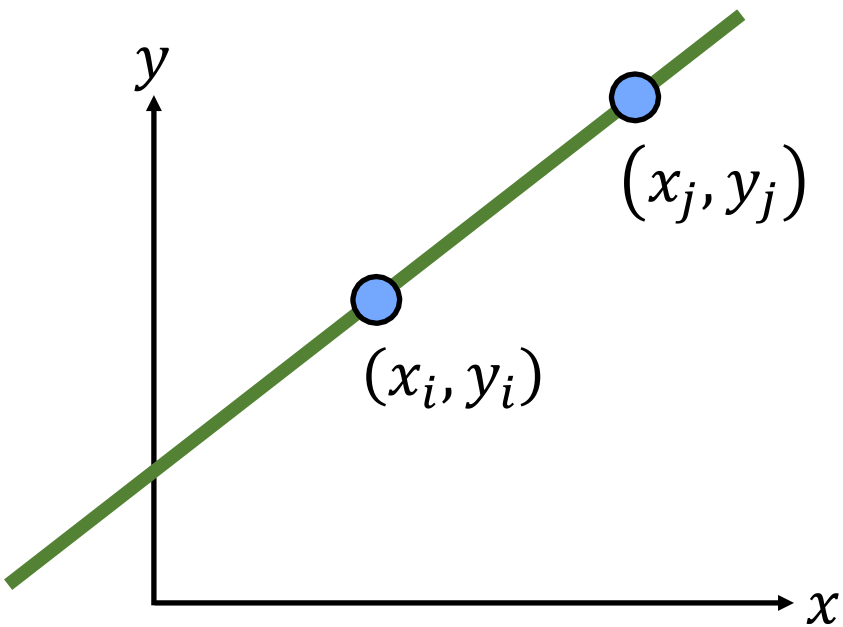


Hough (参数) 空间

Hough 直线



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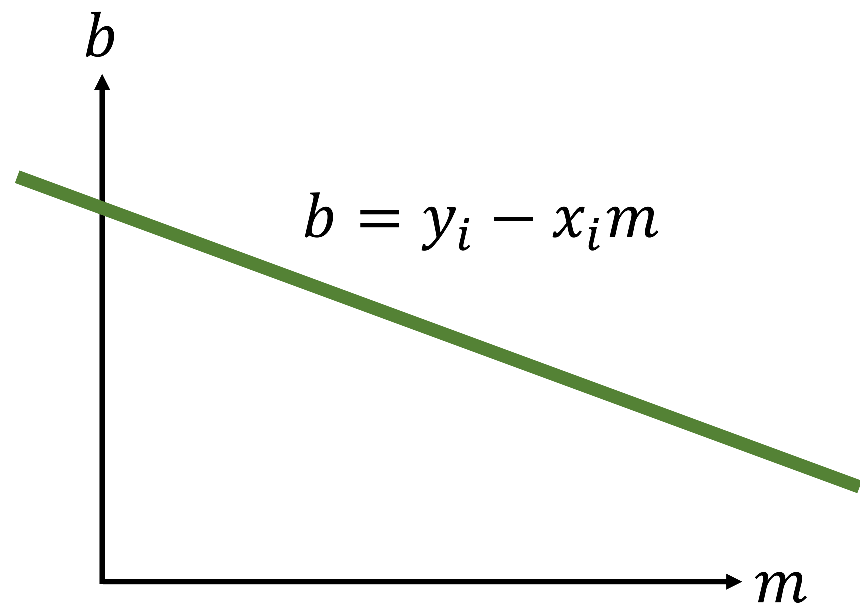

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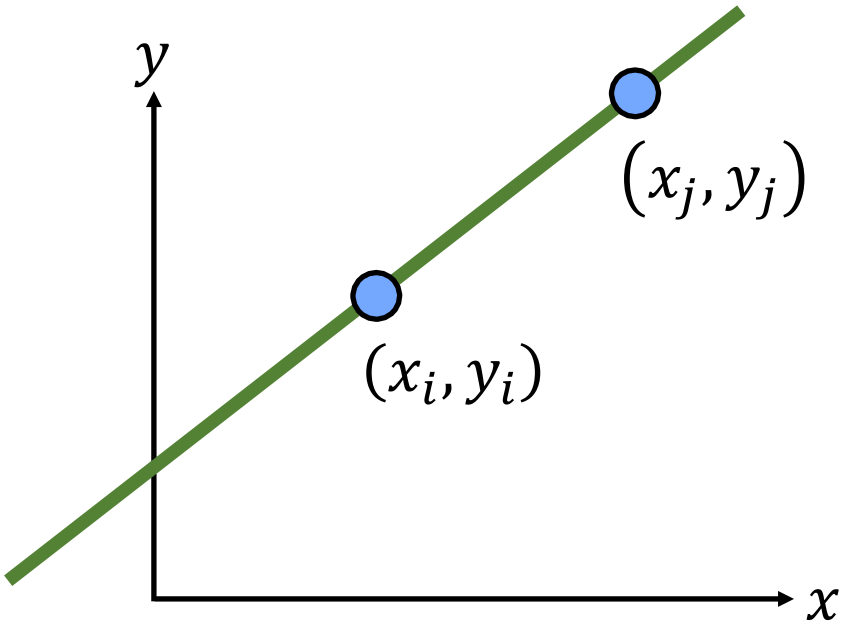

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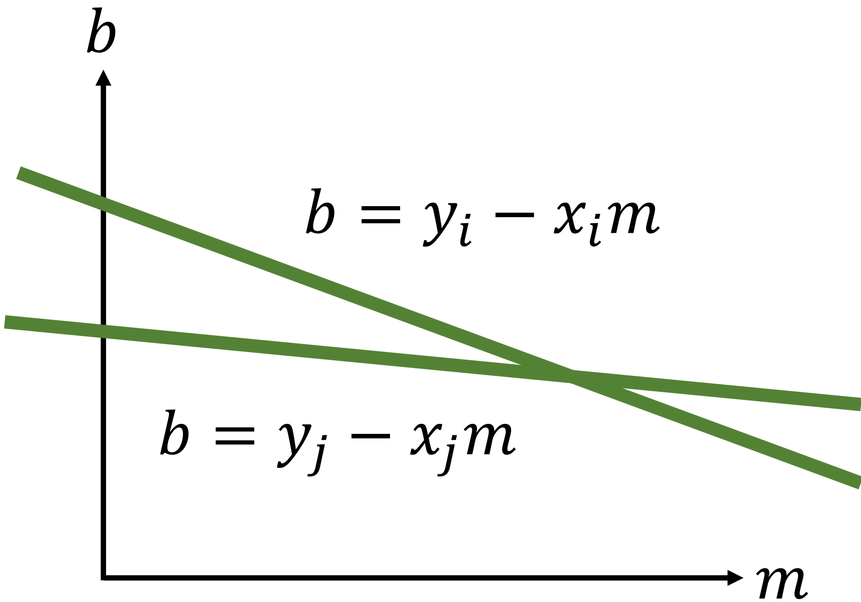
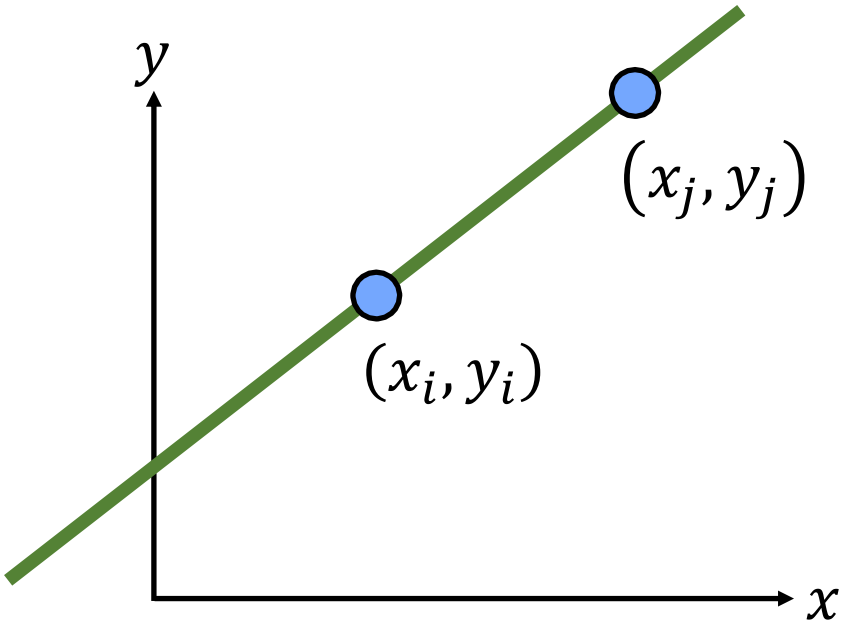


Hough (参数) 空间



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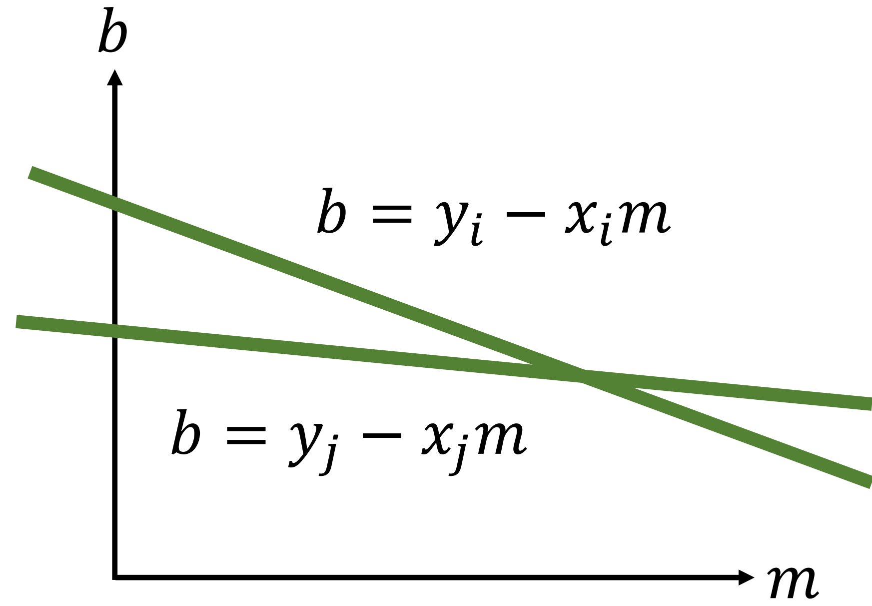
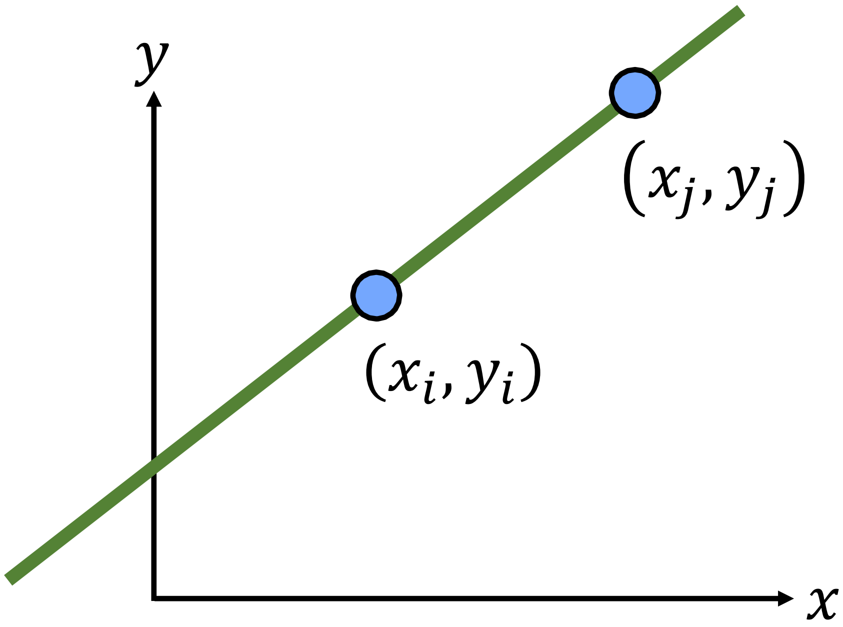
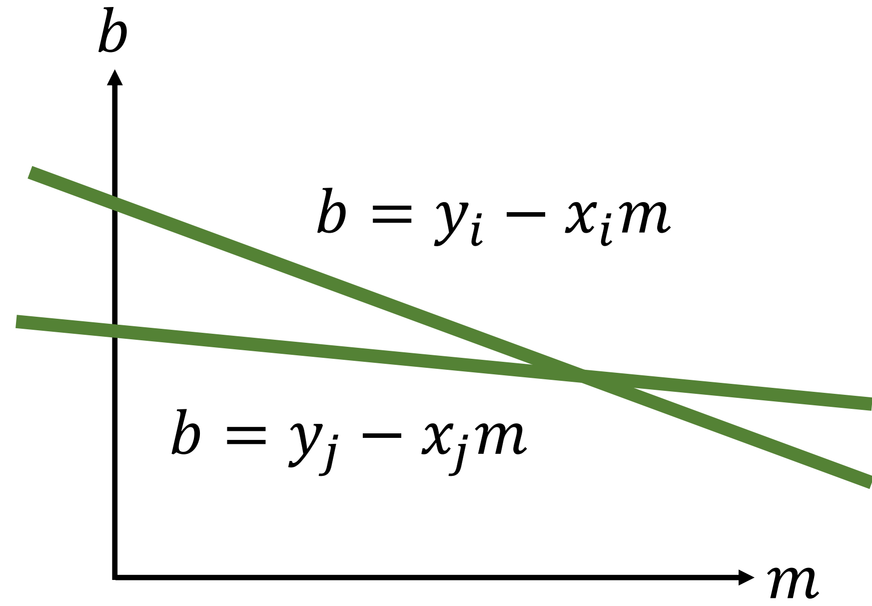


Hough (参数) 空间

同时包含两个点的直线的参数是什么?



图像空间



Hough (参数) 空间

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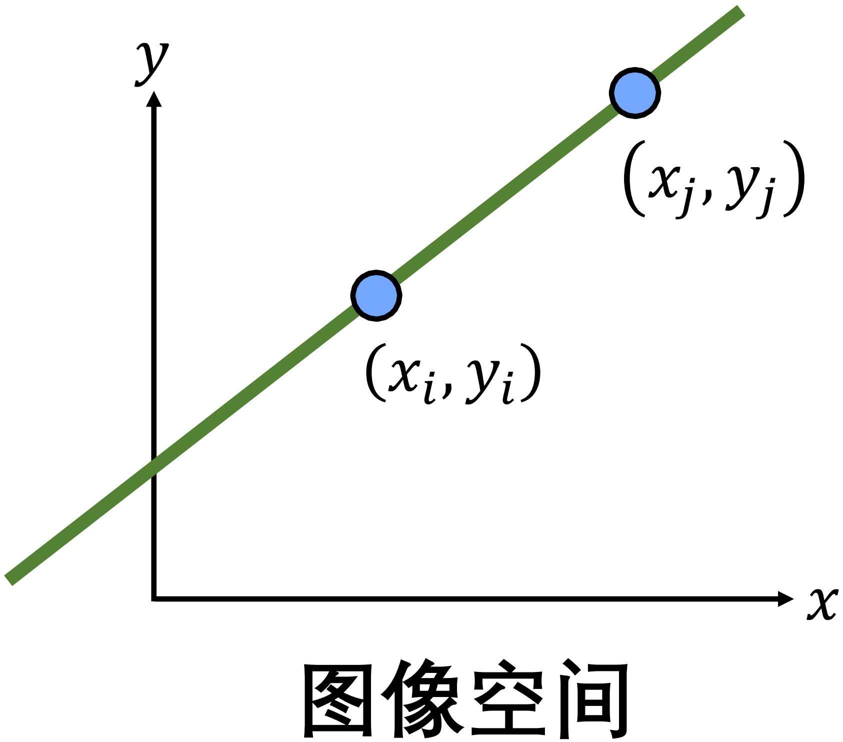
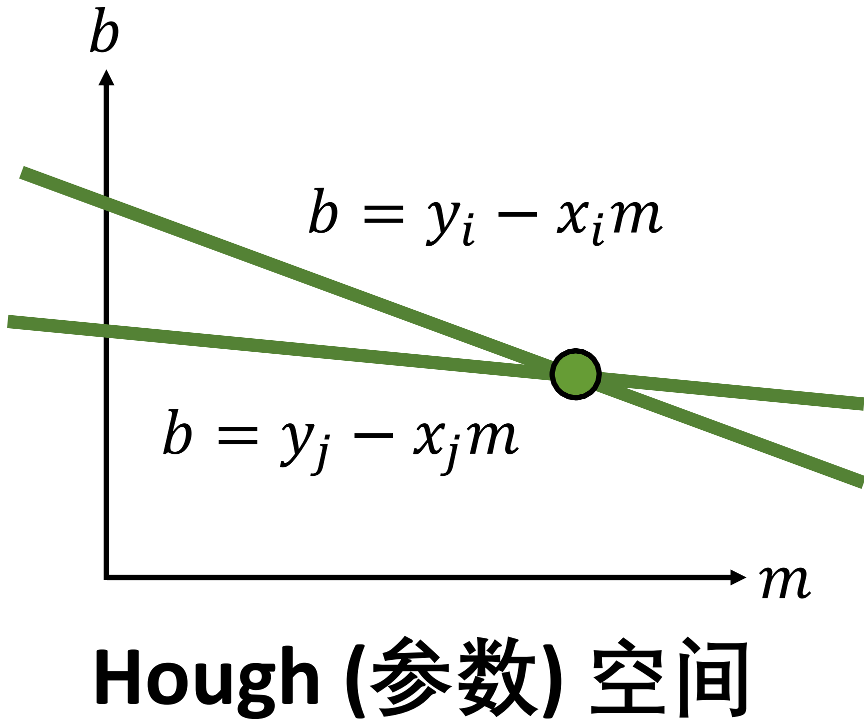
两条线在Hough空间中的交点





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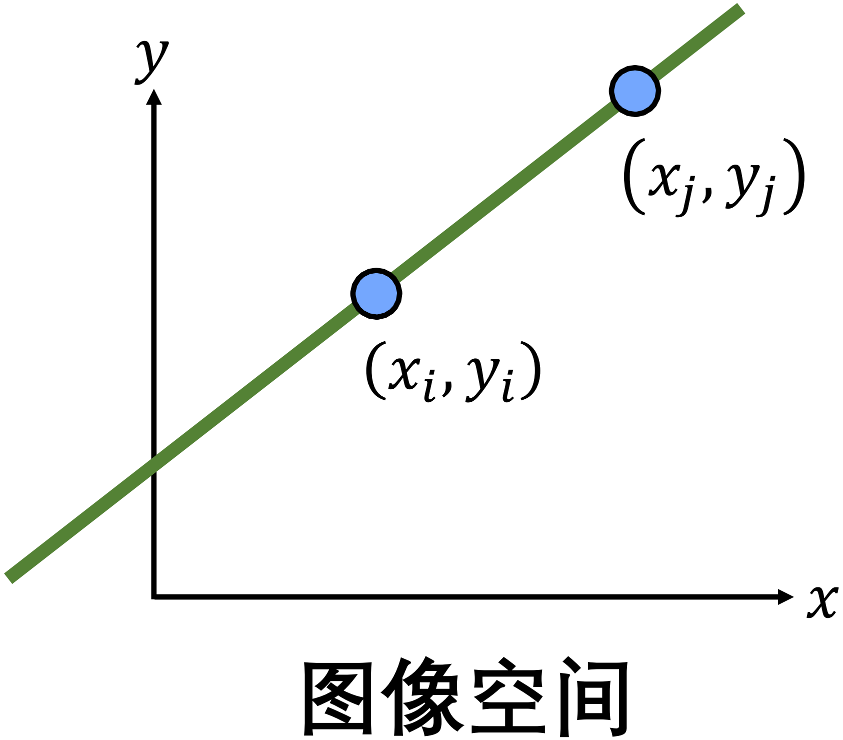
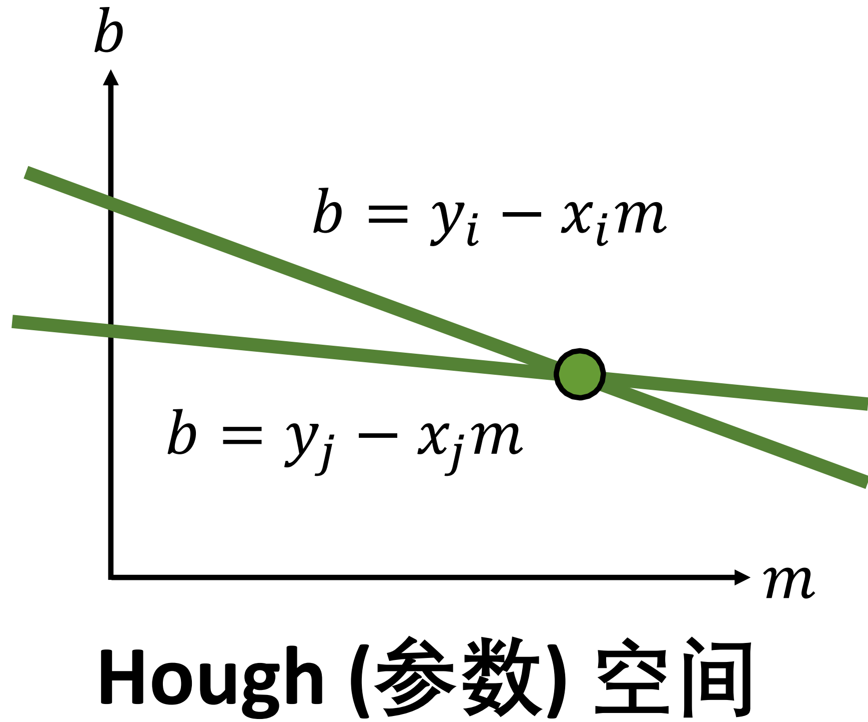
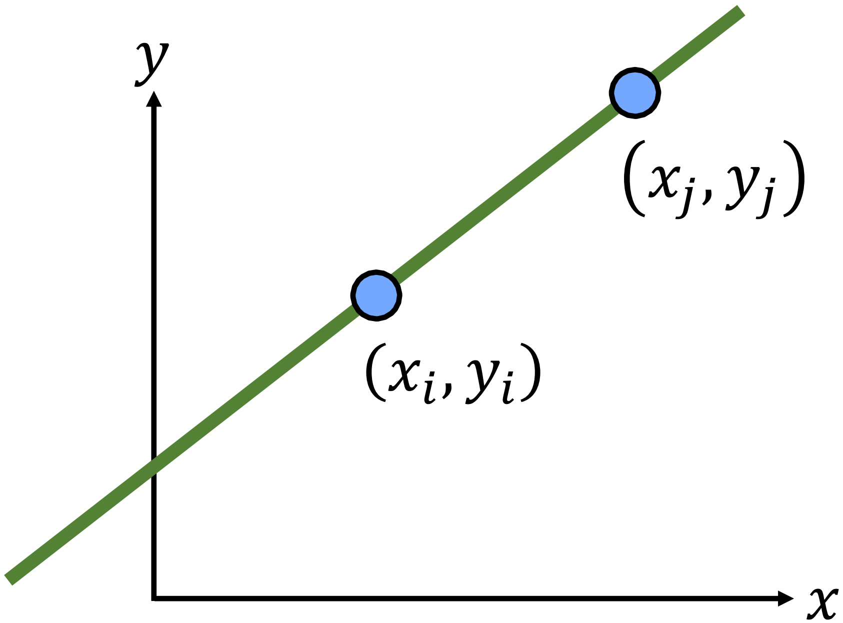
两条线在Hough空间中的交点

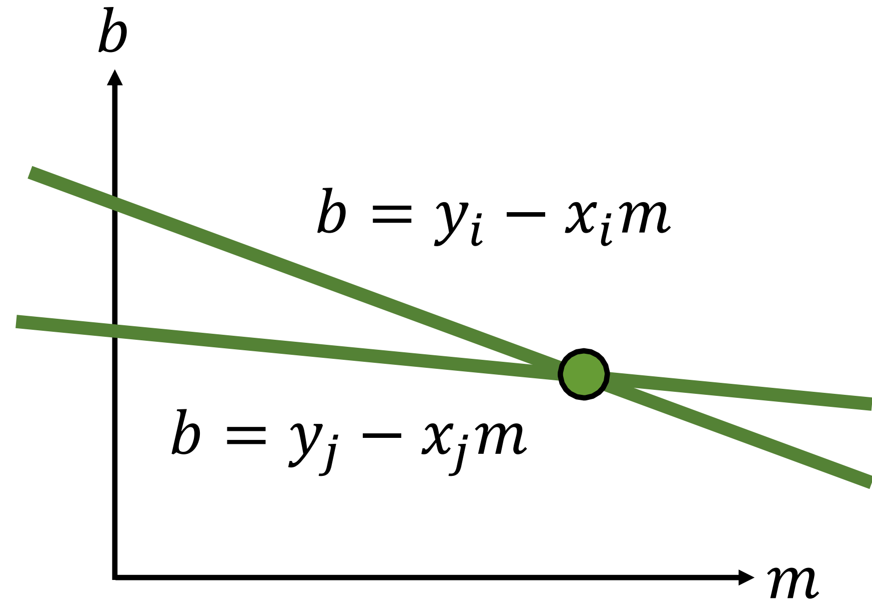




参数空间是连续的！如何应对？



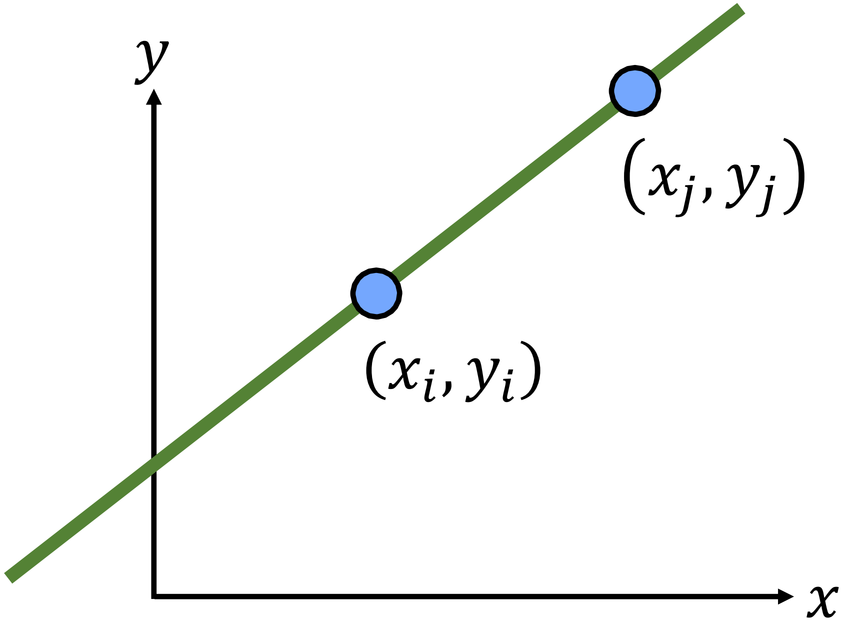
图像空间



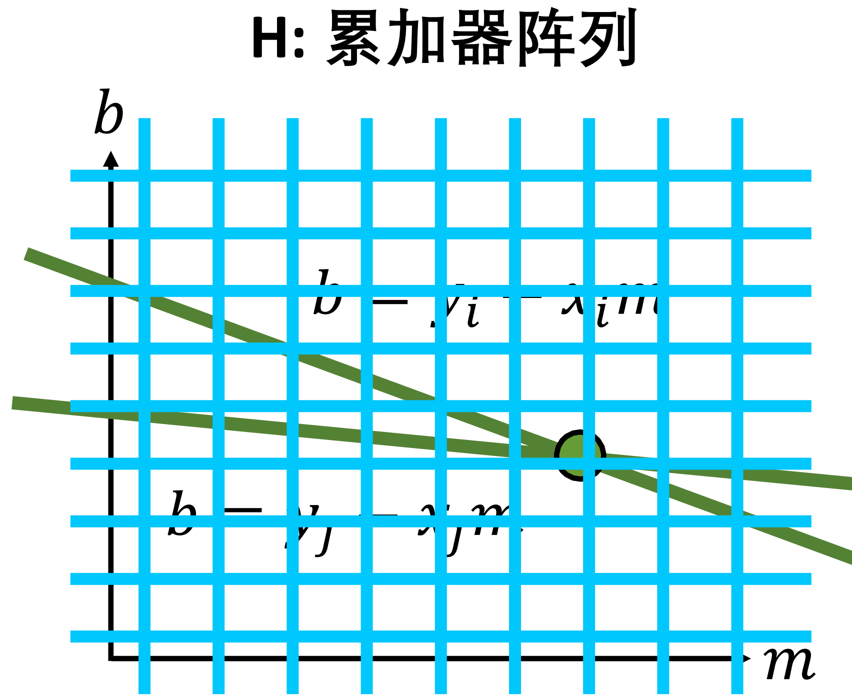
Hough (参数) 空间

参数空间是连续的！如何应对?

在一组离散的“候选人”所在区间内统计选票



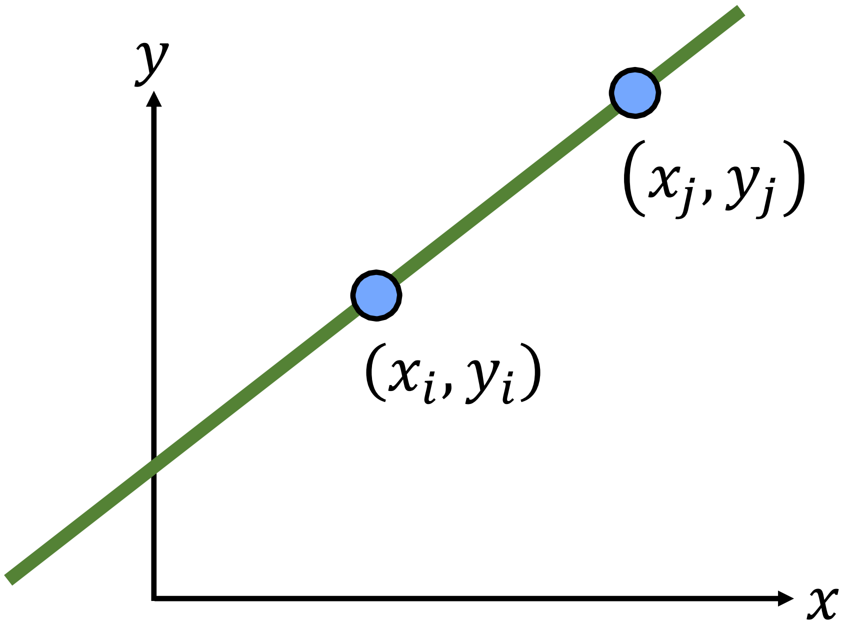
图像空间



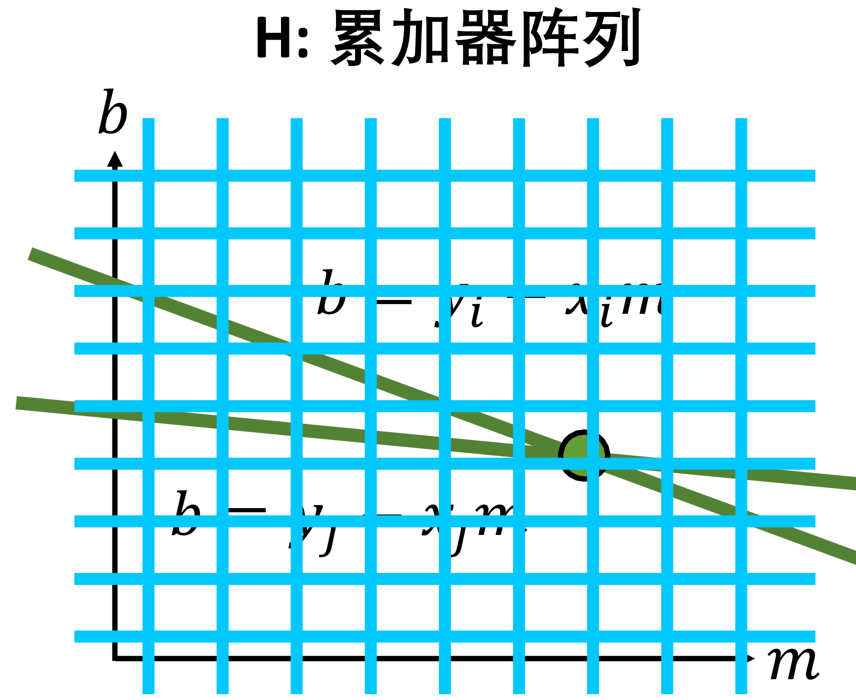
Hough (参数) 空间

参数空间是连续的！如何应对？

在一组离散的“候选人”所在区间内统计选票



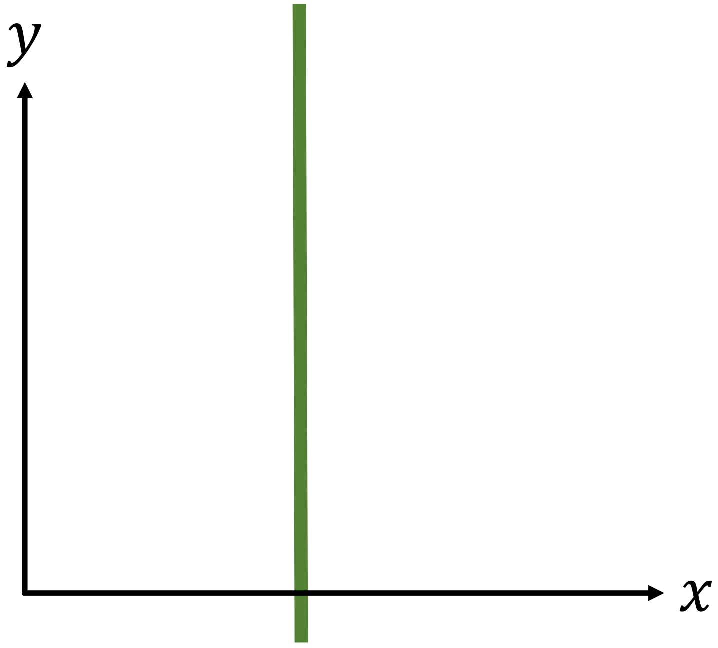
图像空间



Hough (参数) 空间

局部峰值对应于检测到的直线

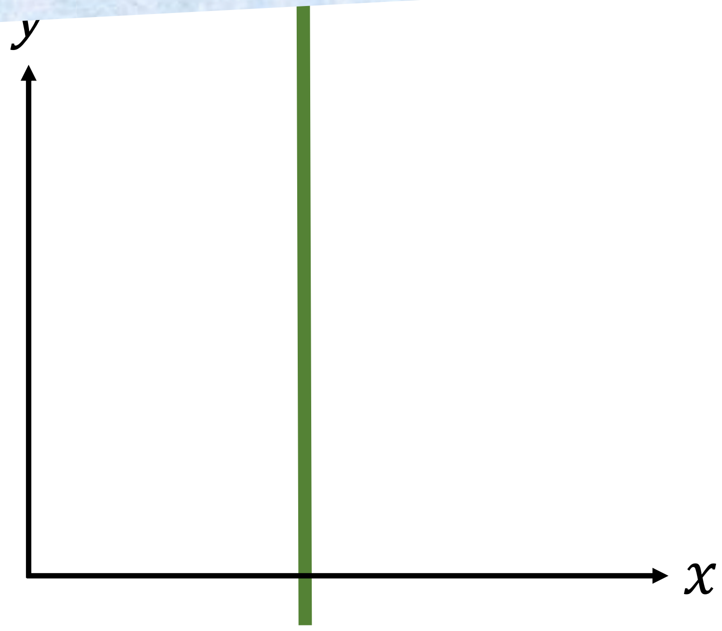
斜截式



图像空间

斜截式

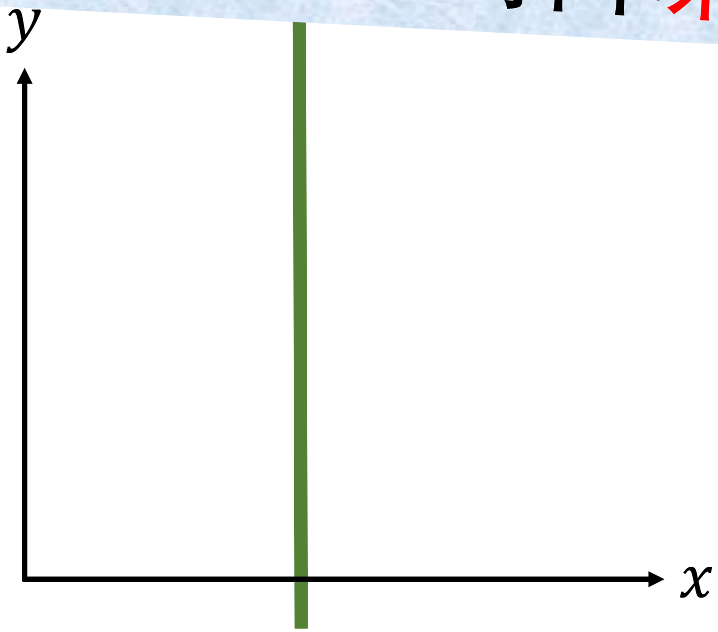
垂直直线的方程是什么？



图像空间

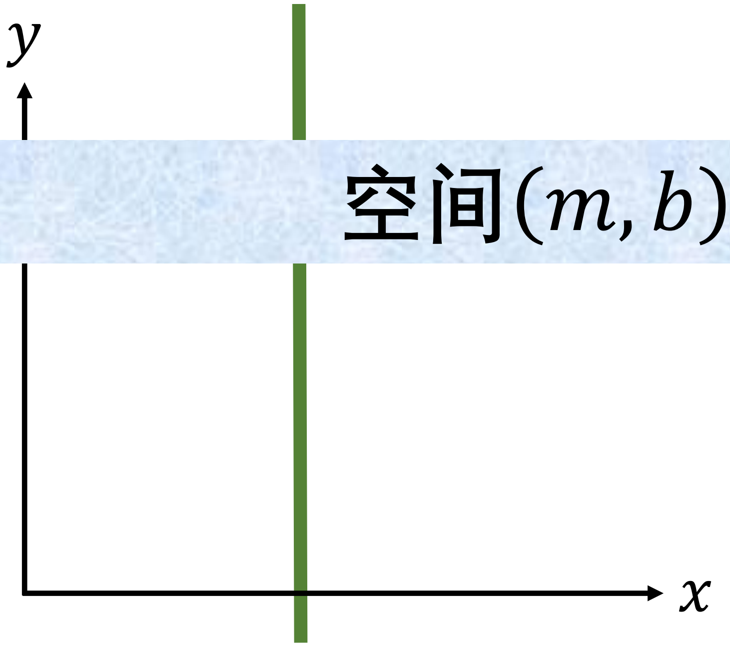
斜截式

斜率未定义



图像空间

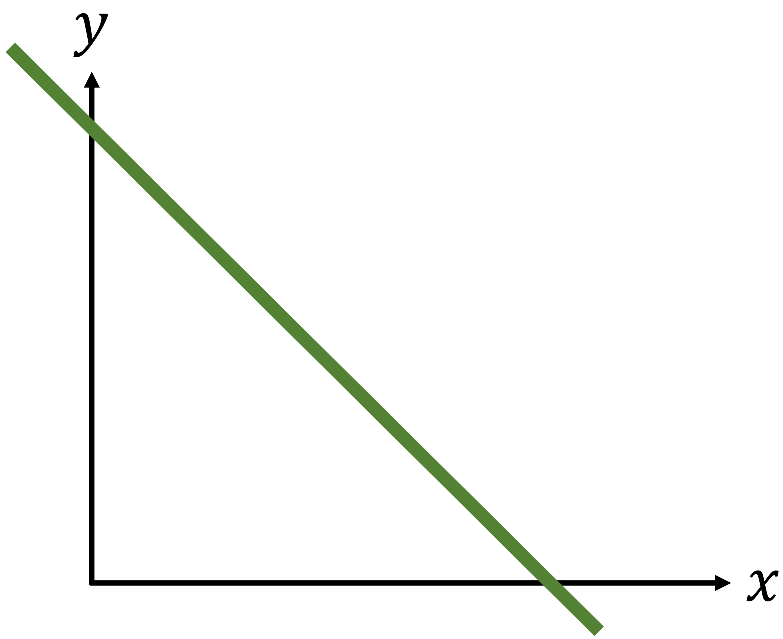
斜截式



空间(m,b)是无界的

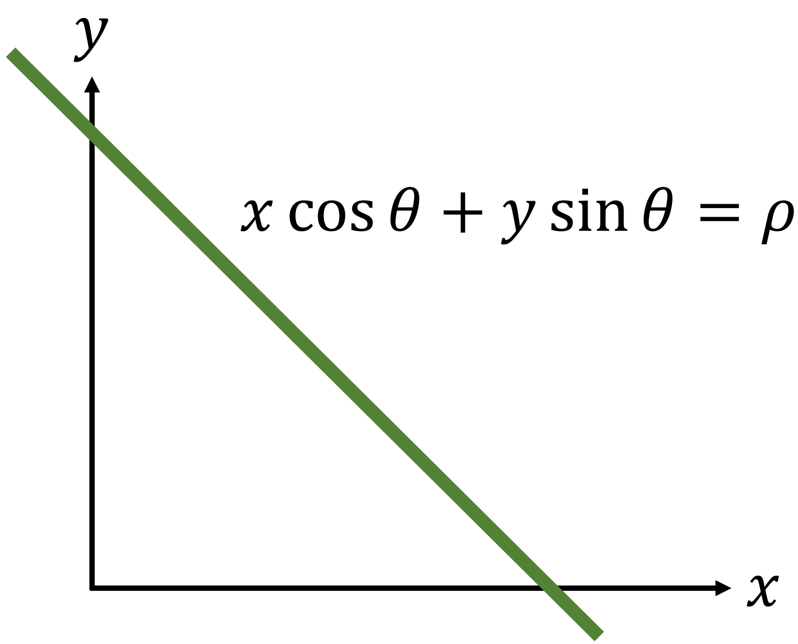
图像空间

极坐标表示



图像空间

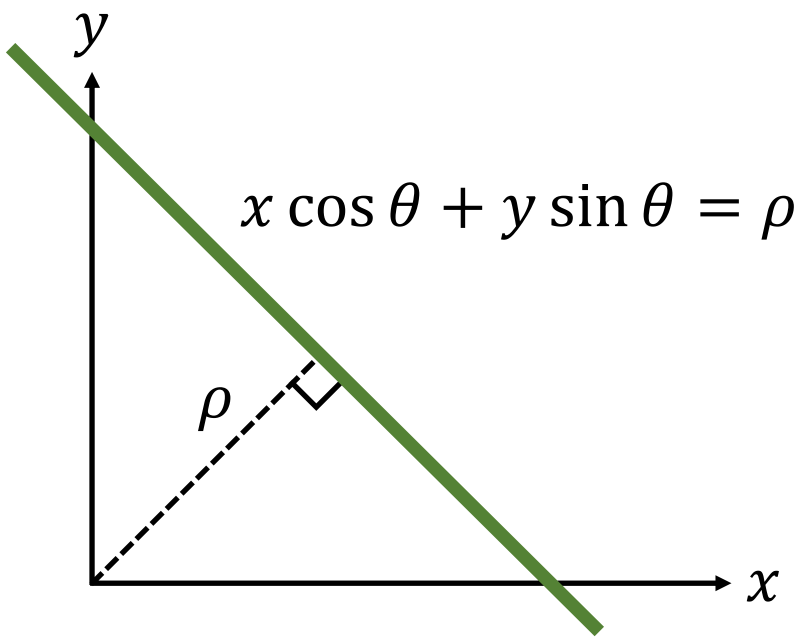
极坐标表示



$$x\cos\theta + y\sin\theta = \rho$$

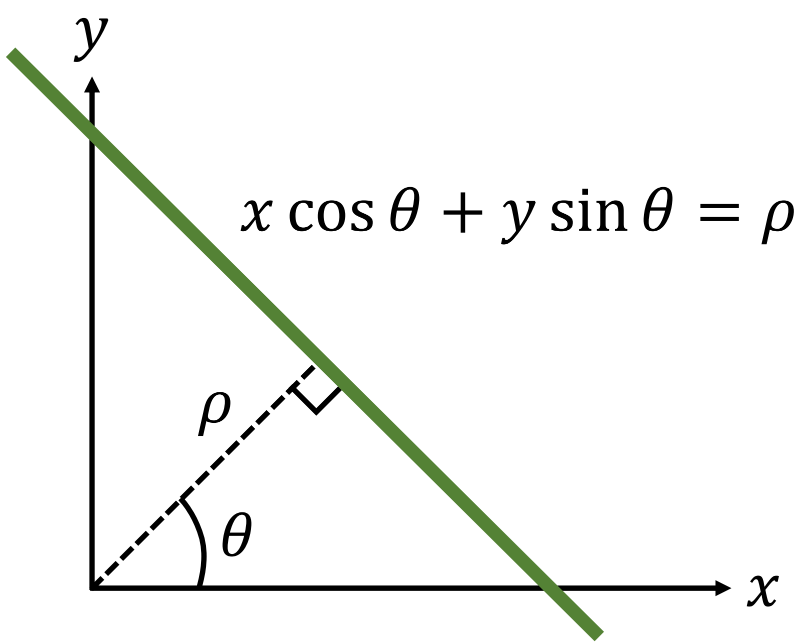
图像空间

极坐标表示



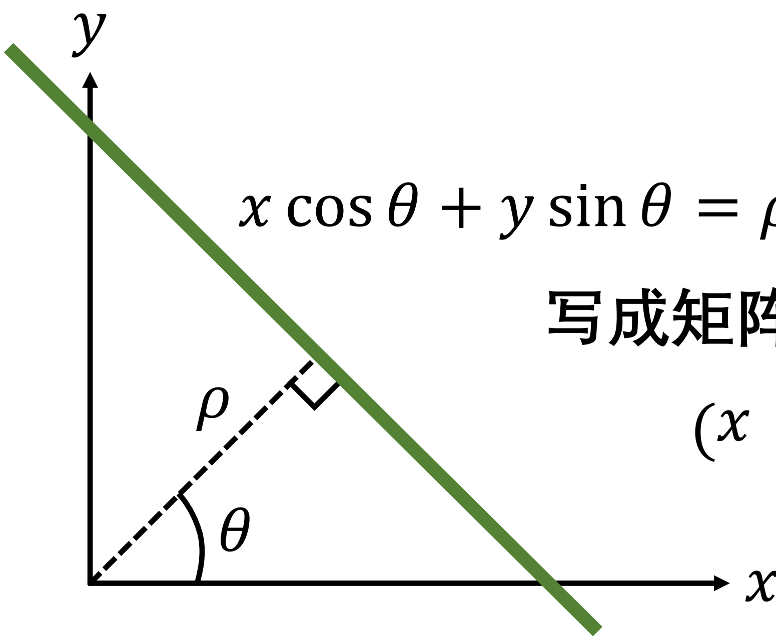
图像空间

极坐标表示



图像空间

极坐标表示



$$x\cos\theta + y\sin\theta = \rho$$

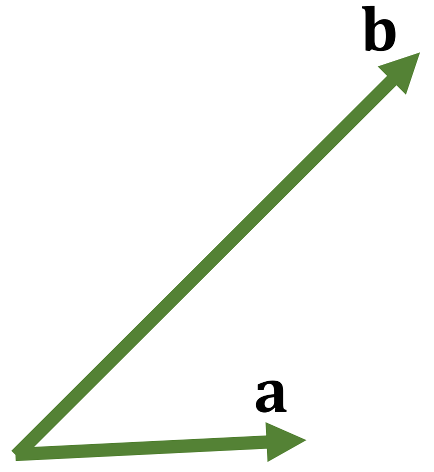
写成矩阵形式

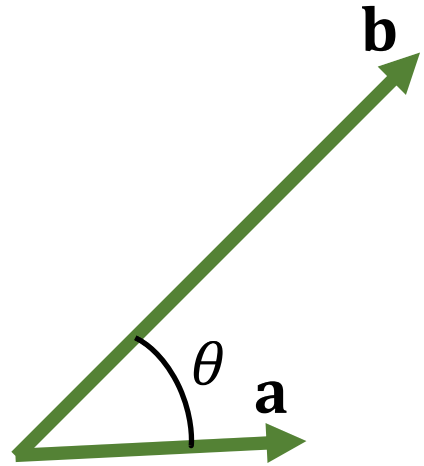
$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

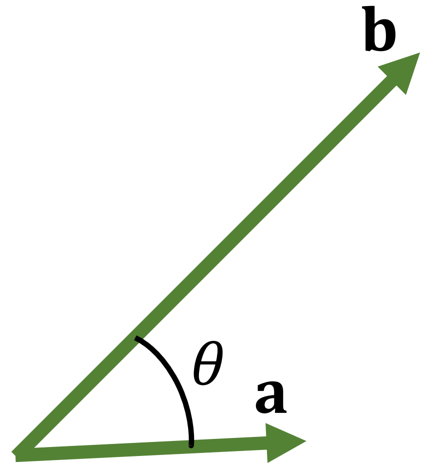
图像空间

回顾： 线性代数

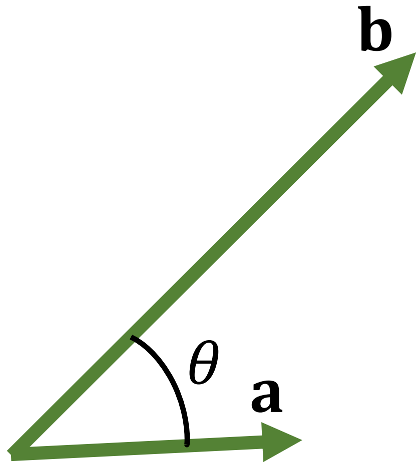

a


b
a

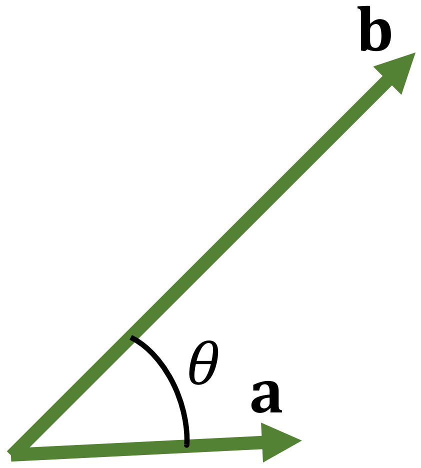

b
θ
a



$$\mathbf{a} \cdot \mathbf{b}$$

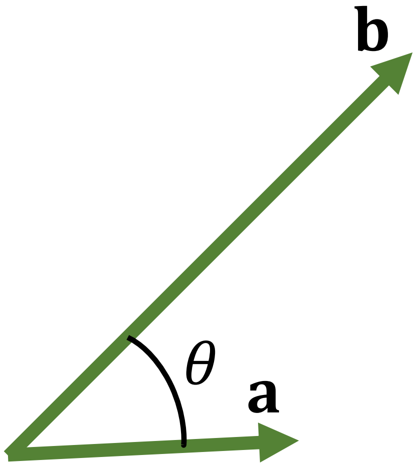


$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$



$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

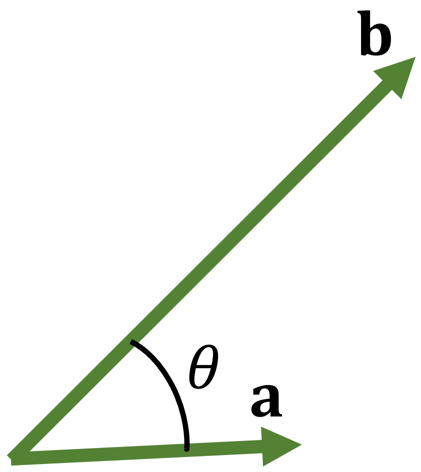
假设 $\|\mathbf{a}\| = 1$



$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

假设 $\|\mathbf{a}\| = 1$

$$= \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

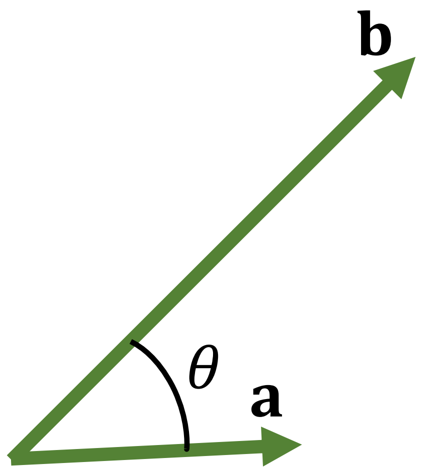


$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos \boldsymbol{\theta}$$

假设 $\|\mathbf{a}\| = 1$

$$= \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

几何解释是什么?

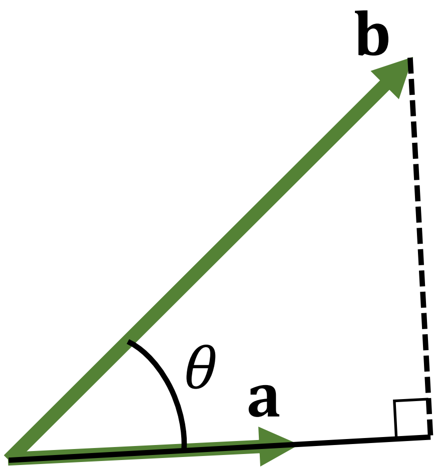


$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

假设 $\|\mathbf{a}\| = 1$

$$= \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

b到a投影的长度

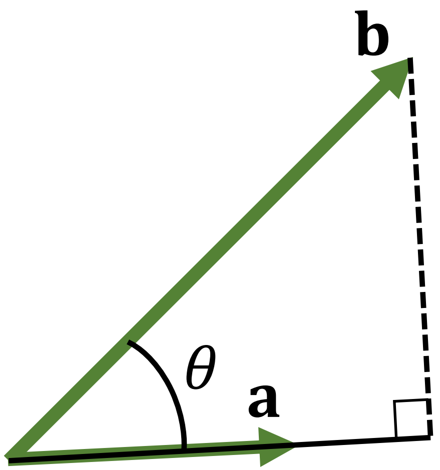


$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

假设 $\|\mathbf{a}\| = 1$

$$= \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

b到a投影的长度



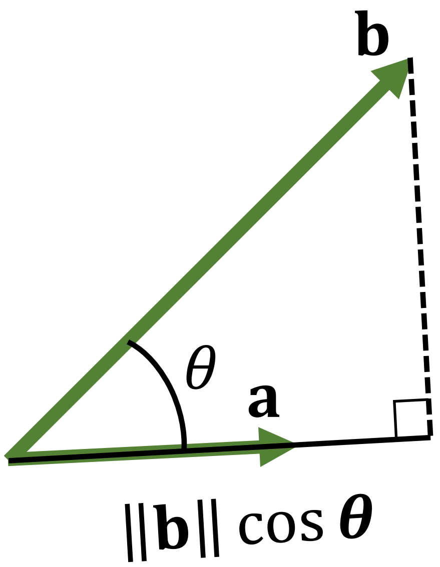
$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

假设 $\|\mathbf{a}\| = 1$

$$= \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

b到a投影的长度

向量投影



$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

假设 $\|\mathbf{a}\| = 1$

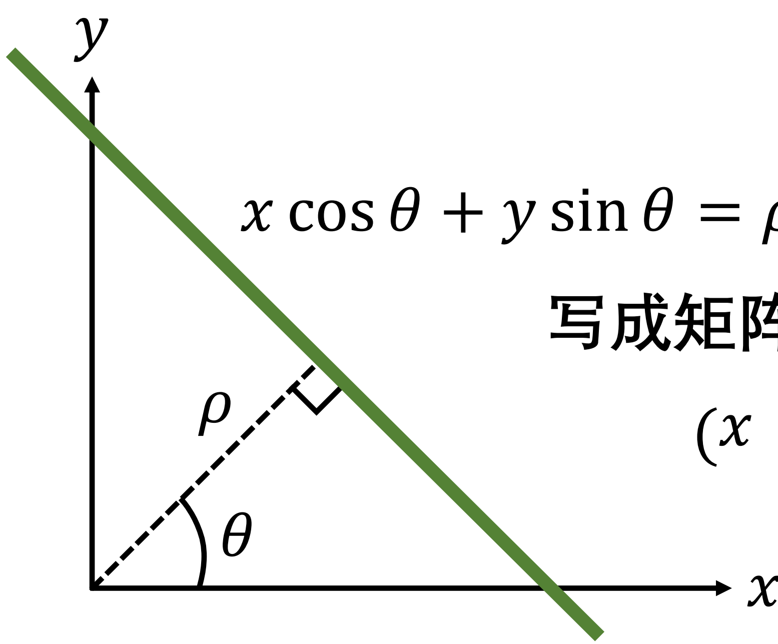
$$= \|\mathbf{b}\| \cos \boldsymbol{\theta}$$

b到a投影的长度

回顾：线性代数

极坐标表示

$$x\cos\theta + y\sin\theta = \rho$$

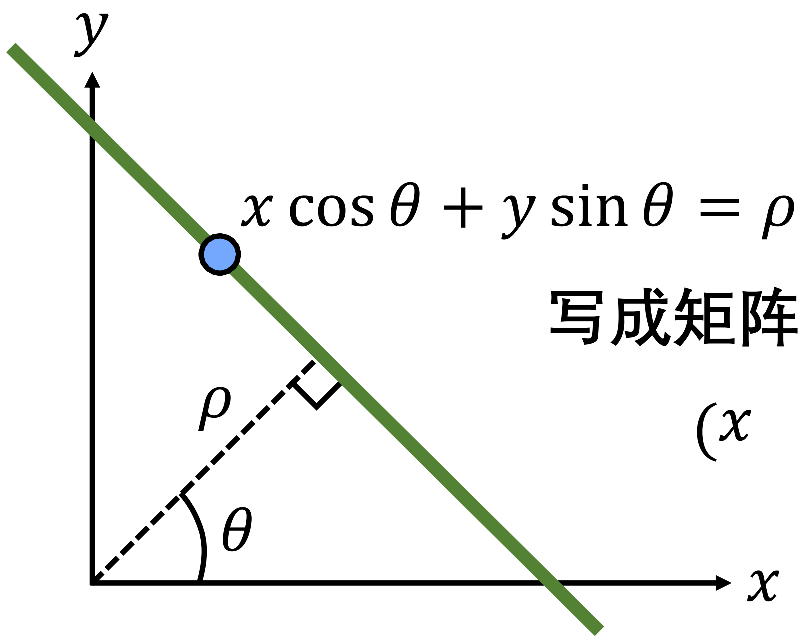


图像空间

写成矩阵形式

$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

极坐标表示



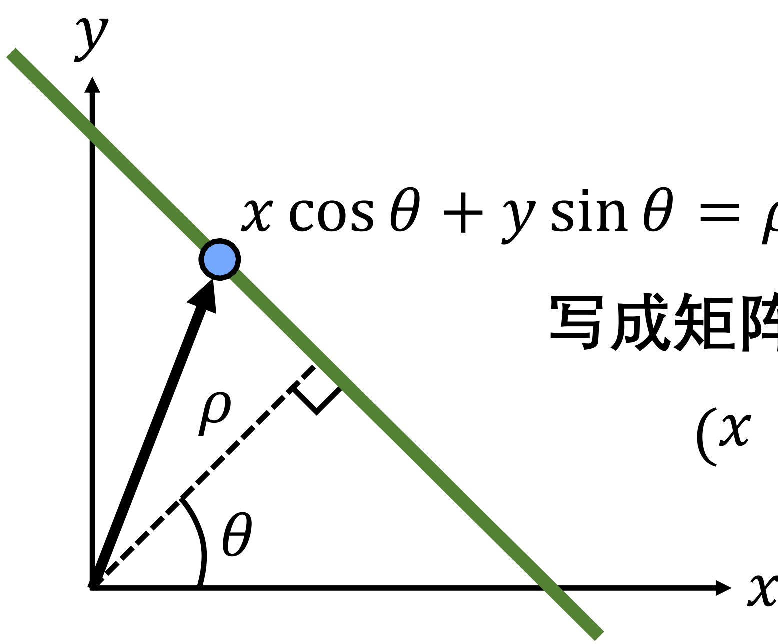
$$x\cos\theta + y\sin\theta = \rho$$

写成矩阵形式

$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

图像空间

极坐标表示

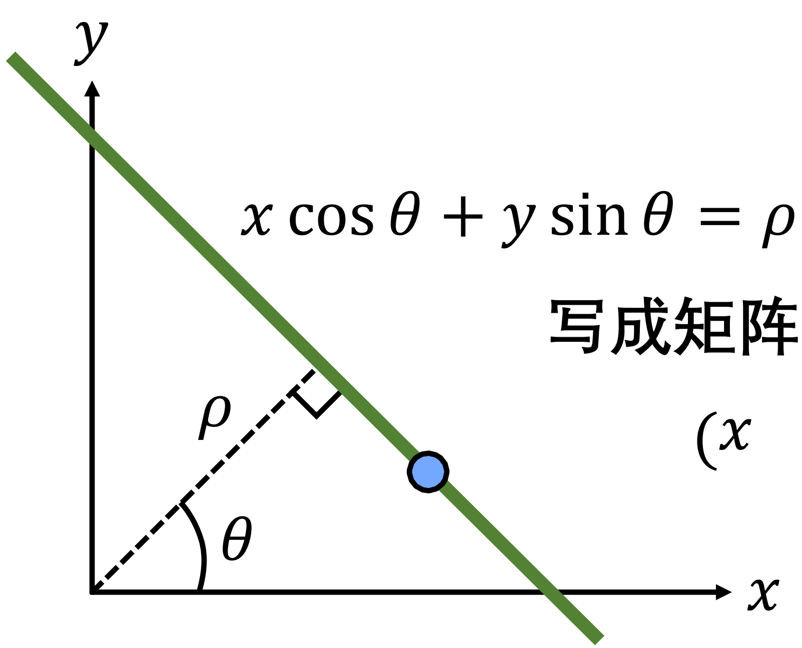


图像空间

写成矩阵形式

$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

极坐标
表示



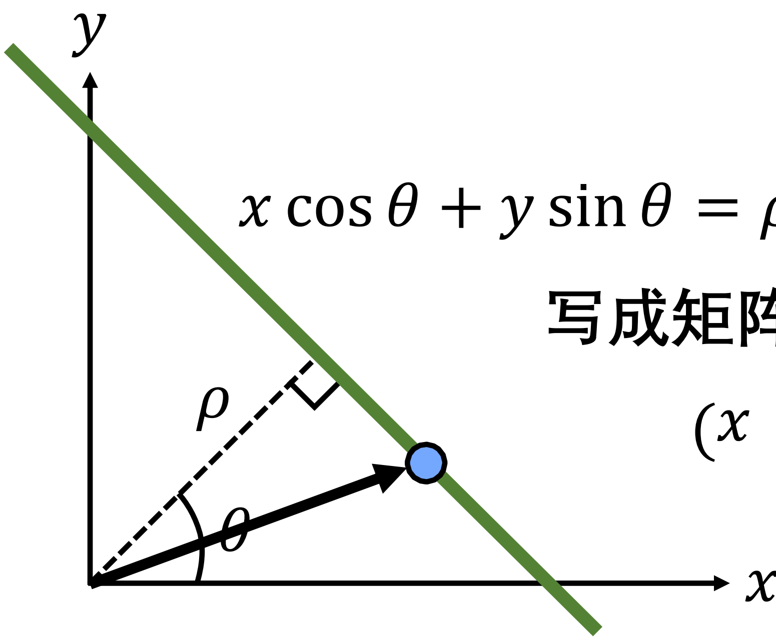
$$x\cos\theta + y\sin\theta = \rho$$

写成矩阵形式

$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

图像空间

极坐标表示



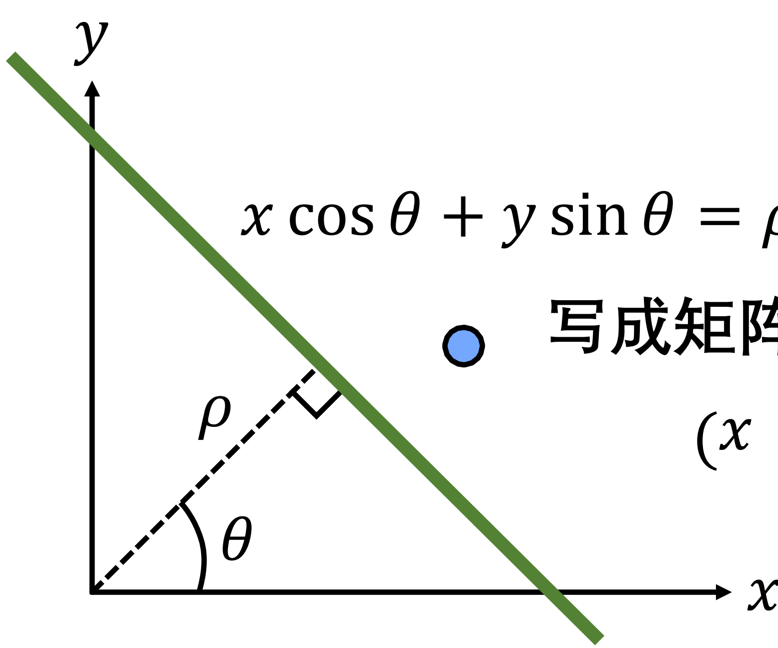
$$x\cos\theta + y\sin\theta = \rho$$

写成矩阵形式

$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

图像空间

极坐标表示



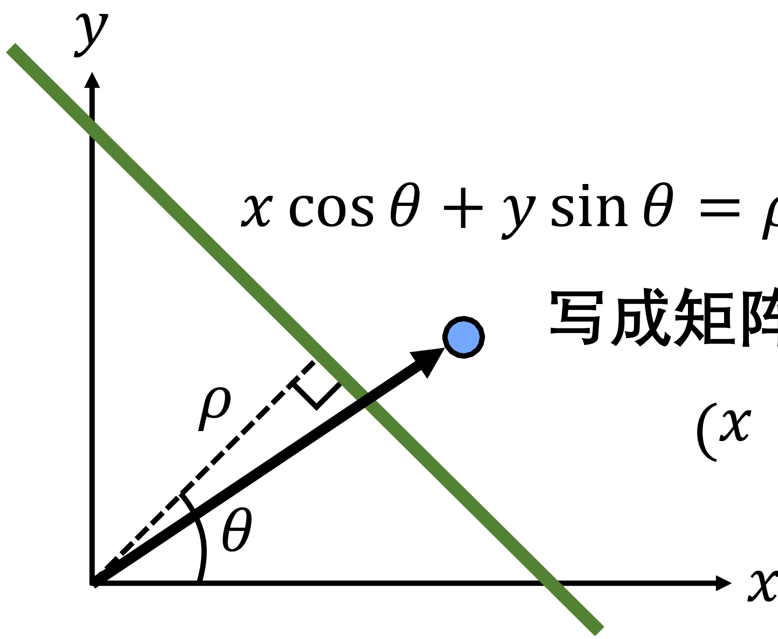
$$x\cos\theta + y\sin\theta = \rho$$

- 写成矩阵形式

$$(x \quad y)(\cos\theta \quad \sin\theta)^{\mathrm{T}} = \rho$$

图像空间

极坐标表示



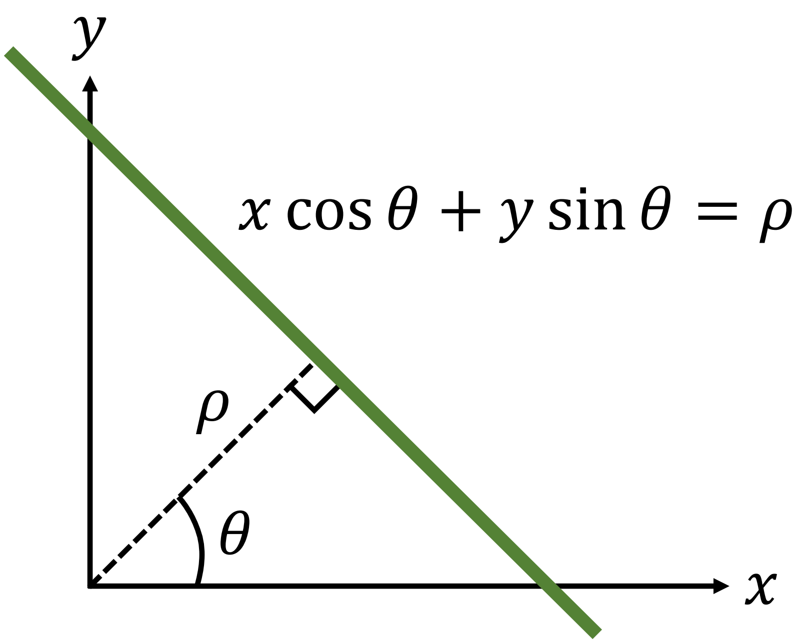
$$x\cos\theta + y\sin\theta = \rho$$

图像空间

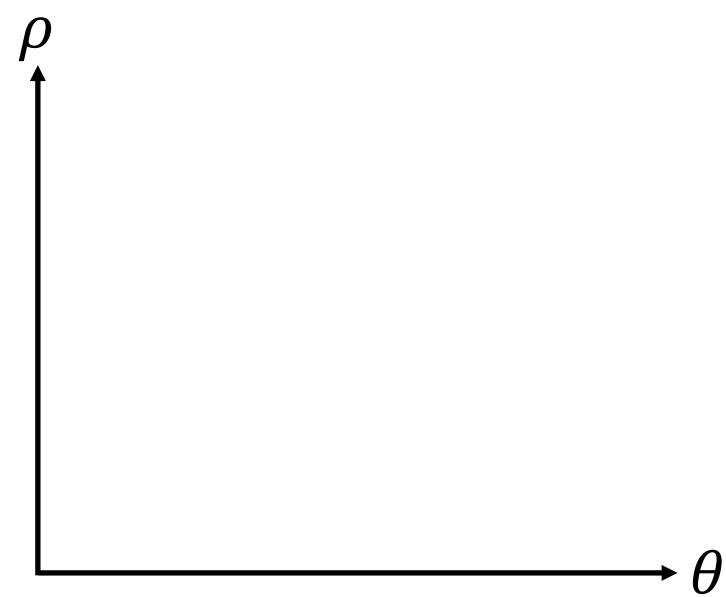
写成矩阵形式

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix}^{\mathrm{T}} = \rho$$

极坐标表示

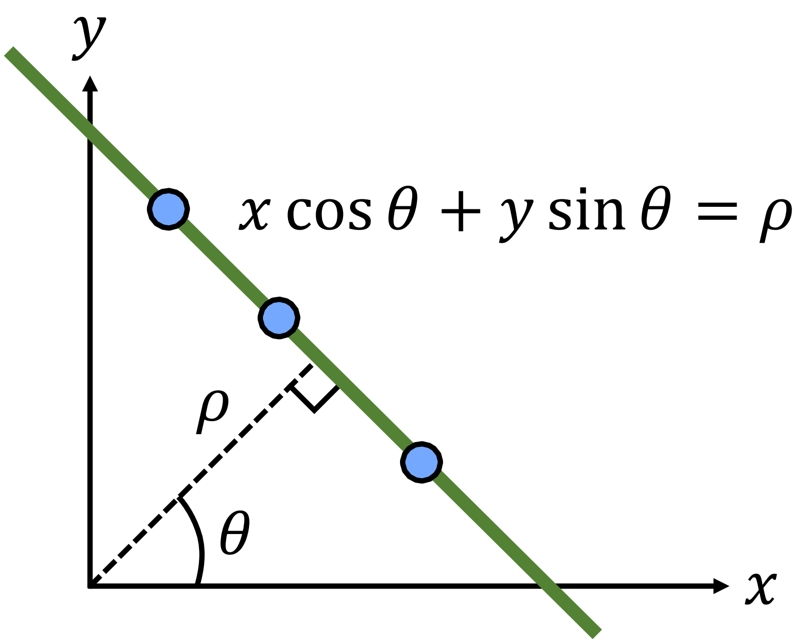


图像空间



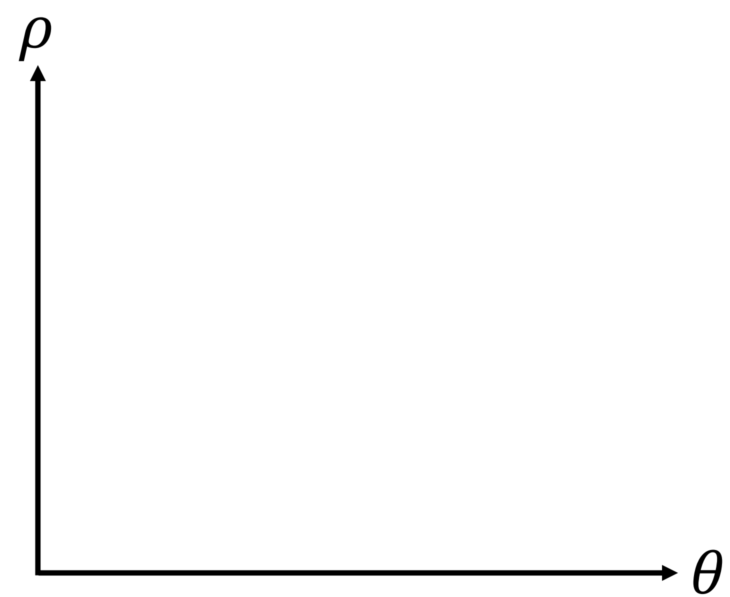
Hough (参数) 空间

极坐标
表示



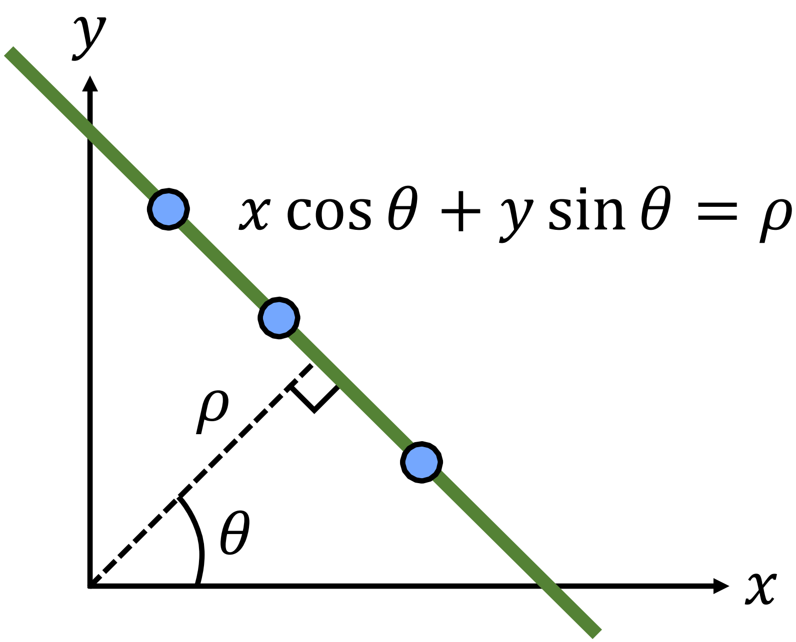
$x\cos\theta + y\sin\theta = \rho$

图像空间



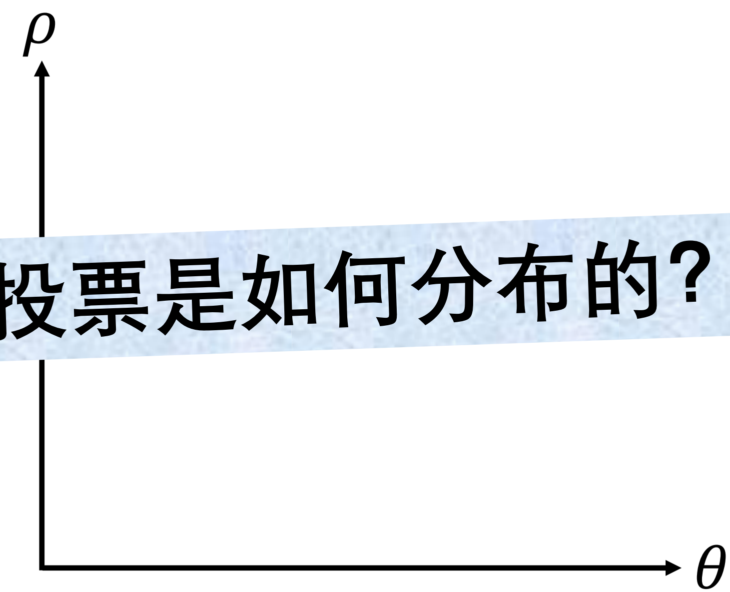
Hough (参数) 空间

极坐标表示



$$x\cos\theta + y\sin\theta = \rho$$

图像空间



投票是如何分布的?

Hough (参数) 空间

$$x_0 \cos\theta + y_0 \sin\theta = \rho$$

$$x_0 \cos\theta + y_0 \sin\theta = \rho$$

$$\rho = \sqrt{x_0^2 + y_0^2}\left(\frac{x_0}{\sqrt{x_0^2 + y_0^2}}\cos\theta + \frac{y_0}{\sqrt{x_0^2 + y_0^2}}\sin\theta\right)$$

$$x_0 \cos\theta + y_0 \sin\theta = \rho$$

$$\rho = \sqrt{x_0^2 + y_0^2}\left(\frac{x_0}{\sqrt{x_0^2 + y_0^2}}\cos\theta + \frac{y_0}{\sqrt{x_0^2 + y_0^2}}\sin\theta\right)$$

令 $\phi = \tan^{-1}\left(\frac{y_0}{x_0}\right)$

$$x_0 \cos\theta + y_0 \sin\theta = \rho$$

$$\rho = \sqrt{x_0^2 + y_0^2}\left(\frac{x_0}{\sqrt{x_0^2 + y_0^2}}\cos\theta + \frac{y_0}{\sqrt{x_0^2 + y_0^2}}\sin\theta\right)$$

$$\cos\phi \qquad \sin\phi$$

令 $\phi = \tan^{-1}\left(\frac{y_0}{x_0}\right)$

$$x_0 \cos\theta + y_0 \sin\theta = \rho$$

$$\rho = \sqrt{x_0^2 + y_0^2}\left(\frac{x_0}{\sqrt{x_0^2 + y_0^2}}\cos\theta + \frac{y_0}{\sqrt{x_0^2 + y_0^2}}\sin\theta\right)$$

令 $\phi = \tan^{-1}\left(\frac{y_0}{x_0}\right)$

$$\rho = \sqrt{x_0^2 + y_0^2}(\cos\phi \cos\theta + \sin\phi \sin\theta)$$

$$x_0 \cos\theta + y_0 \sin\theta = \rho$$

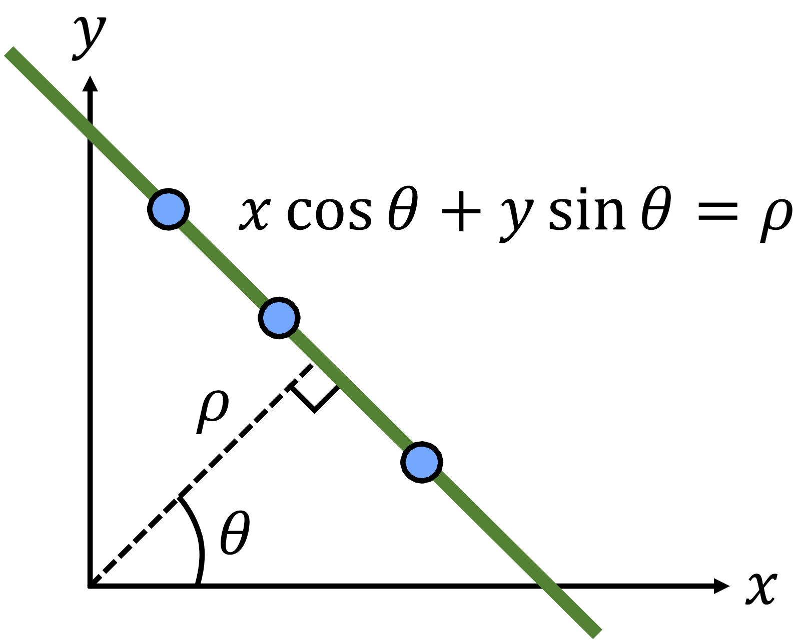
$$\rho = \sqrt{x_0^2 + y_0^2}\left(\frac{x_0}{\sqrt{x_0^2 + y_0^2}}\cos\theta + \frac{y_0}{\sqrt{x_0^2 + y_0^2}}\sin\theta\right)$$

令 $\phi = \tan^{-1}\left(\frac{y_0}{x_0}\right)$

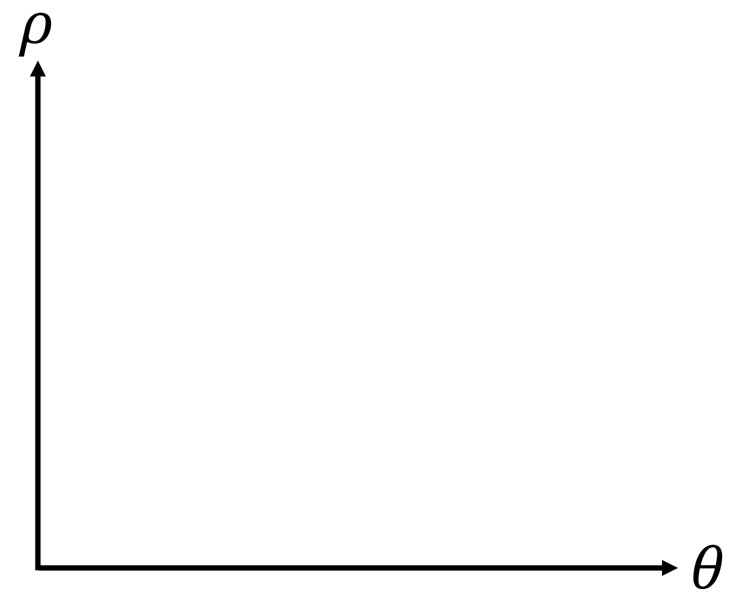
$$\rho = \sqrt{x_0^2 + y_0^2}(\cos\phi\cos\theta + \sin\phi\sin\theta)$$

$$\rho = \sqrt{x_0^2 + y_0^2}\cos(\theta - \phi)$$

极坐标
表示

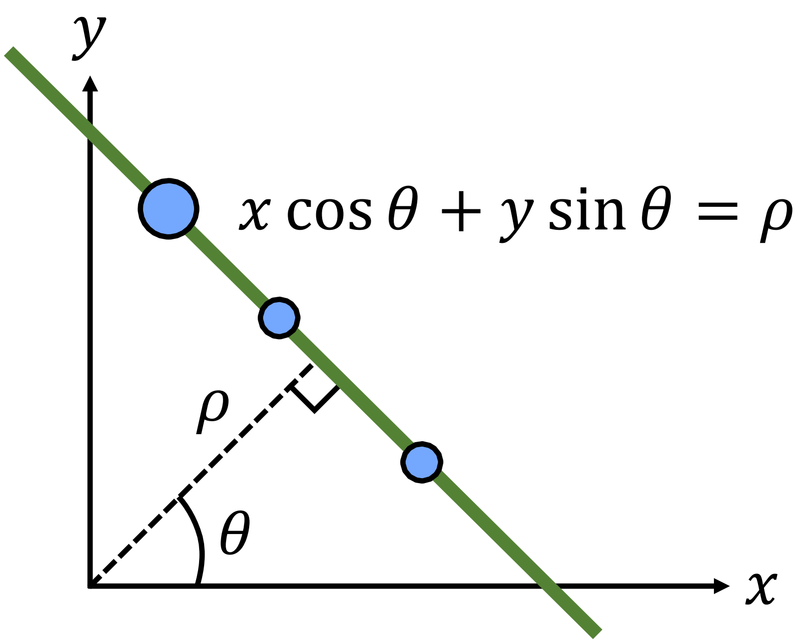


图像空间



Hough (参数) 空间

极坐标表示



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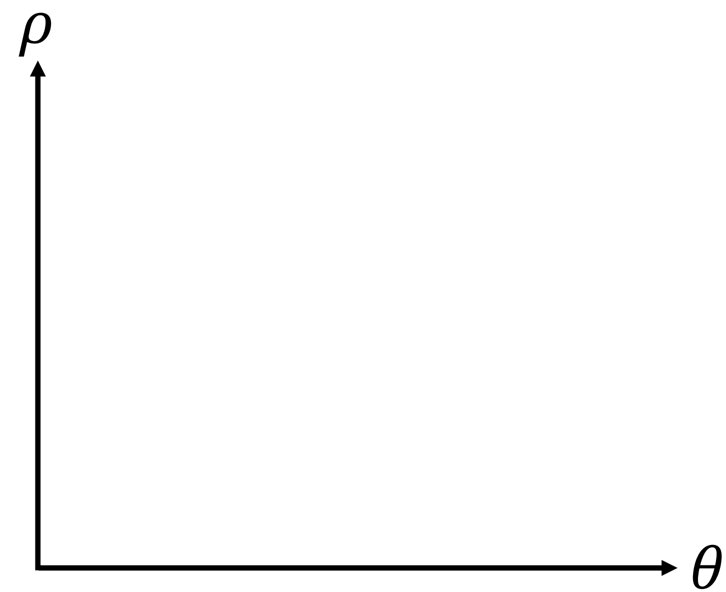
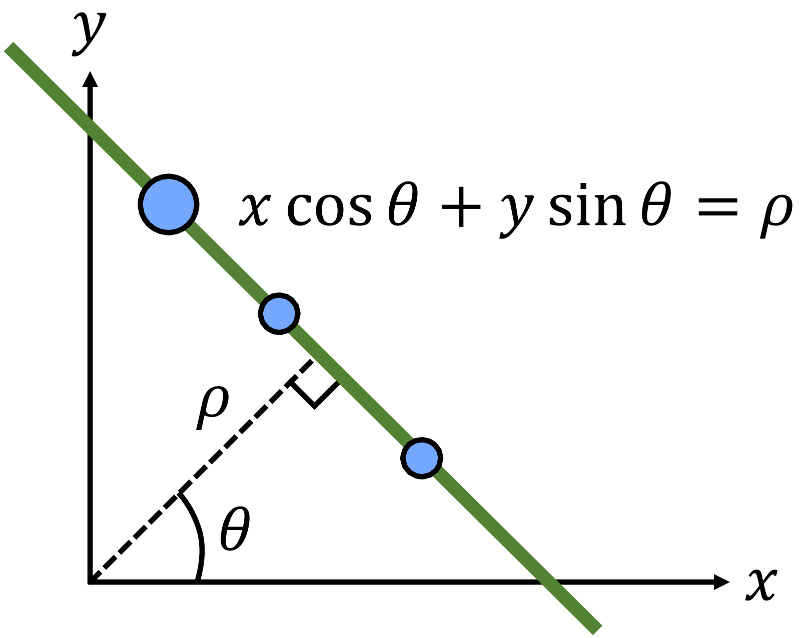
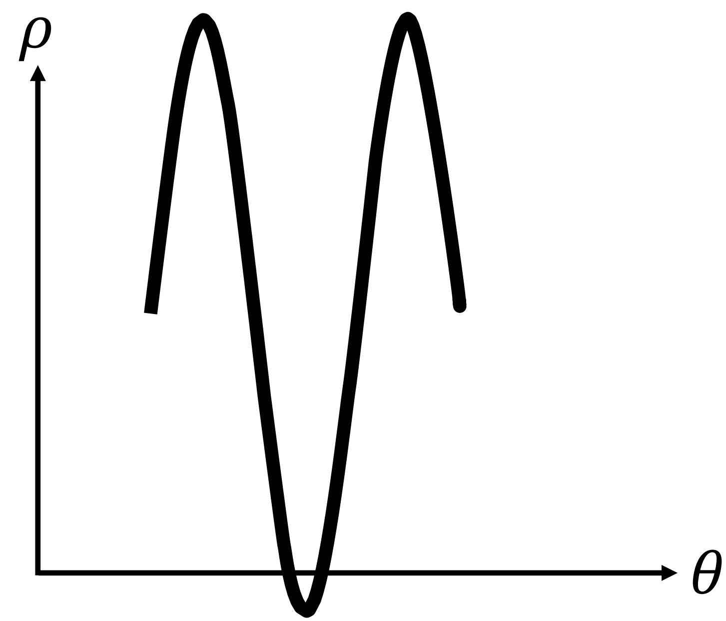


Hough (参数) 空间

极坐标表示

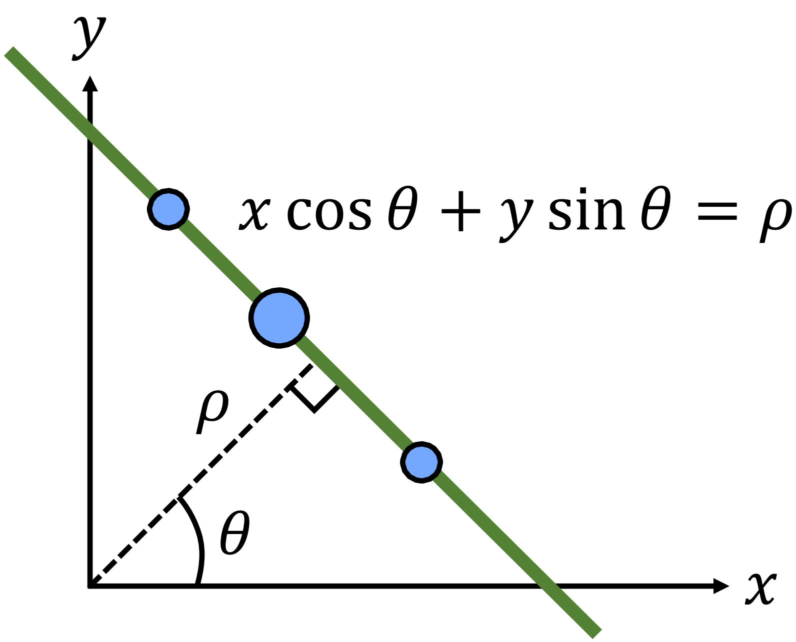


图像空间



Hough (参数) 空间

极坐标表示



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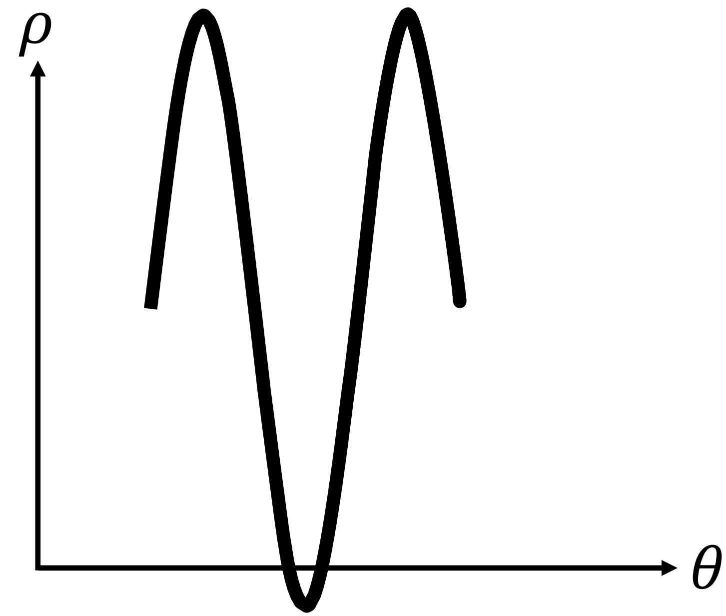
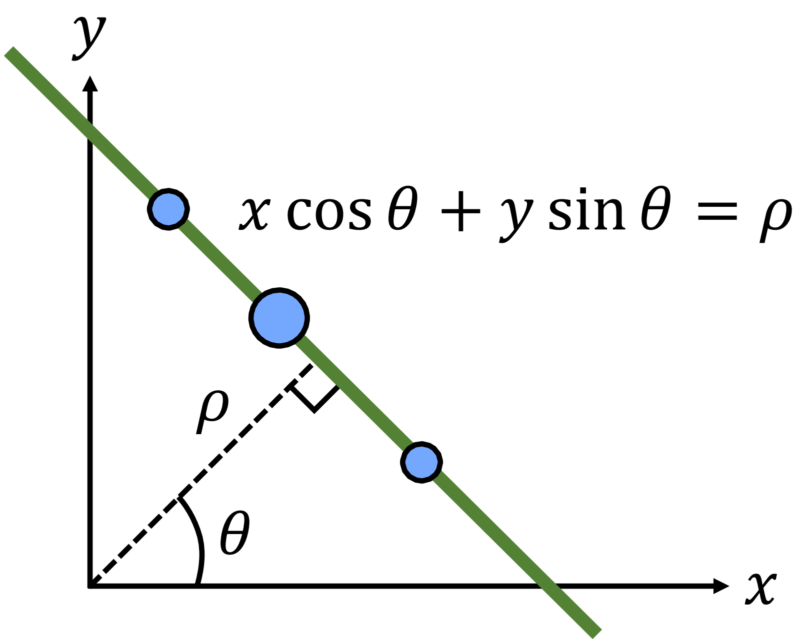


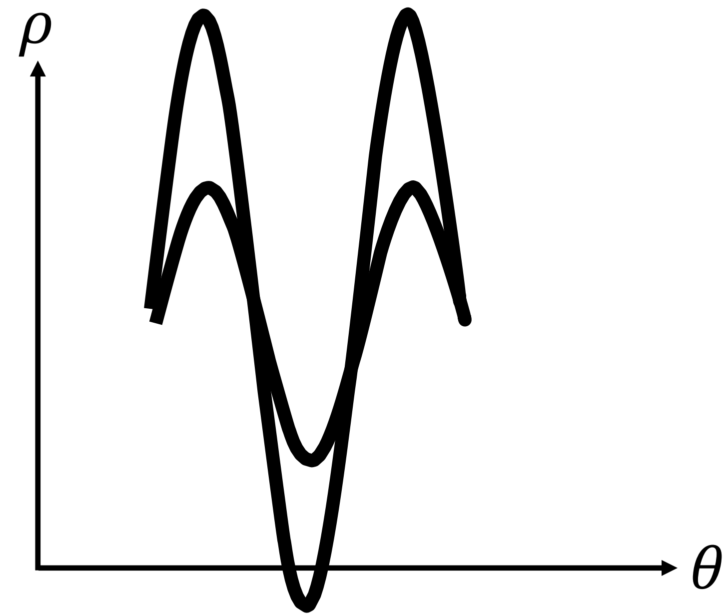
Hough (参数) 空间

极坐标
表示



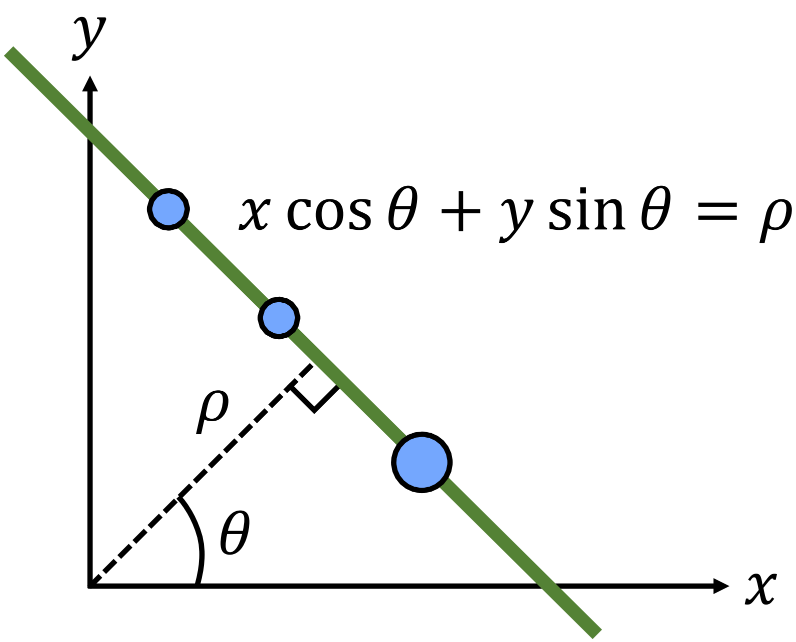
$x\cos\theta + y\sin\theta = \rho$

图像空间

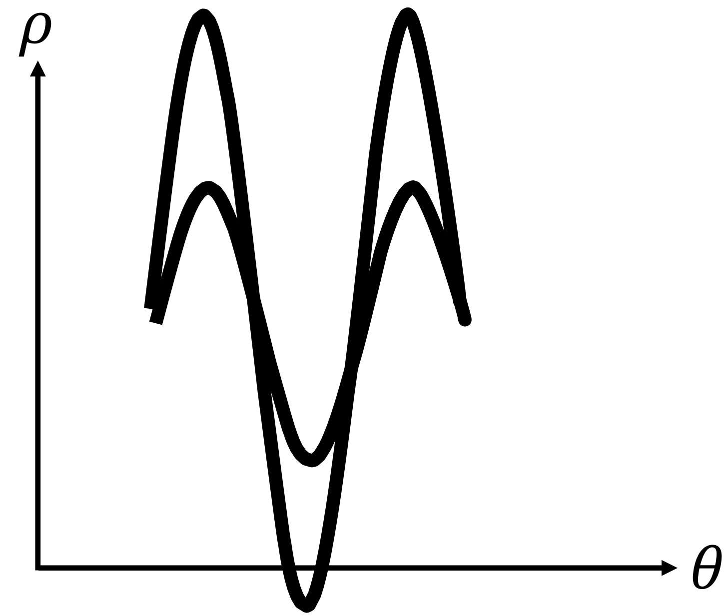


Hough (参数) 空间

极坐标表示

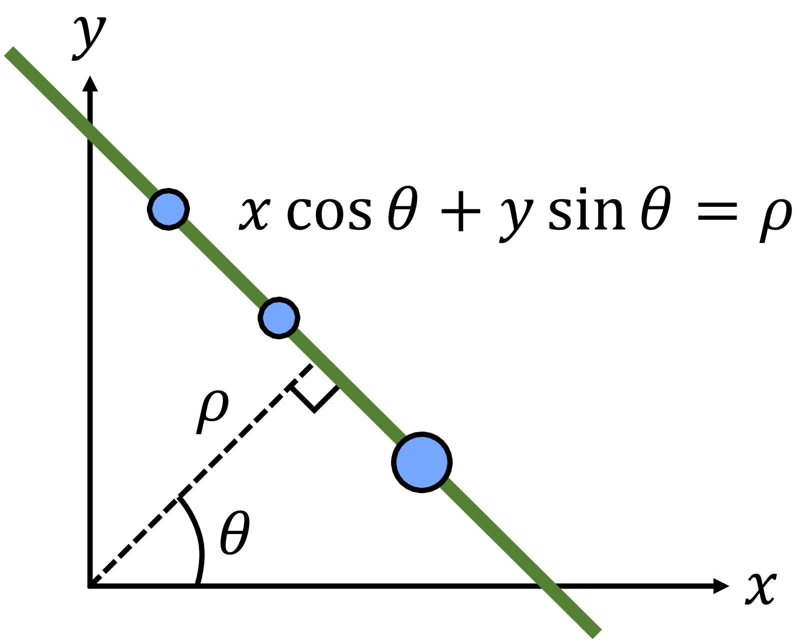


图像空间

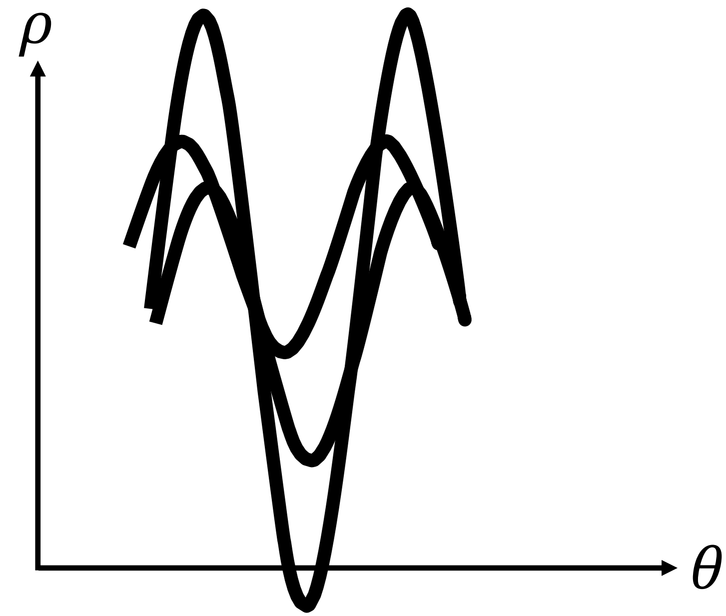


Hough (参数) 空间

极坐标表示

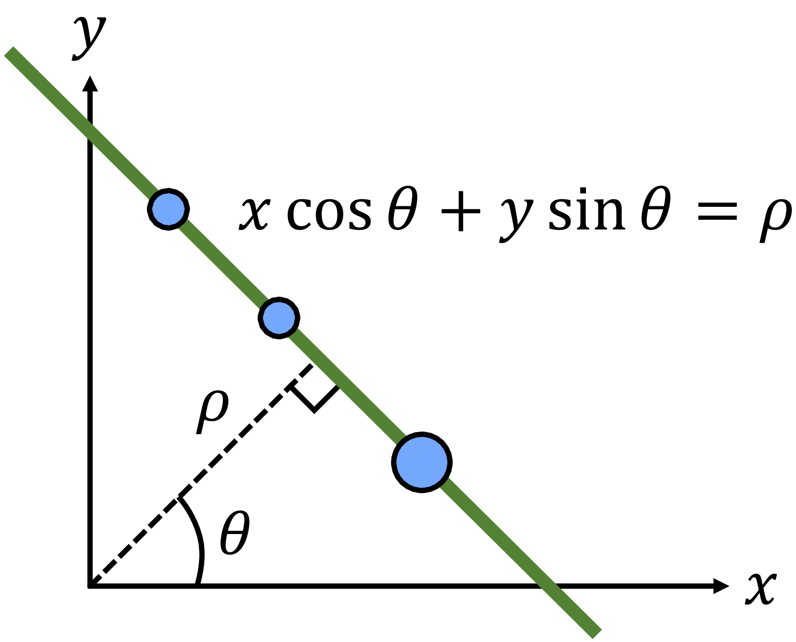


图像空间

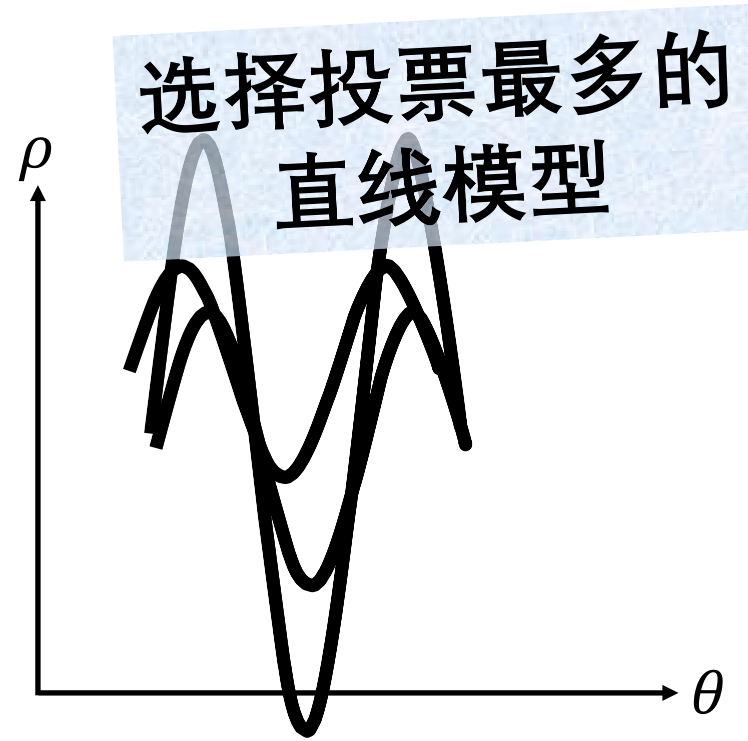


Hough (参数) 空间

极坐标表示

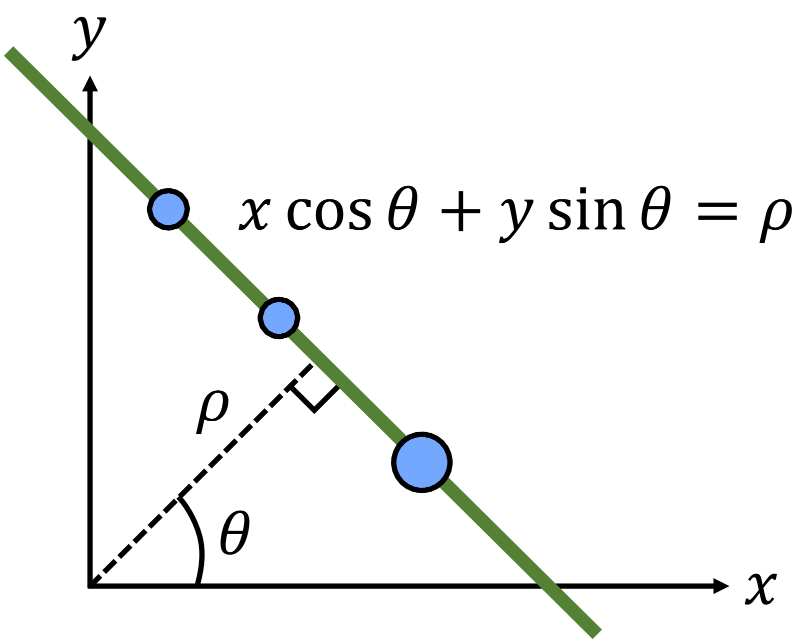


图像空间



Hough (参数) 空间

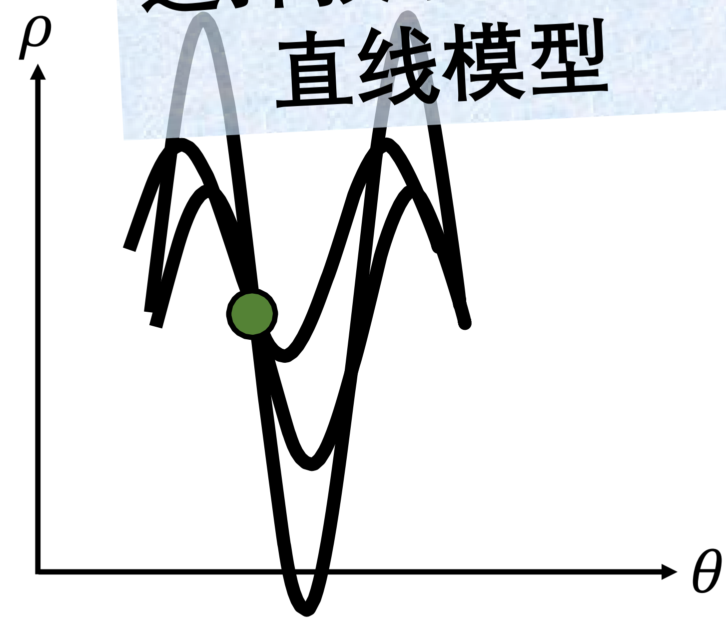
极坐标表示



$$x\cos\theta + y\sin\theta = \rho$$

图像空间

选择投票最多的直线模型



Hough (参数) 空间

Hough 直线

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

    foreach $\theta = \theta_{\min}$ to $\theta_{\max}$ **do**

        $\rho = x \cos\theta + y \sin\theta$

        $H[\rho, \theta] \mathrel{+}= 1$

    end

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho, \theta]$

Hough
直线

```
Initialize H[ρ, θ] = 0
foreach edgel e ∈ I[x, y] do
    foreach θ = θ_min to θ_max do
        ρ = x cos θ + y sin θ
        H[ρ, θ] += 1
    end
end
ρ*, θ* = argmax_{ρ,θ} H[ρ, θ]
```

每点投票数的时间复杂度?

Hough
直线

改进 #1

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

    foreach $\theta = \theta_{\text{min}}$ to θ_{max} **do**

        $\rho = x \cos\theta + y \sin\theta$

        $H[\rho, \theta] \mathrel{+}= 1$

    end

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho, \theta]$

Hough 直线

改进 #1

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

$\rho = x \cos\theta + y \sin\theta$

$H[\rho, \theta] \mathrel{+}= 1$

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho, \theta]$

Hough 直线

改进 #1

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

$\quad\quad \theta = \text{angle}(\nabla I[x, y])$

$\quad\quad \rho = x \cos\theta + y \sin\theta$

$\quad\quad H[\rho, \theta] \mathrel{+}= 1$

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho, \theta]$

Hough 直线

改进 #2

Initialize $H[\rho,\theta] = 0$

foreach edgel $e \in I[x,y]$ **do**

    $\theta = \mathrm{angle}(\nabla I[x,y])$

    $\rho = x\cos\theta + y\sin\theta$

    $H[\rho,\theta] += 1$

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho,\theta]$

Hough 直线

改进 #2

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

$\quad\theta = \mathrm{angle}(\nabla I[x, y])$

$\quad\rho = x \cos\theta + y \sin\theta$

$\quad H[\rho, \theta] \mathrel{+}= 1$

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho, \theta]$

Hough 直线

改进 #2

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

$\quad \theta = \mathrm{angle}(\nabla I[x, y])$

$\quad \rho = x \cos\theta + y \sin\theta$

$\quad H[\rho, \theta] \mathrel{+}=$

end

$\rho^*, \theta^* = \arg\max_{\rho,\theta} H[\rho, \theta]$

Hough 直线

改进 #2

Initialize $H[\rho, \theta] = 0$

foreach edgel $e \in I[x, y]$ **do**

$\quad \theta = \text{angle}(\nabla I[x, y])$

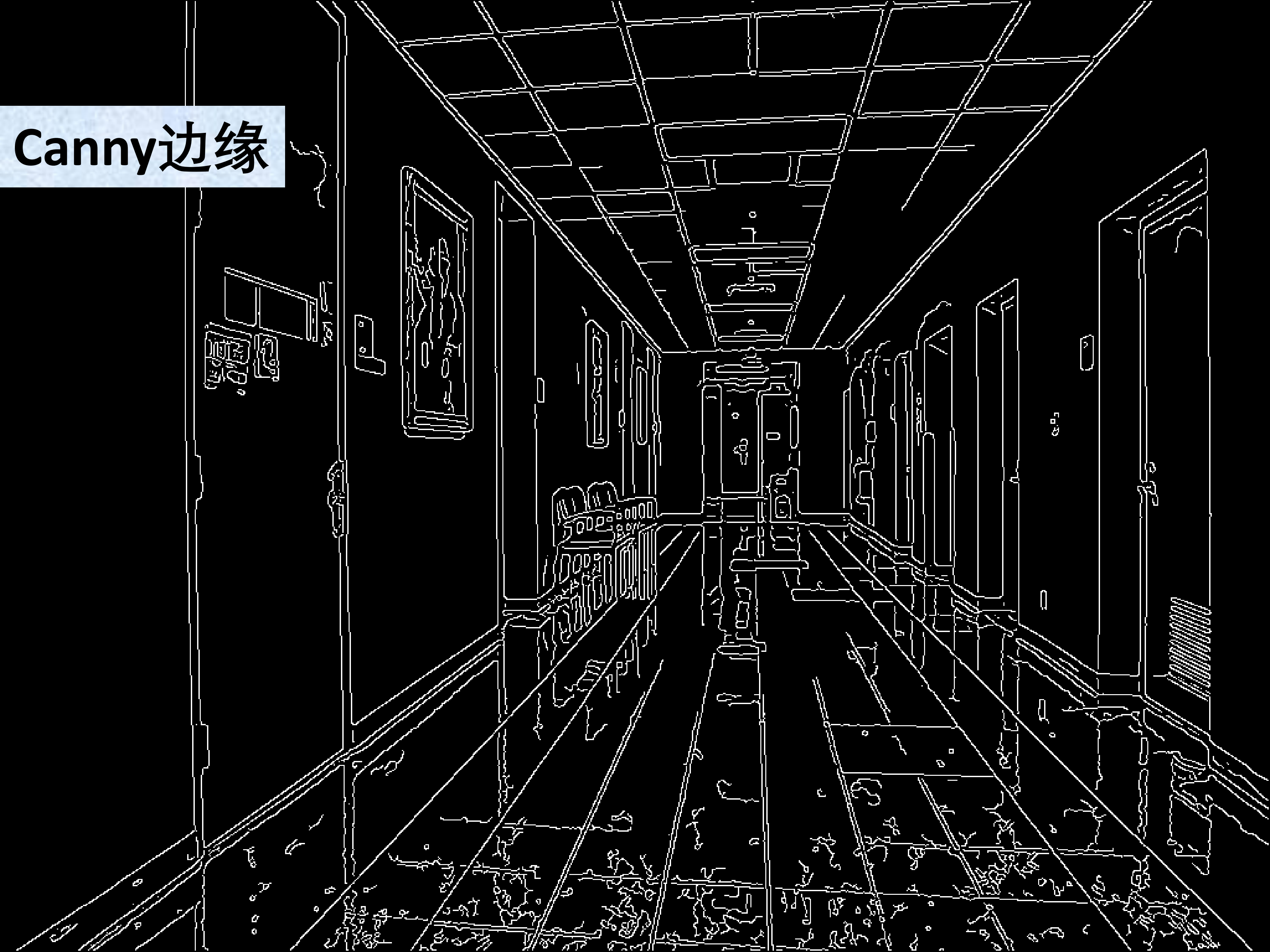
$\quad \rho = x \cos\theta + y \sin\theta$

$\quad H[\rho, \theta] \mathrel{+}= \|\nabla I[x, y]\|$

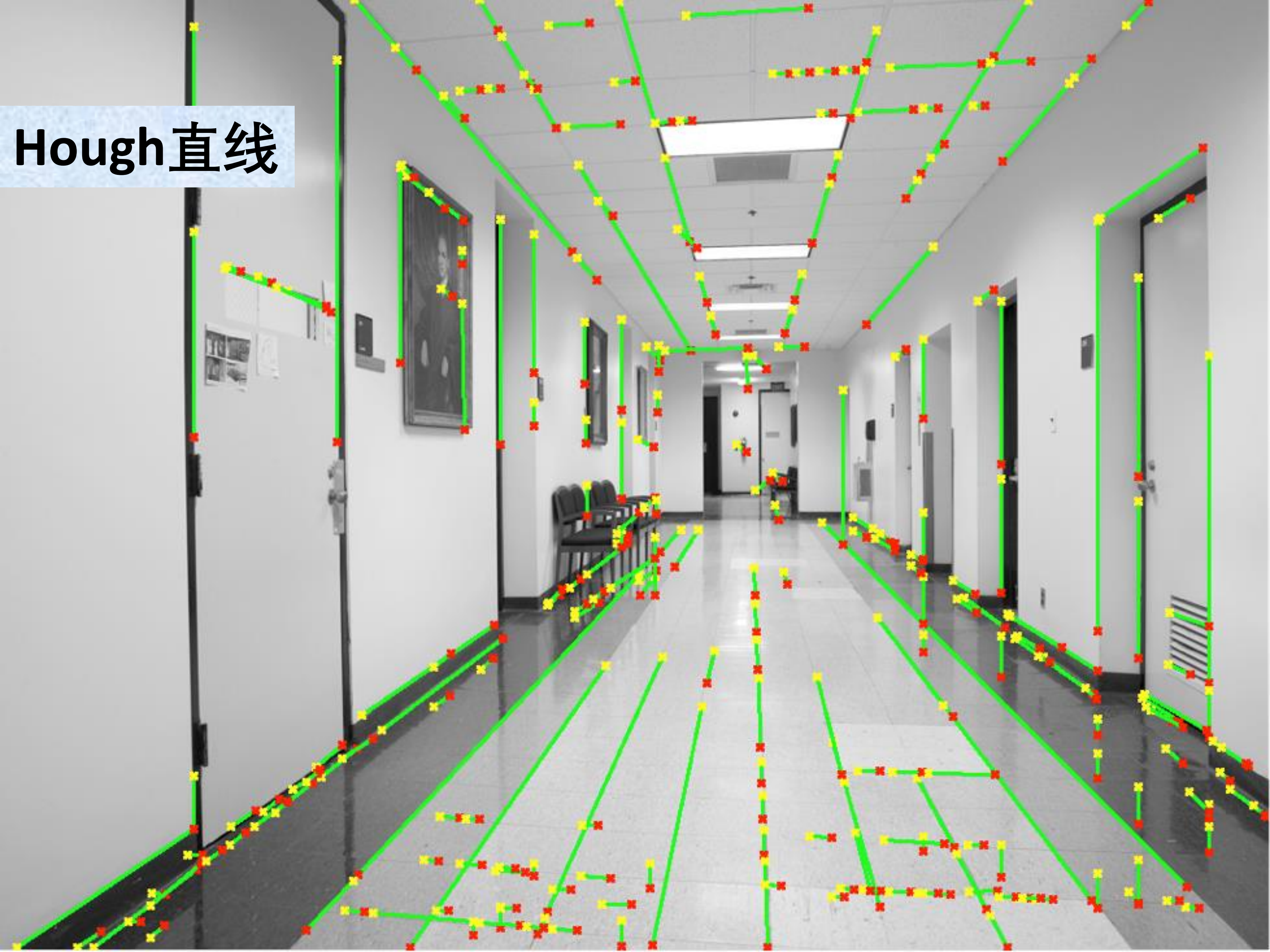
end

$\rho^*, \theta^* = \arg\max_{\rho, \theta} H[\rho, \theta]$

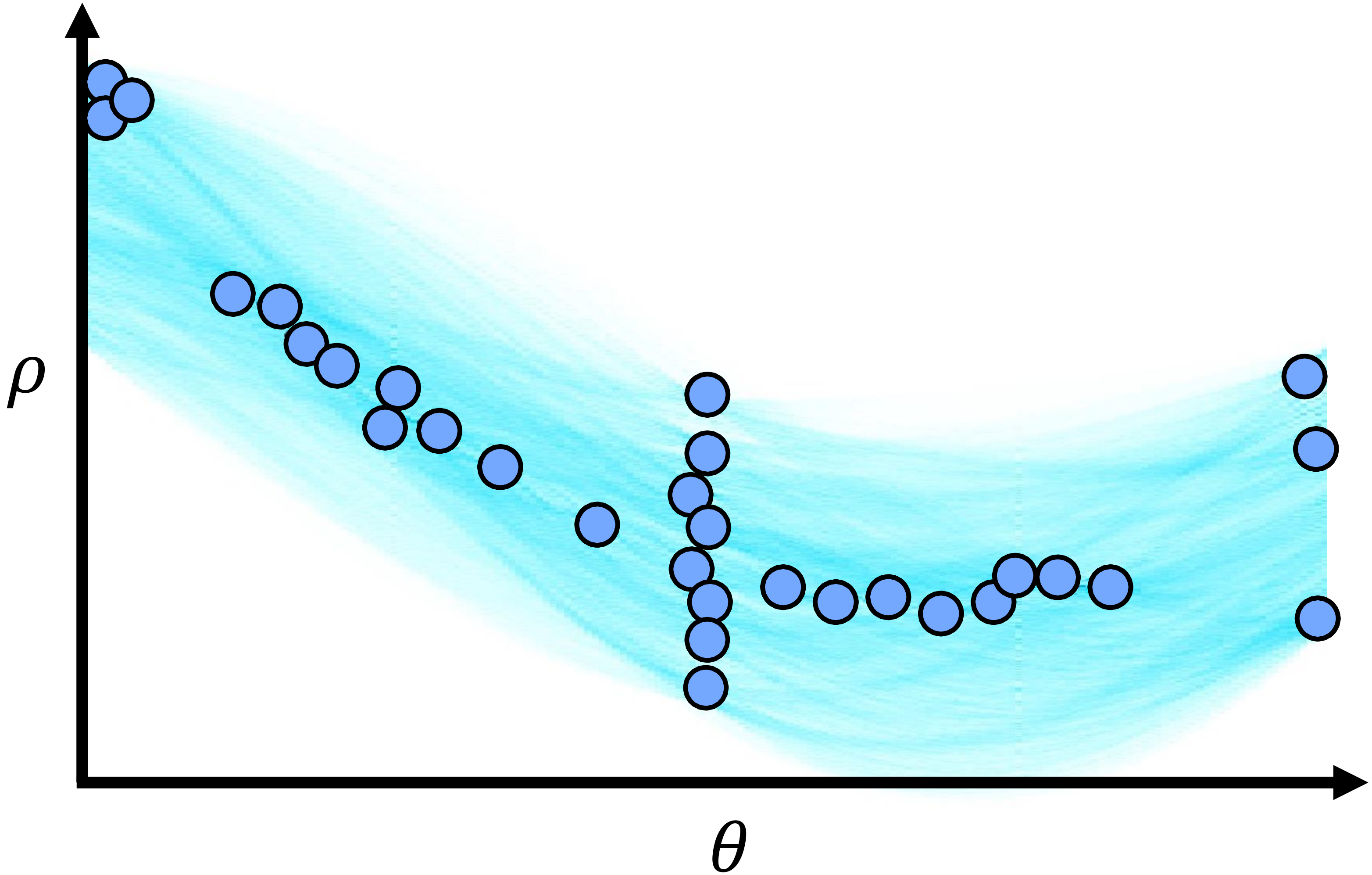
输入图像


Canny边缘

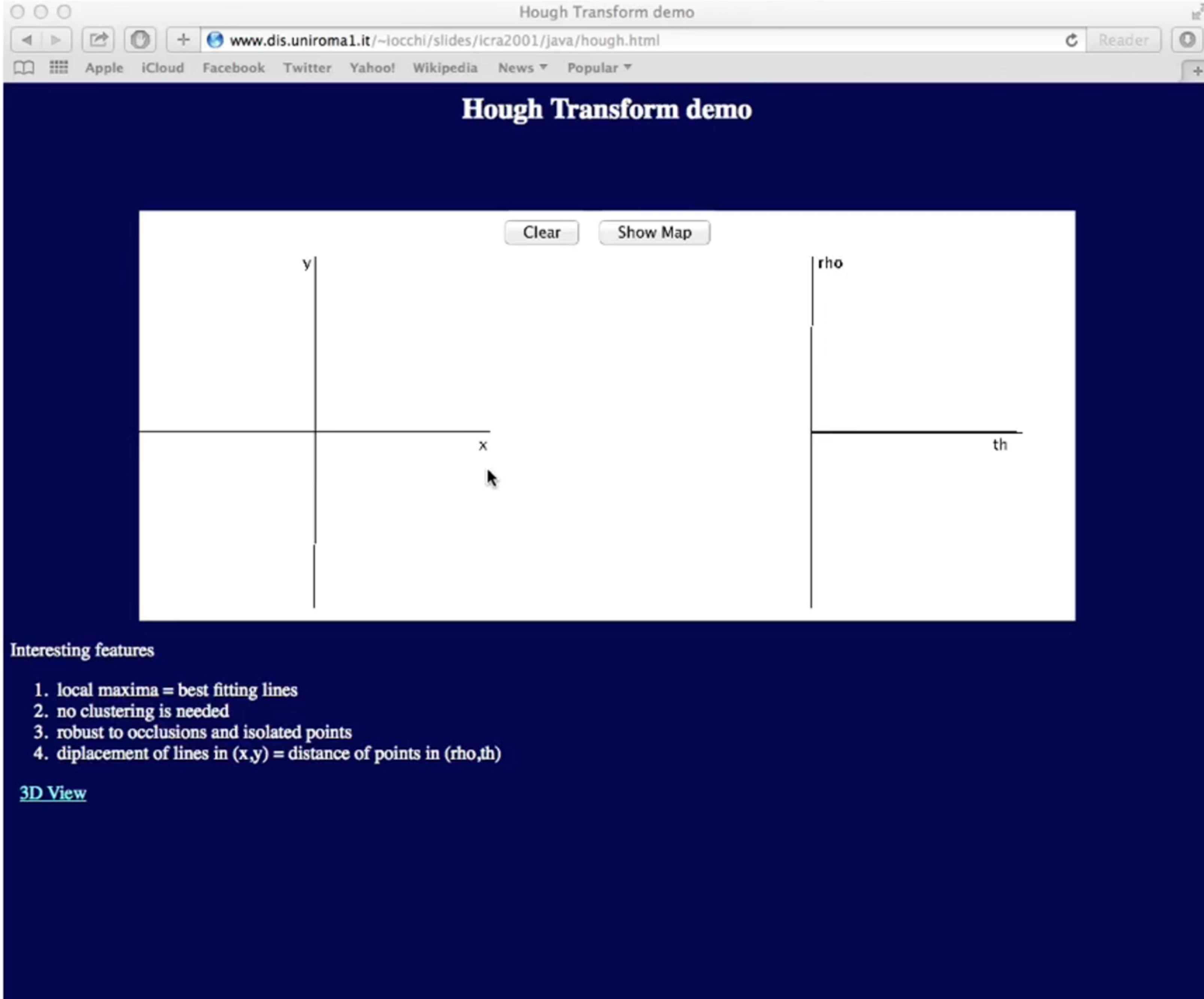
Hough直线



Hough空间




实时车道线检测


Hough Transform demo
www.dis.uniroma1.it/~iocchi/slides/icra2001/java/hough.html
Apple
iCloud
Facebook
Twitter
Yahoo!
Wikipedia
News
Popular
Hough Transform demo
Clear
Show Map
y
x
rho
th
Interesting features
1. local maxima = best fitting lines
2. no clustering is needed
3. robust to occlusions and isolated points
4. diplacement of lines in (x,y) = distance of points in (rho,th)
3D View

Hough圆变换

假设
固定半径

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$

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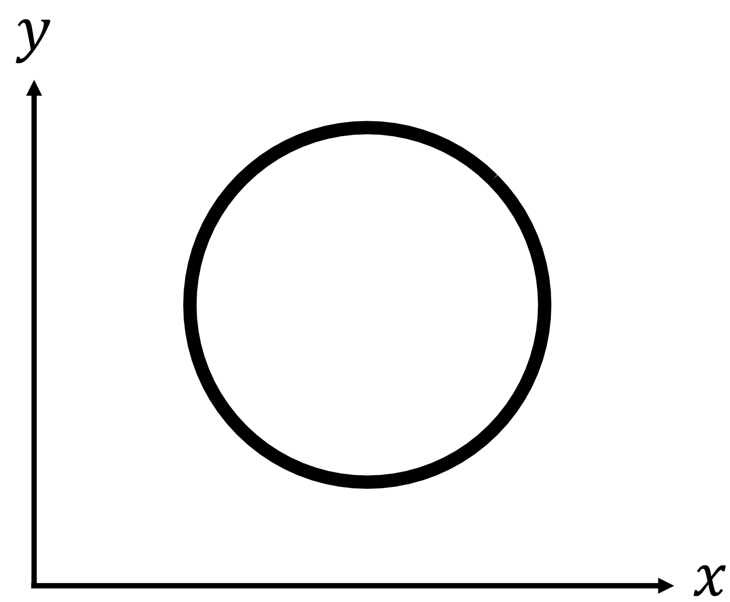

假设
固定半径

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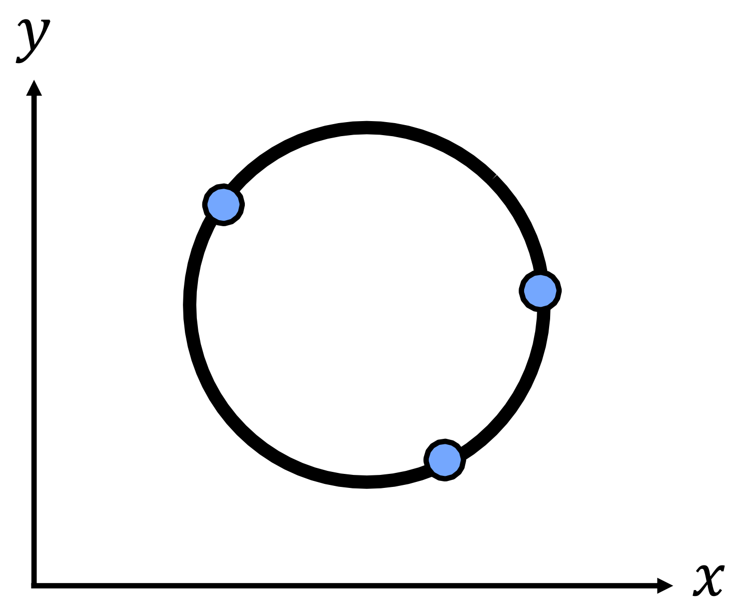

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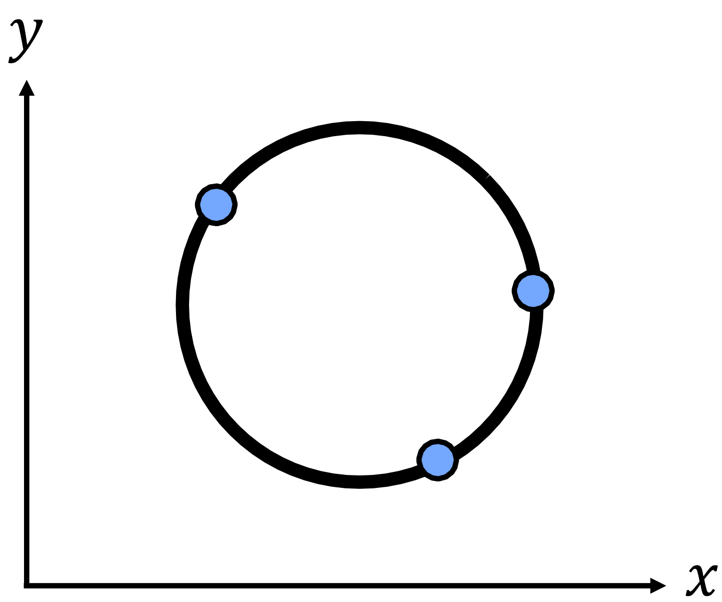
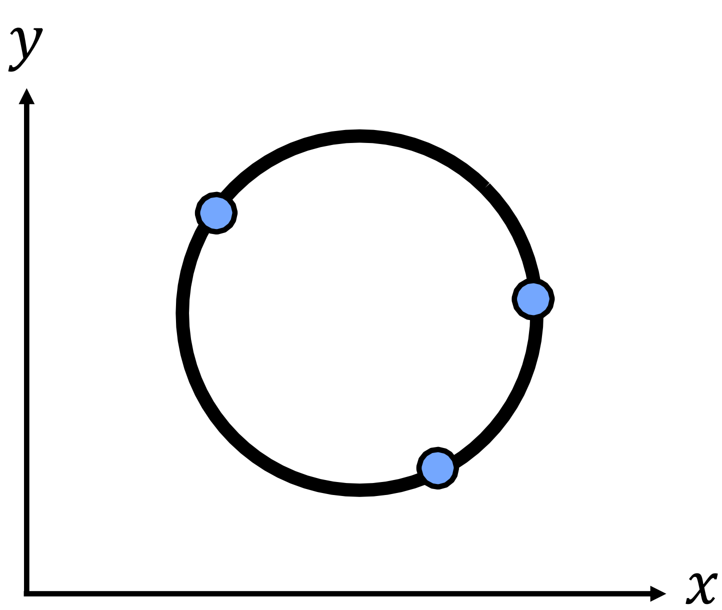
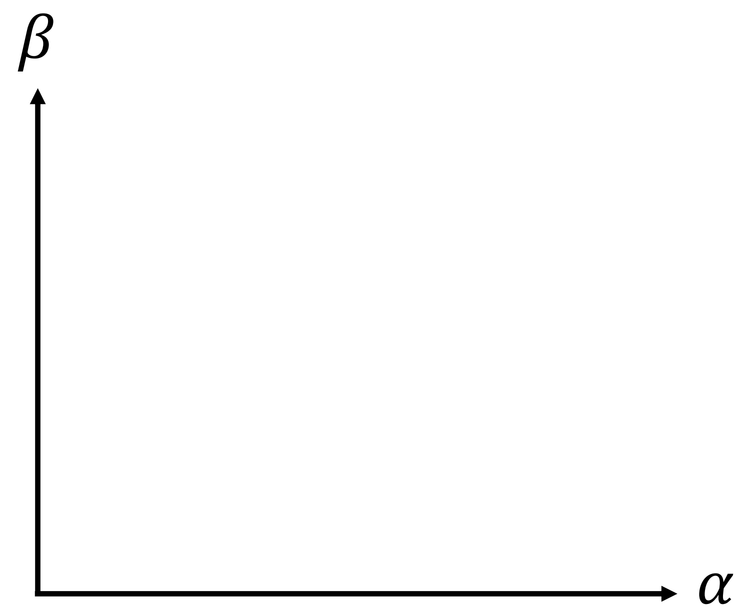


参数空间?

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$





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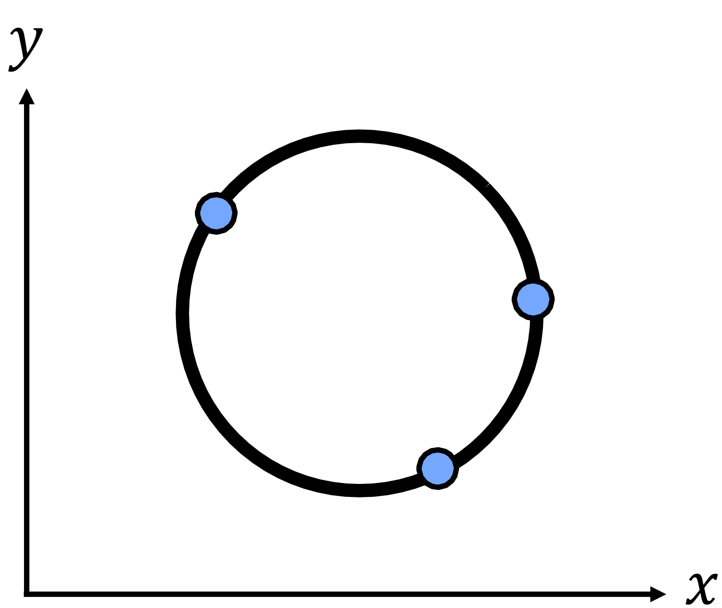

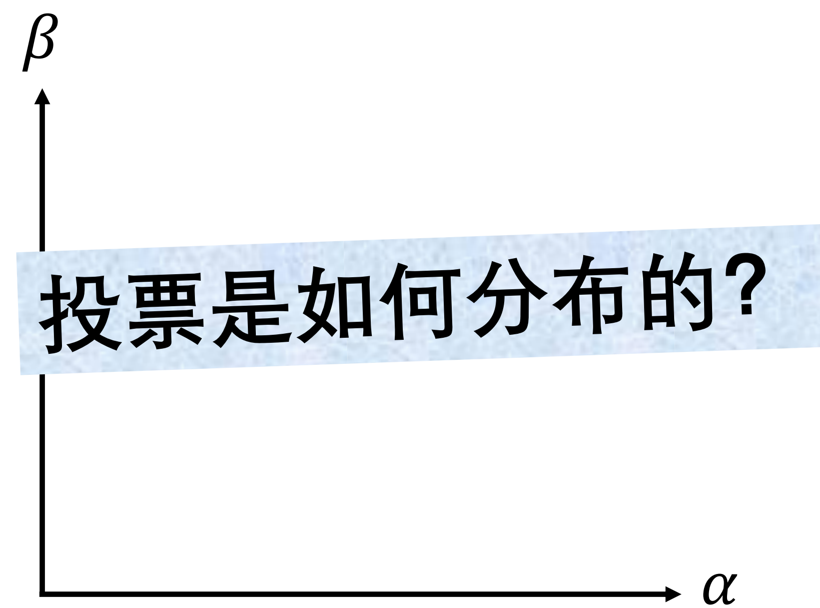

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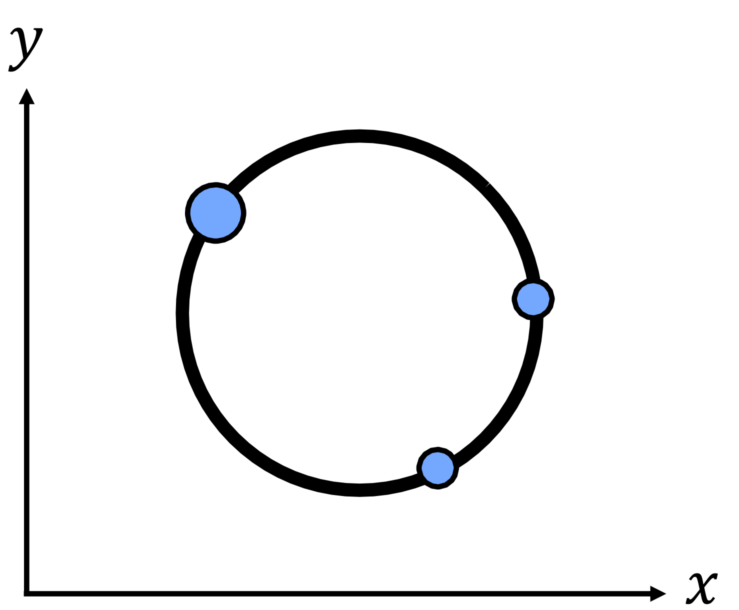

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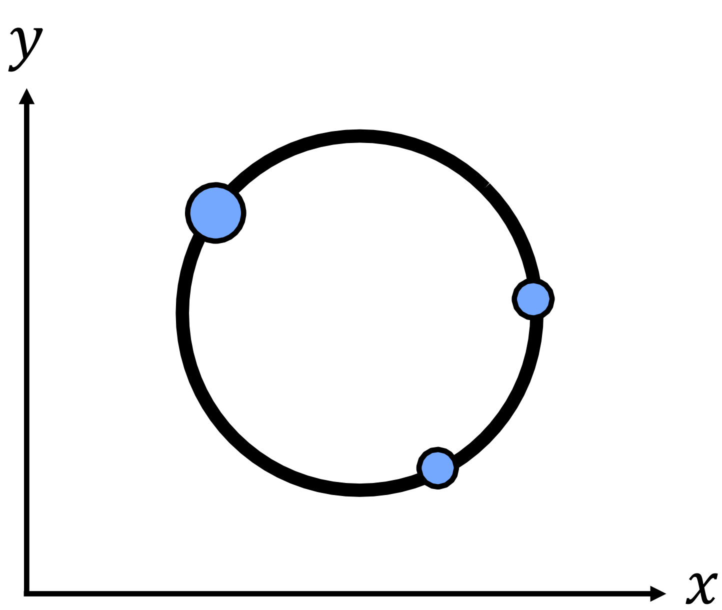
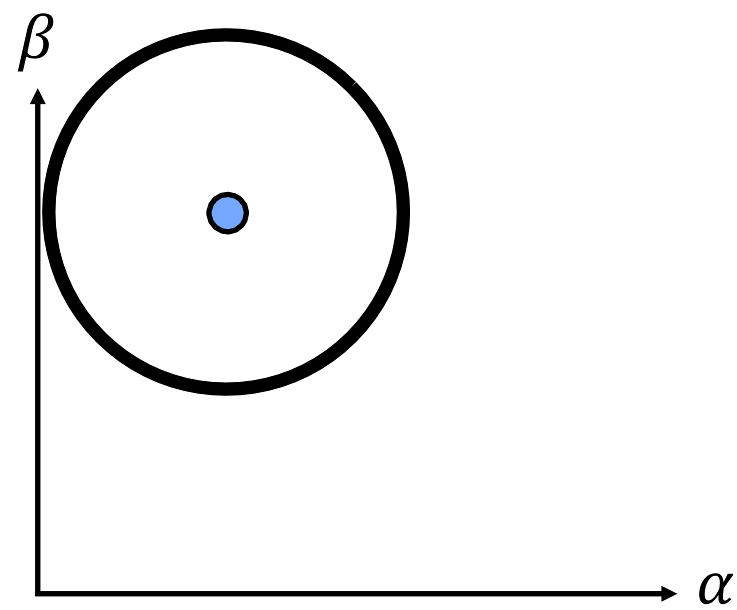
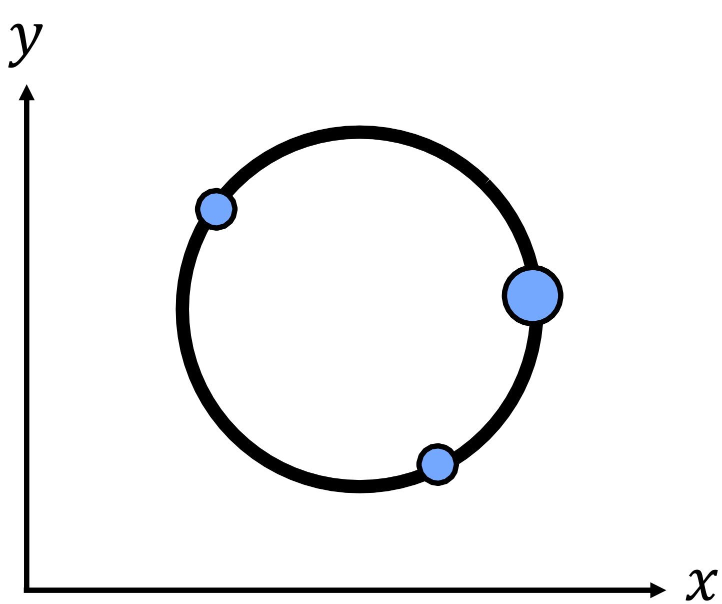
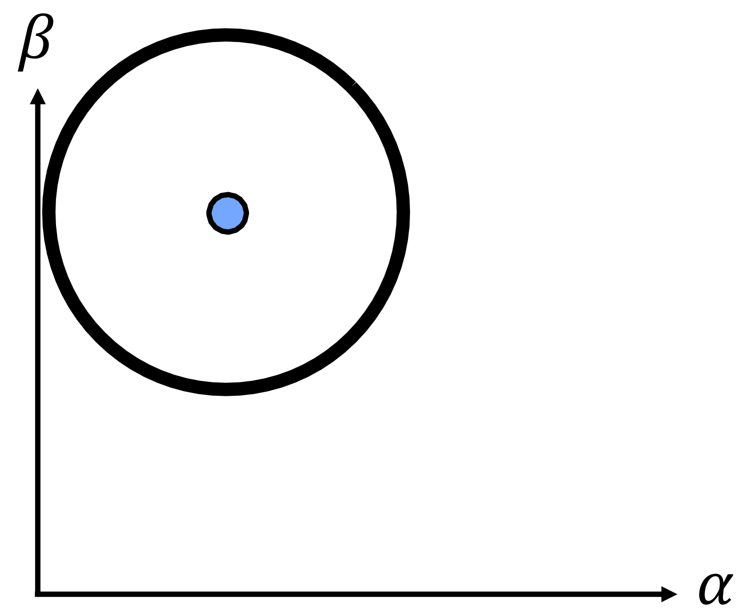




假设
固定半径

$$(x_i-\alpha)^2+(y_i-\beta)^2=r^2$$





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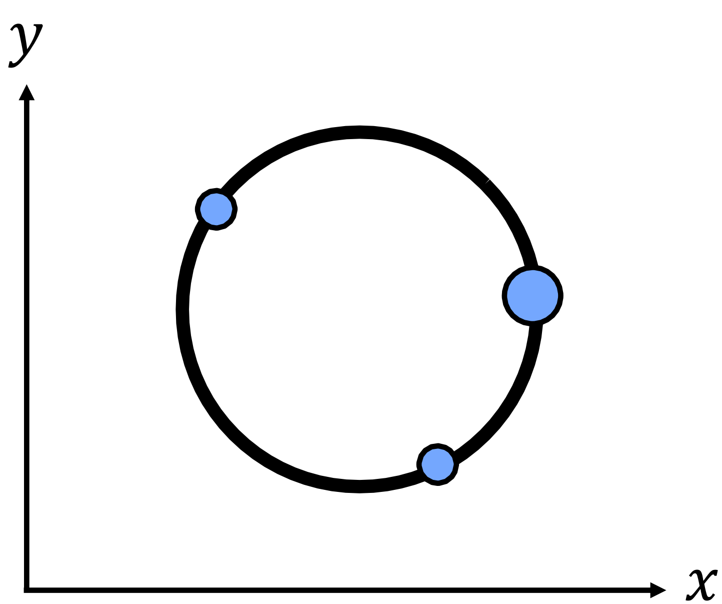

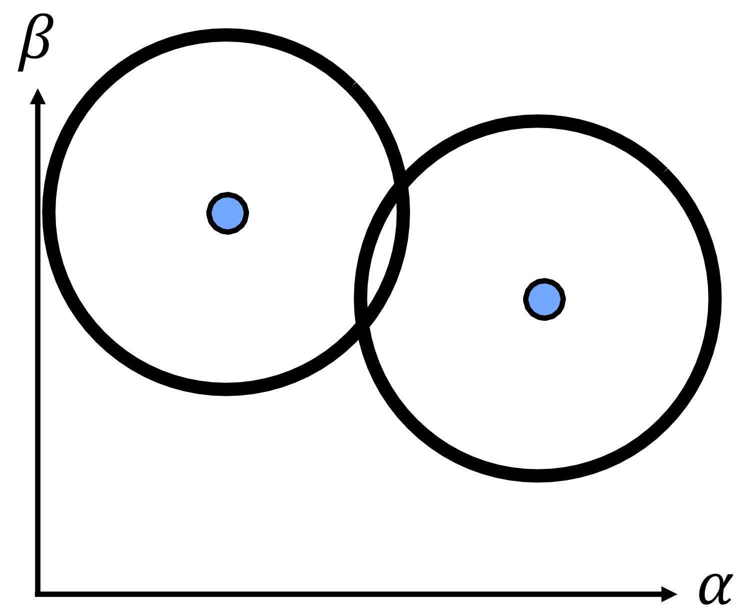

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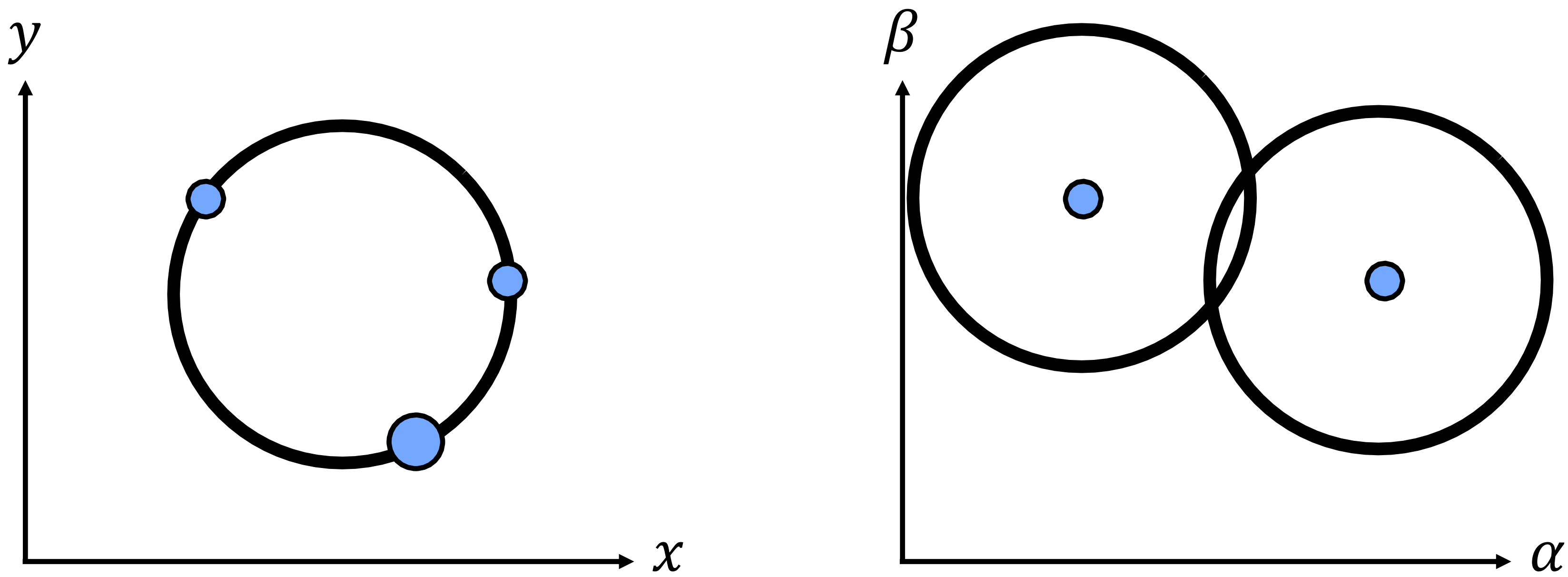

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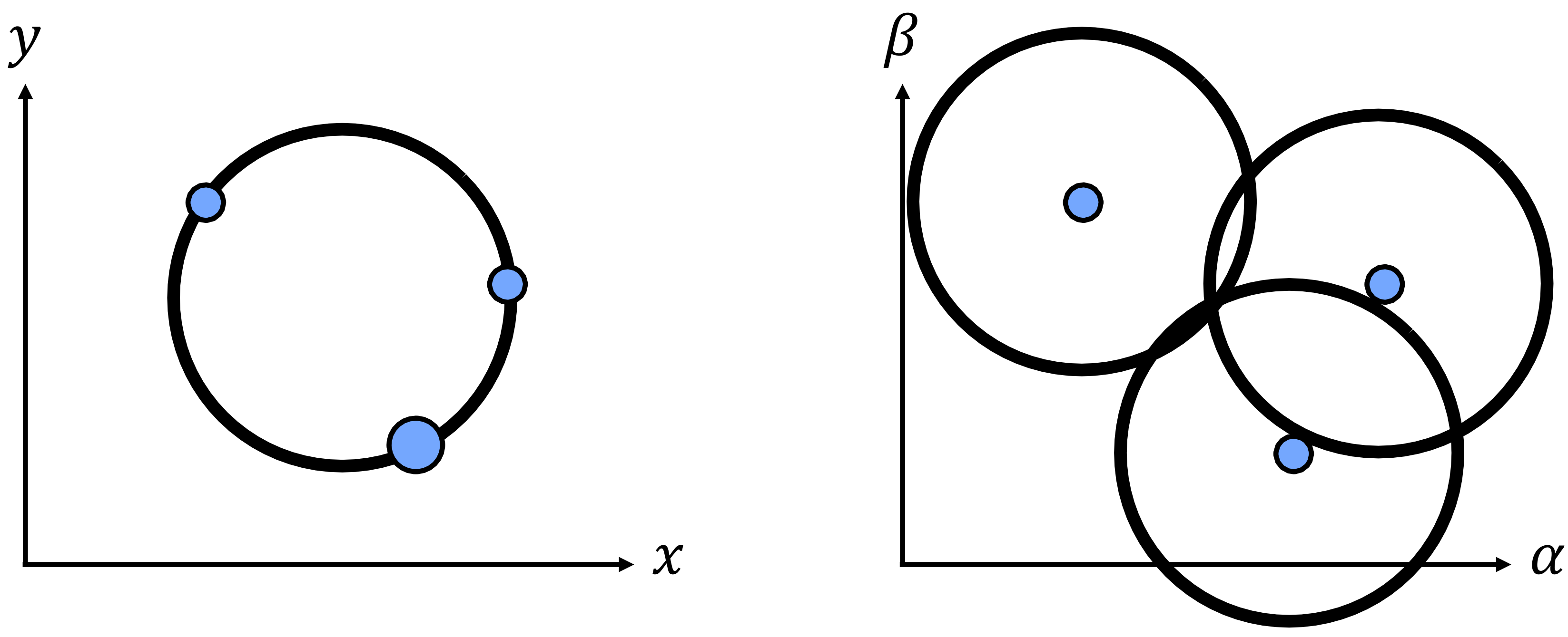

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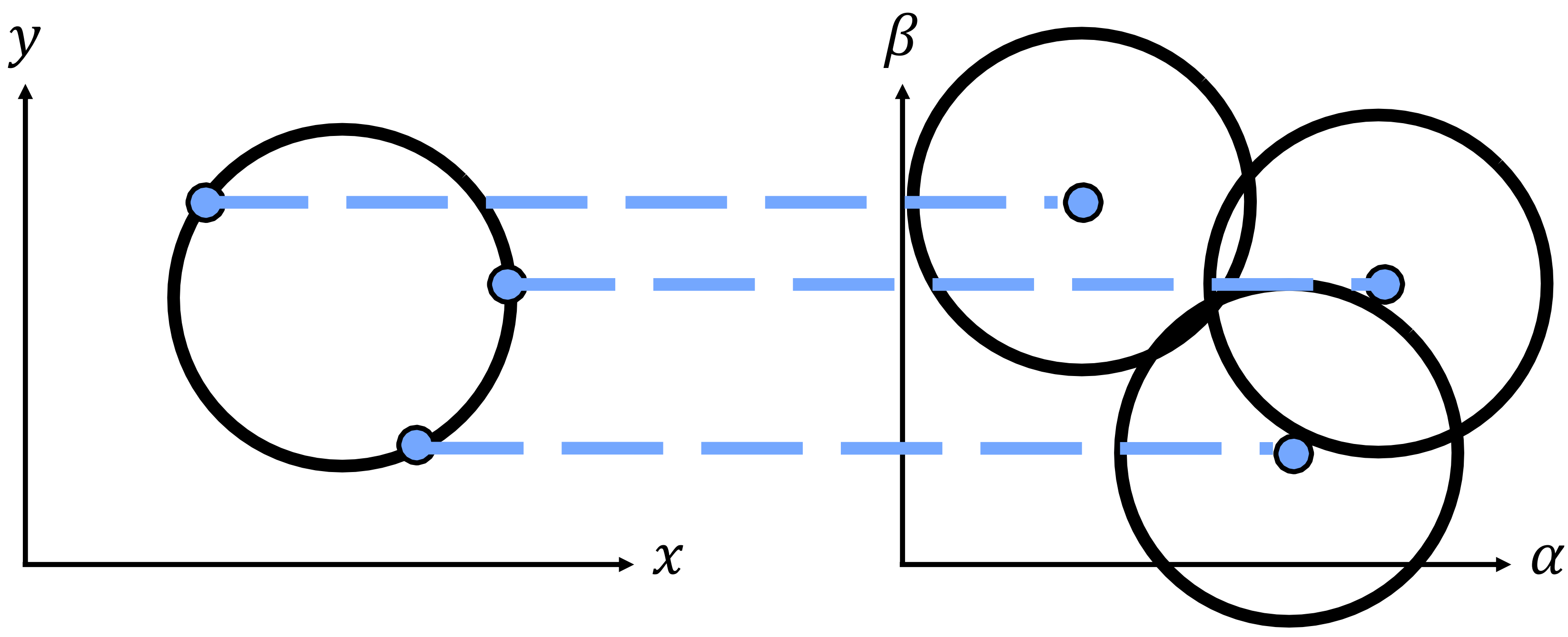

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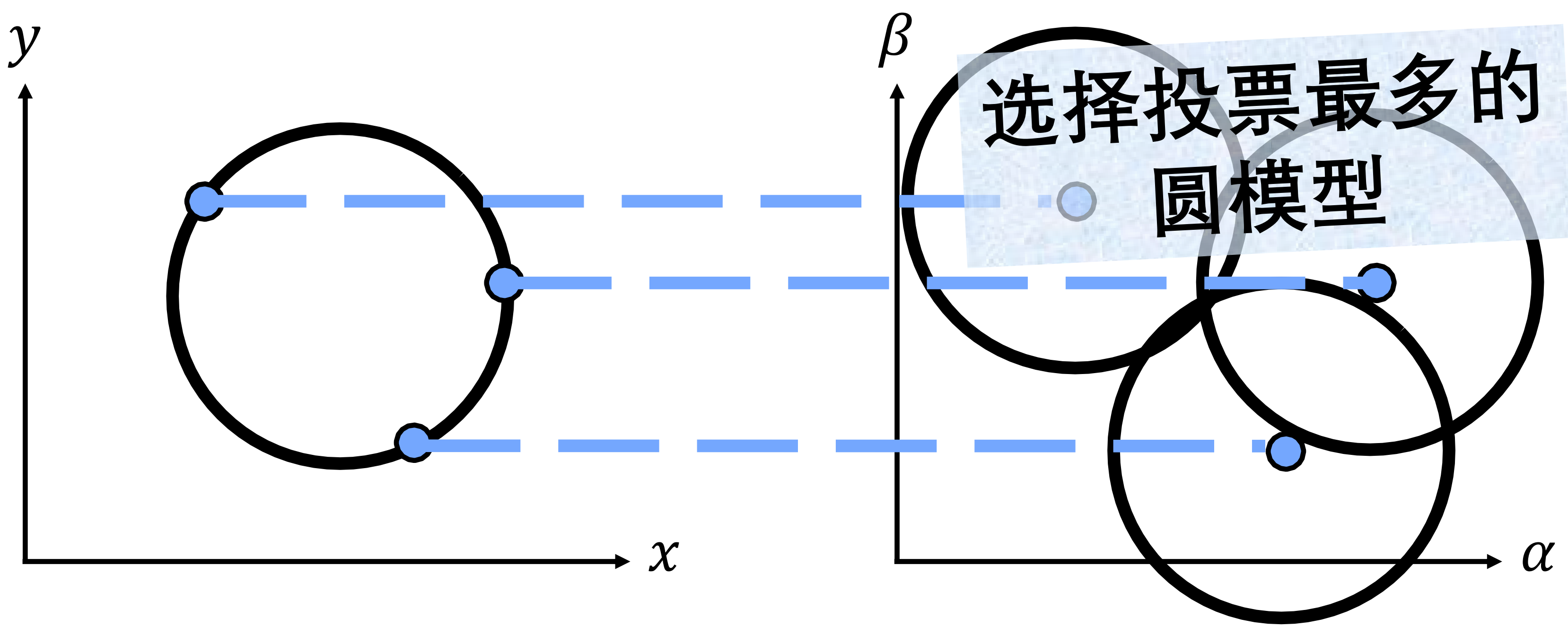

假设
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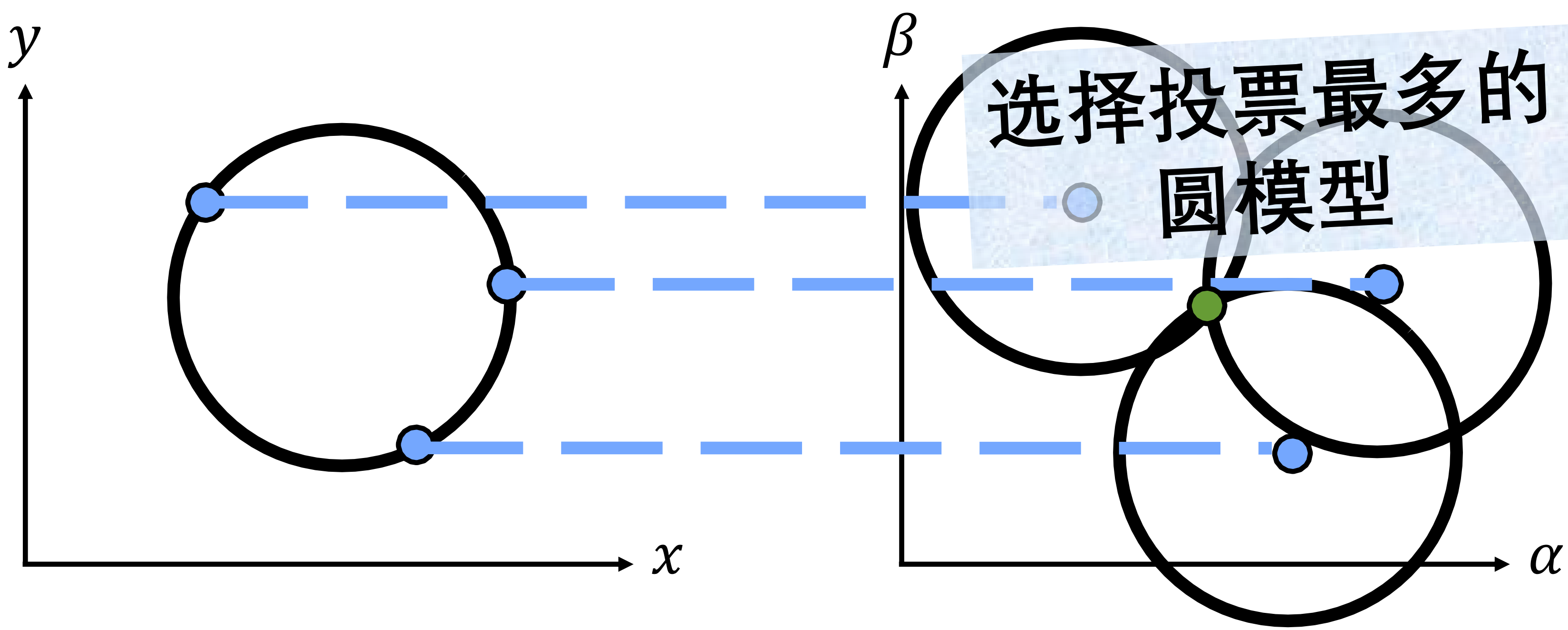

假设
未知半径

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$

假设
未知半径

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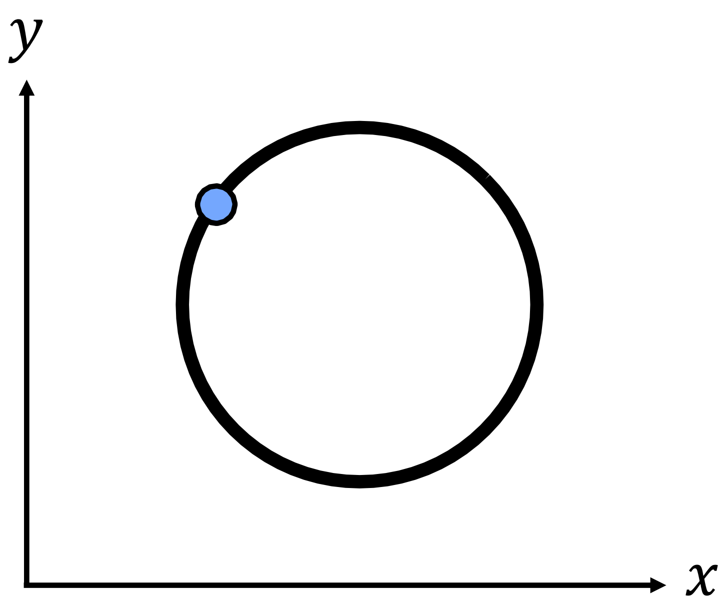
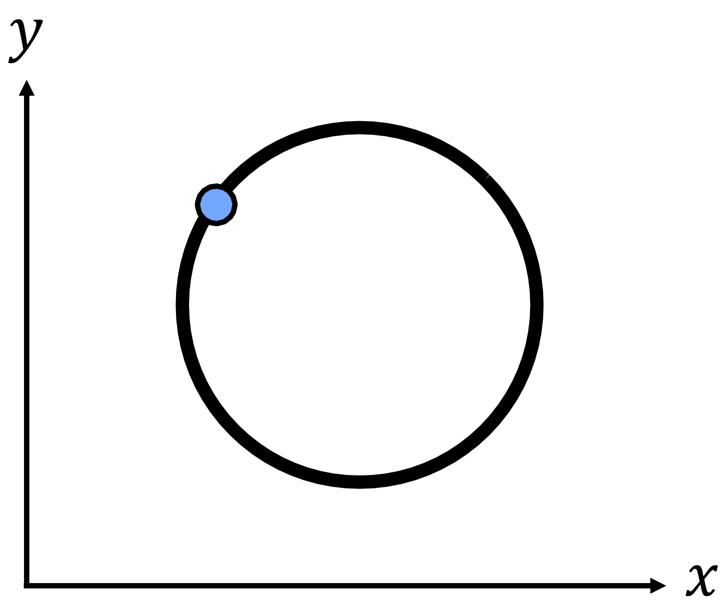


假设
未知半径

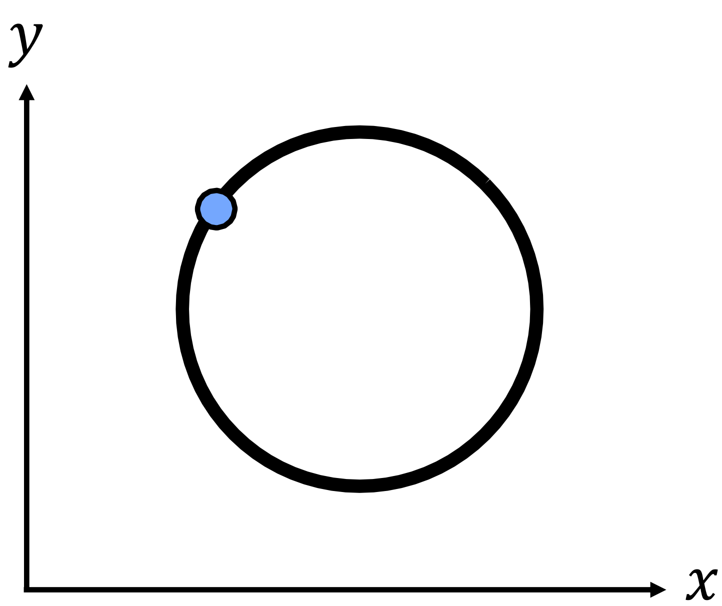
$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$

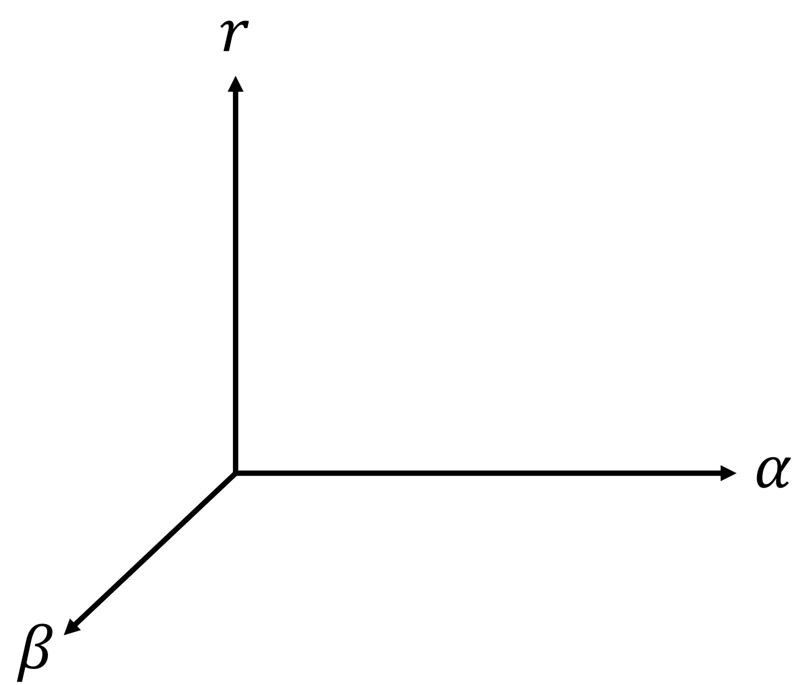


参数空间?

假设
未知半径

$$(x_i-\alpha)^2+(y_i-\beta)^2=r^2$$





假设
未知半径

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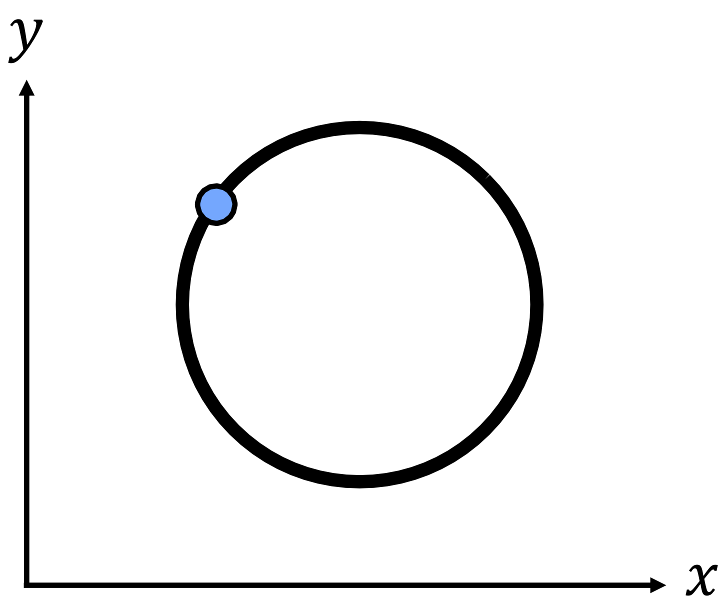
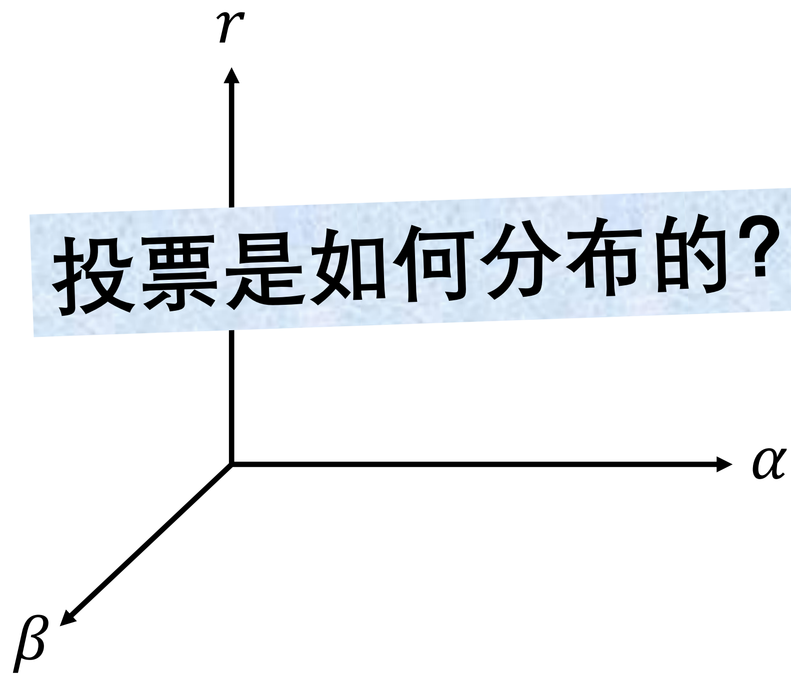
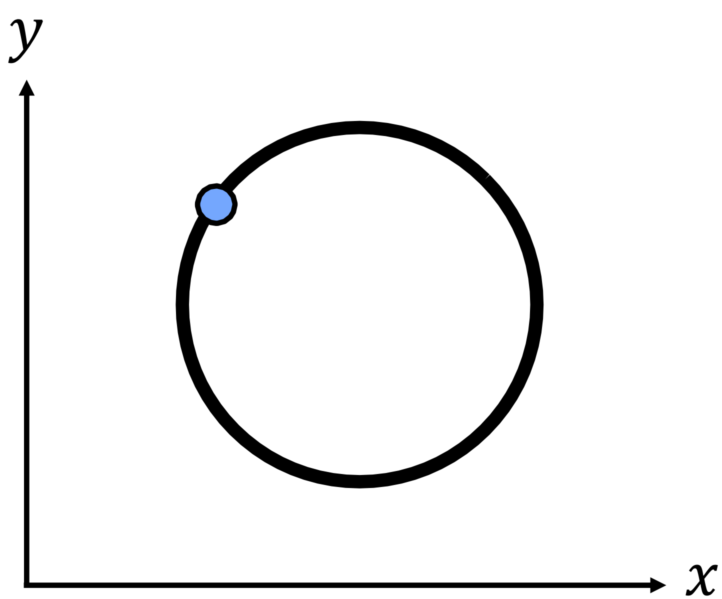


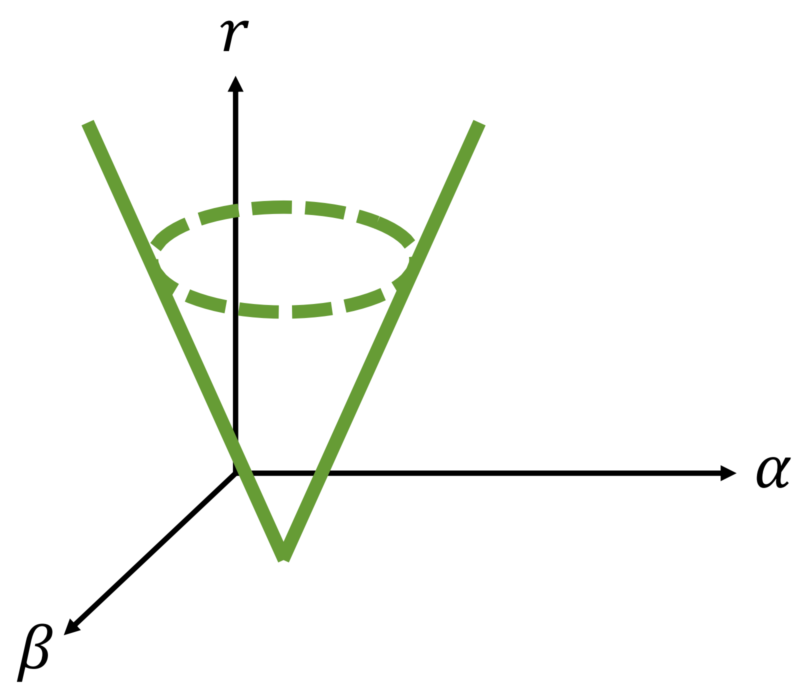


投票是如何分布的?

假设
未知半径

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$





假设
未知半径

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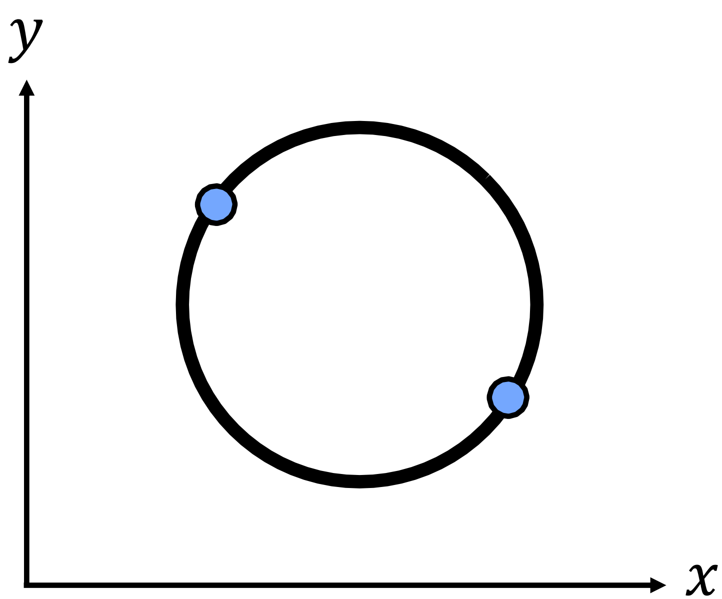
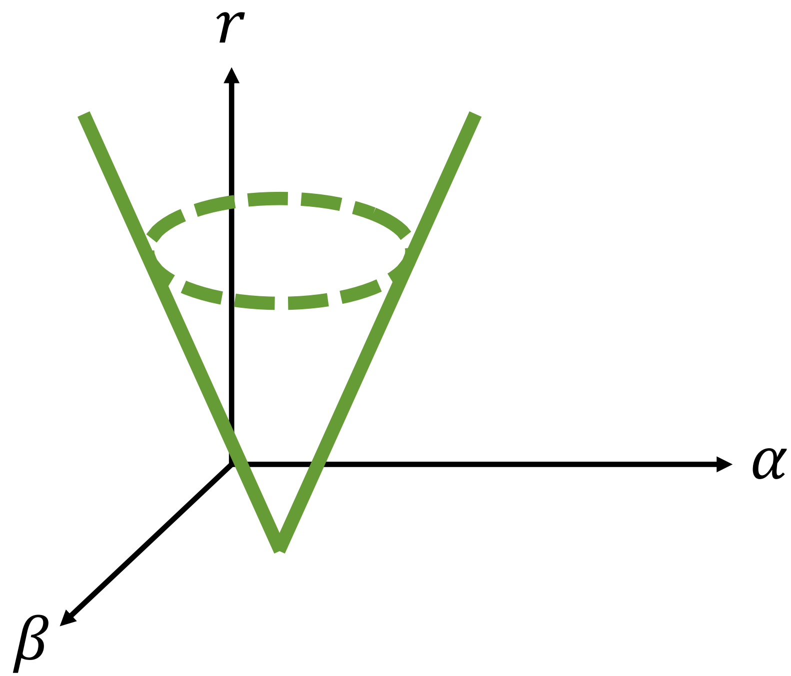
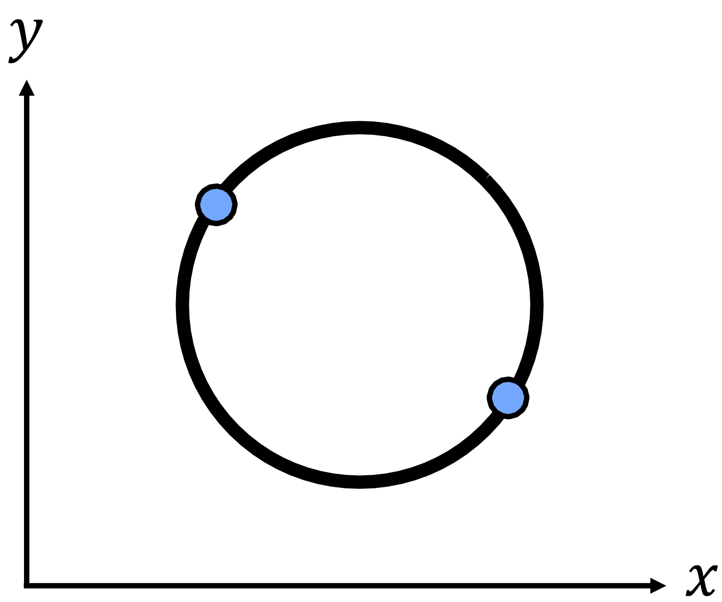
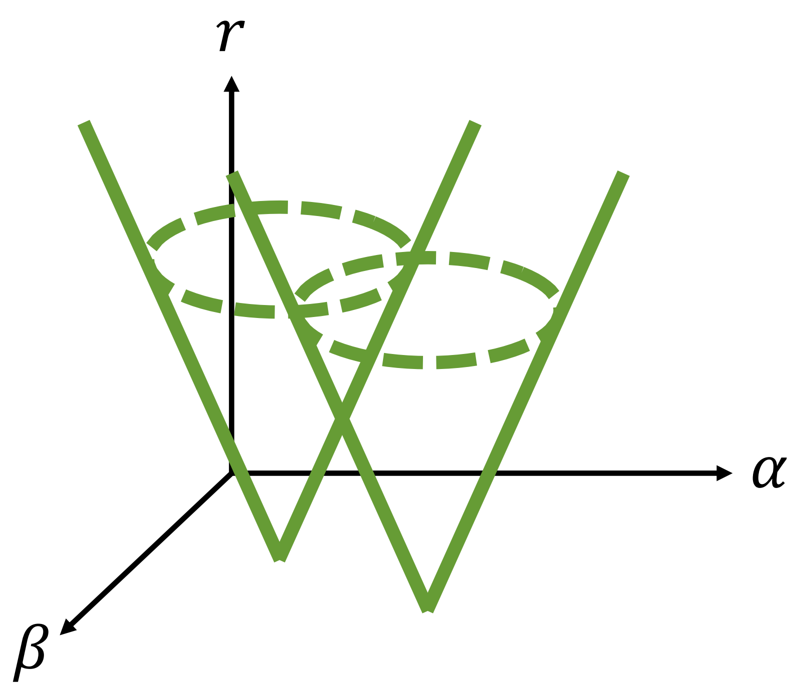




假设
未知半径

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$





未知半径
已知梯度
方向

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$

未知半径
已知梯度
方向

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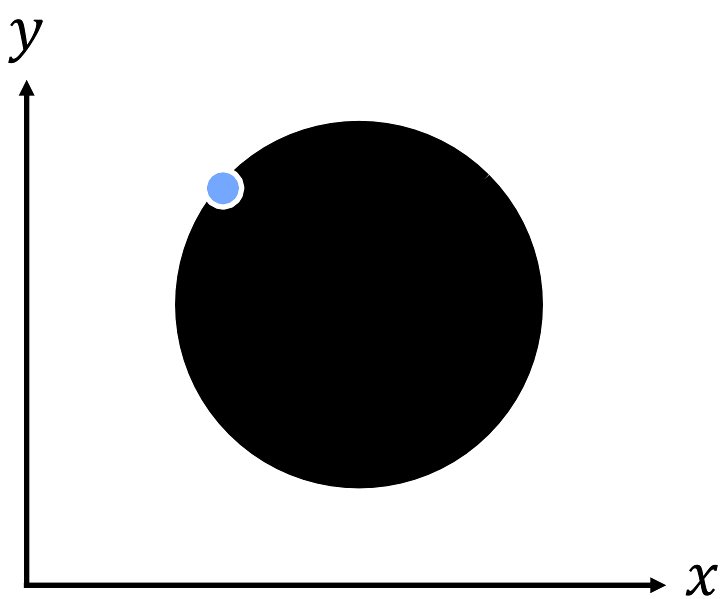
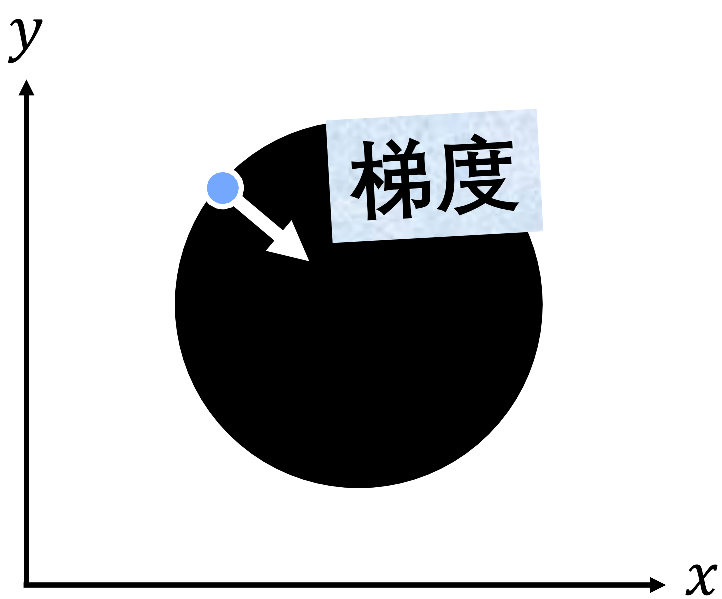


未知半径
已知梯度
方向

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$



未知半径
已知梯度
方向

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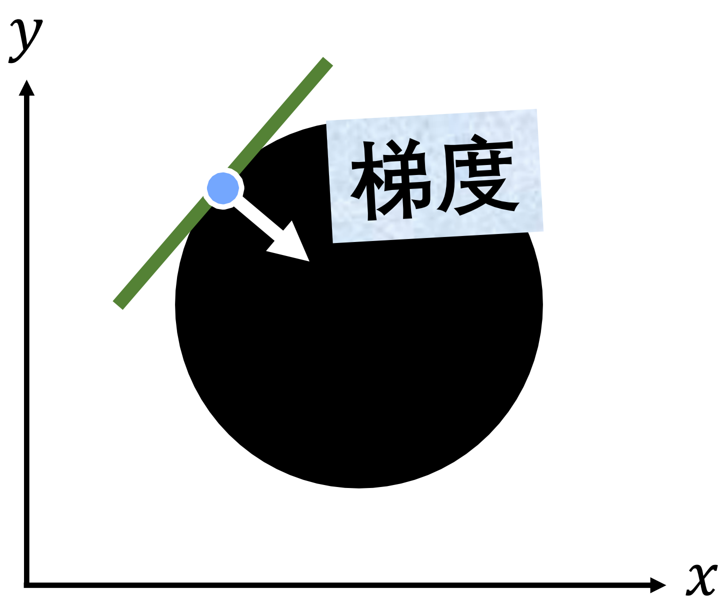
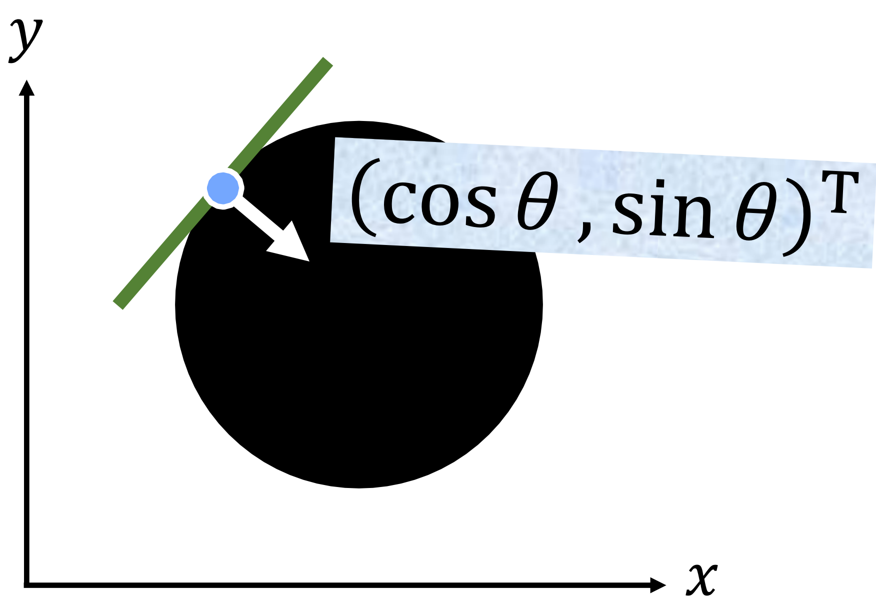


未知半径
已知梯度
方向

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$



未知半径
已知梯度
方向

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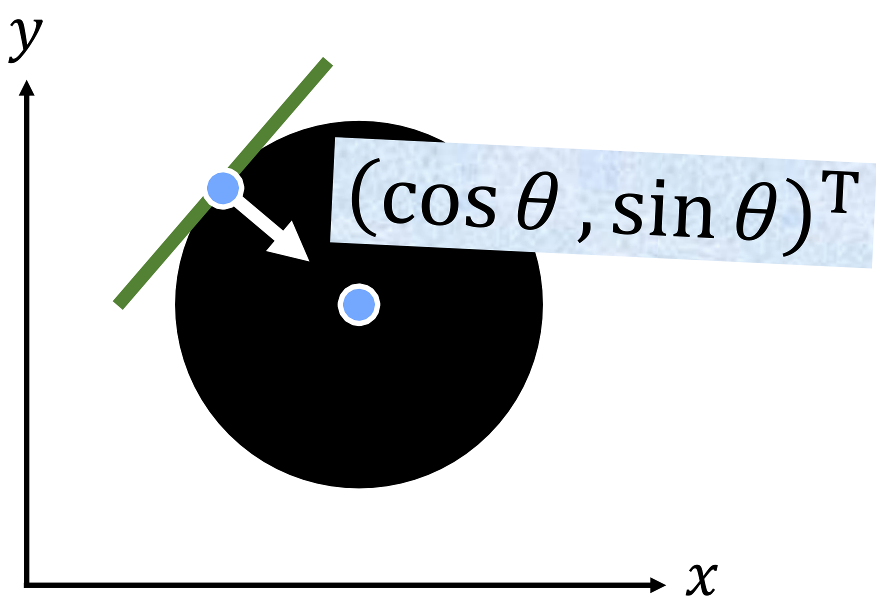
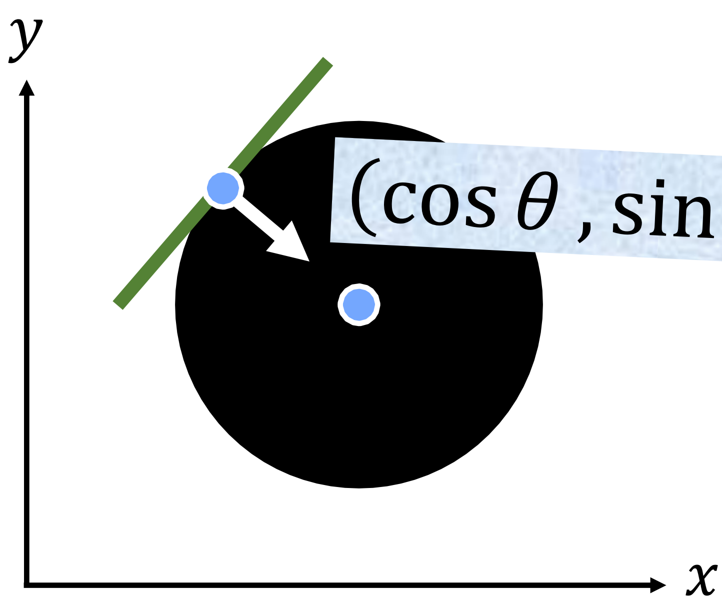


未知半径
已知梯度
方向

$$(x_i-\alpha)^2+(y_i-\beta)^2=r^2$$

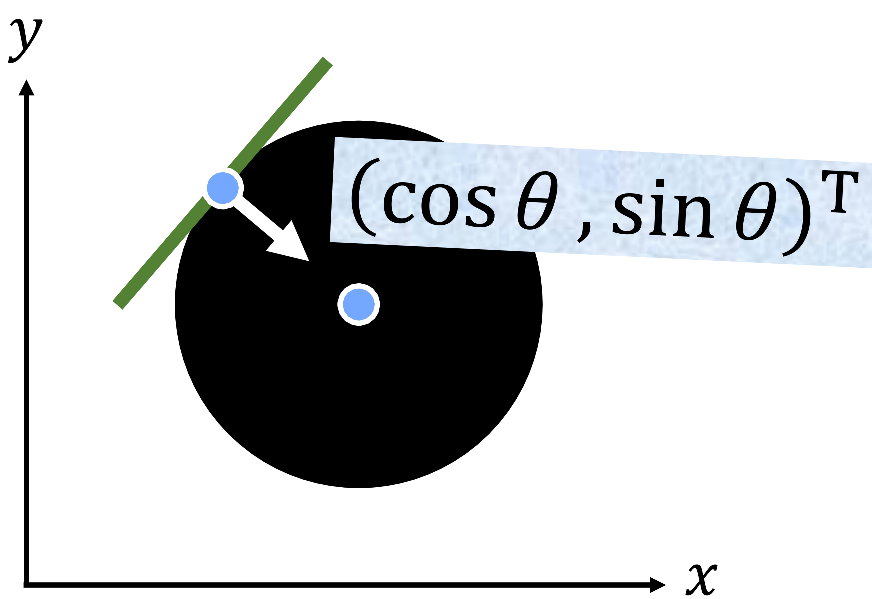


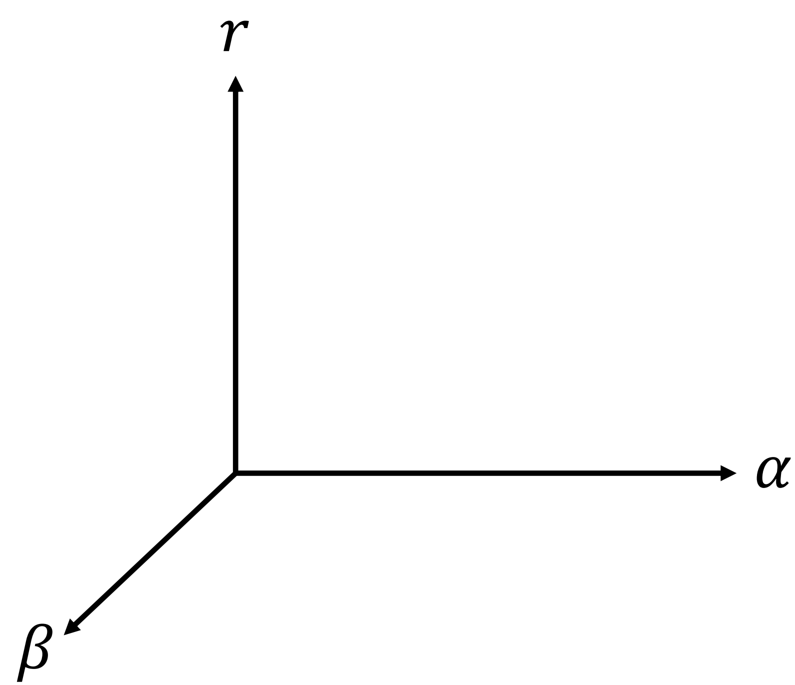
$(\cos\theta, \sin\theta)^{\mathrm{T}}$

参数空间?

未知半径
已知梯度
方向

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$





未知半径
已知梯度
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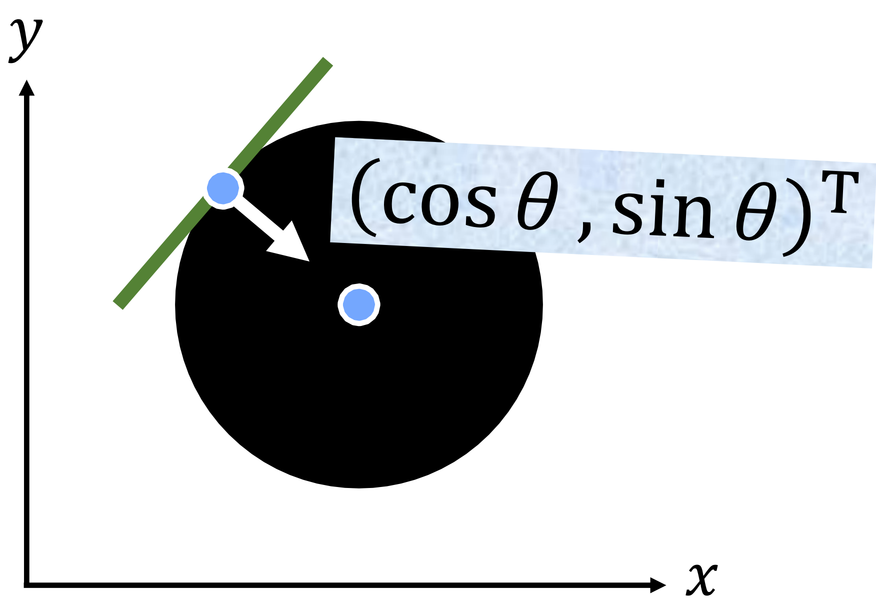
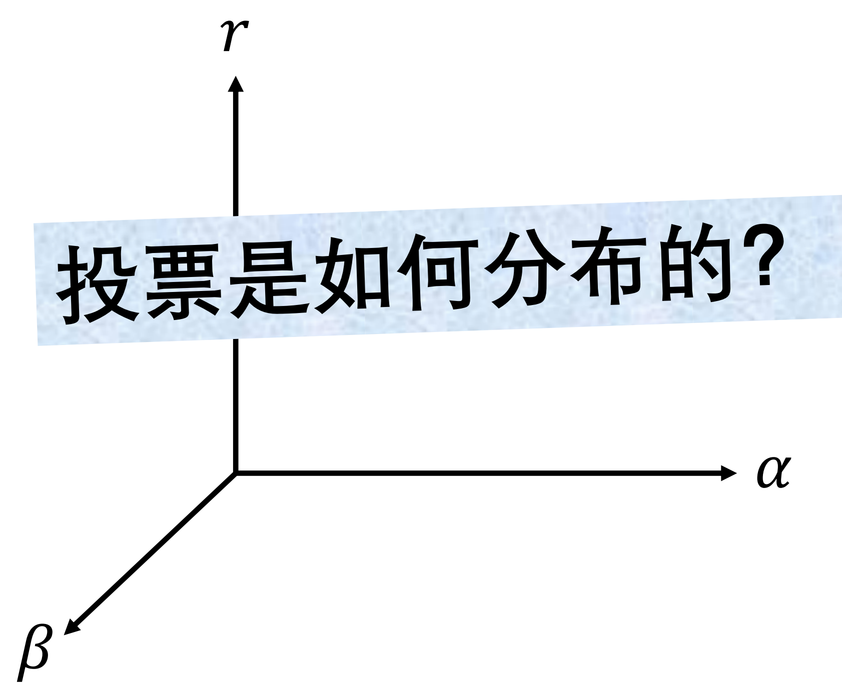
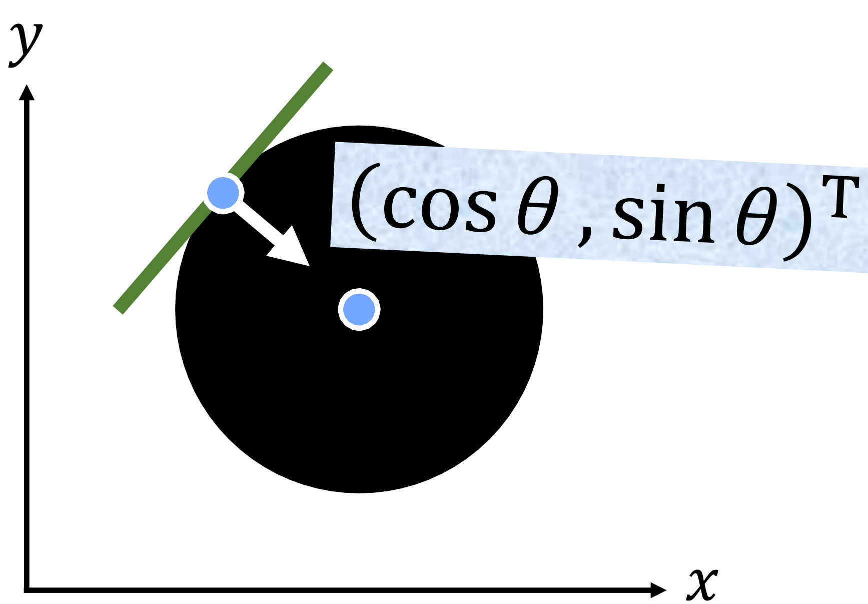


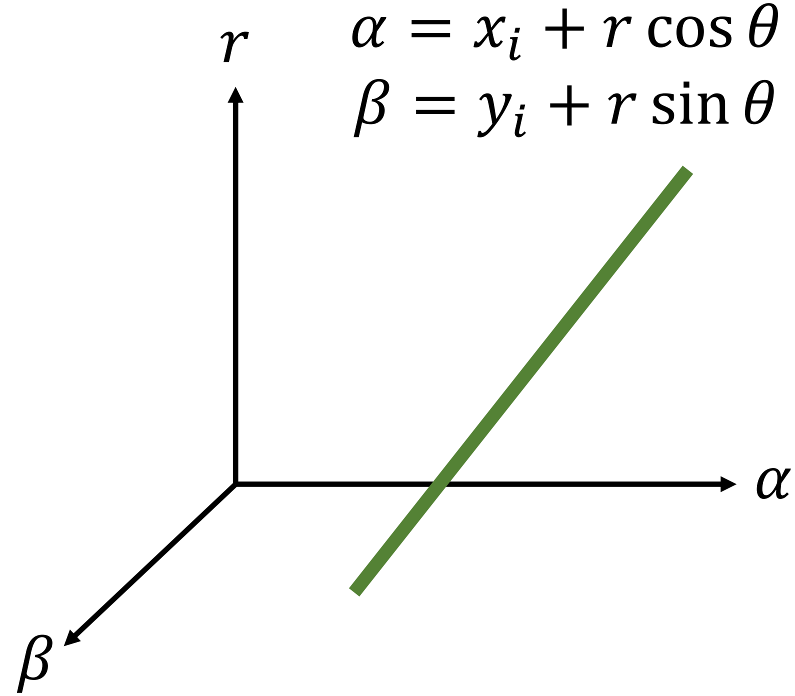


未知半径
已知梯度
方向

$$(x_i - \alpha)^2 + (y_i - \beta)^2 = r^2$$



$$\alpha = x_i + r\cos\theta$$
$$\beta = y_i + r\sin\theta$$



Hough
圆圈

Initialize $H[\alpha, \beta, r] = 0$
foreach edgel $e \in I[x, y]$ **do**
foreach possible radius r **do**
foreach possible gradient direction θ **do**
$\alpha = x + r\cos\theta$
$\beta = y + r\cos\theta$
$H[\alpha, \beta, r] \mathrel{+}= 1$
end
end
end
$\alpha^*, \beta^*, r^* = \arg\max_{\alpha,\beta,r} H[\alpha, \beta, r]$

Hough
圆圈

Initialize $H[\alpha,\beta,r]=0$
foreach edgel $e\in I[x,y]$ **do**
  foreach possible radius r **do**
    foreach possible gradient direction θ **do**
      $\alpha = x + r\cos\theta$
      $\beta = y + r\cos\theta$
      $H[\alpha,\beta,r] \mathrel{+}= 1$
    end
  end
end
$\alpha^*,\beta^*,r^* = \arg\max H[\alpha,\beta,r]$

每点投票数的时间复杂度?


Yellow

优点

优点

所有点都是独立处理的

优点

所有点都是独立处理的

对外点具有鲁棒性

优点

所有点都是独立处理的

对外点具有鲁棒性

可以在一次处理过程中检测多个模型实例

缺点

缺点

搜索时间复杂度随参数个数呈指数增长

缺点

搜索时间复杂度随参数个数呈指数增长

非目标形状会在参数空间产生伪峰

缺点

搜索时间复杂度随参数个数呈指数增长

非目标形状会在参数空间产生伪峰

难以选择最佳量化

任意形状

怎么办?

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Pergamon Press Ltd.

GENERALIZING THE HOUGH TRANSFORM TO DETECT ARBITRARY SHAPES*

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(*Received* 10 *October* 1979; *in revised form* 9 *September* 1980; *received for publication* 23 *September* 1980)

Abstract– The Hough transform is a method for detecting curves by exploiting the duality between points on a curve and parameters of that curve. The initial work showed how to detect both analytic curves[1,2] and non-analytic curves,[3] but these methods were restricted to binary edge images. This work was generalized to the detection of some analytic curves in grey level images, specifically lines,[4] circles[5] and parabolas.[6] The line detection case is the best known of these and has been ingeniously exploited in several applications.[7,8,9]

We show how the boundaries of an *arbitrary* non-analytic shape can be used to construct a mapping between image space and Hough transform space. Such a mapping can be exploited to detect instances of that particular shape in an image. Furthermore, variations in the shape such as rotations, scale changes or figure–ground reversals correspond to straightforward transformations of this mapping. However, the most remarkable property is that such mappings can be composed to build mappings for complex shapes from the mappings of simpler component shapes. This makes the generalized Hough transform a kind of universal transform which can be used to find arbitrarily complex shapes.

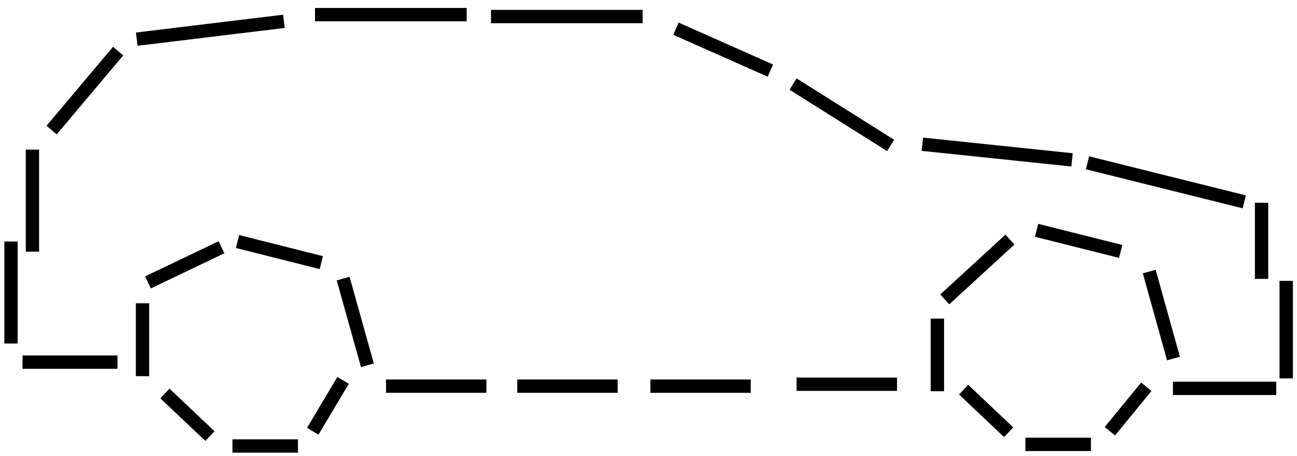
Pattern Recognition, 1981

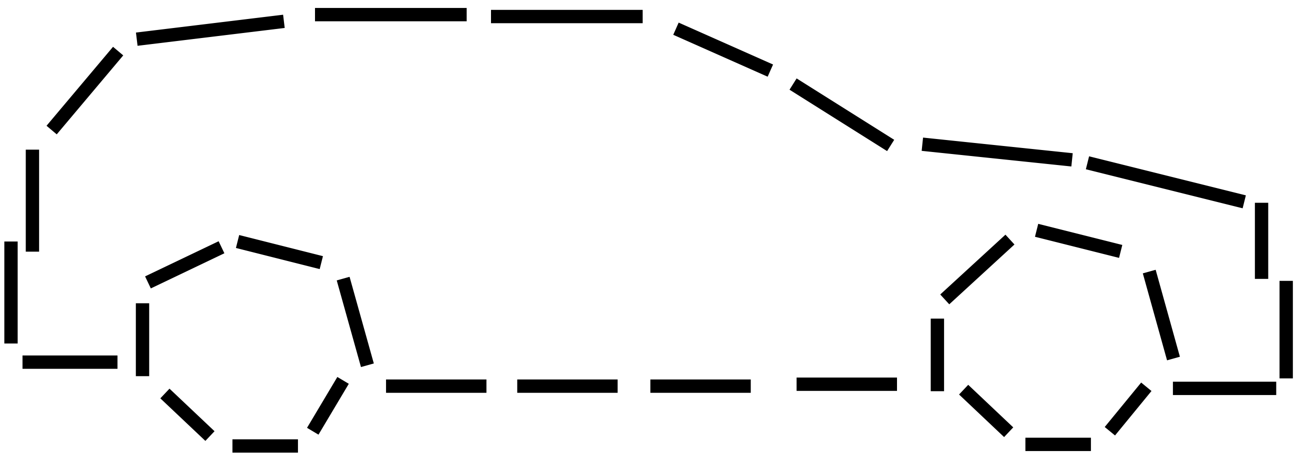
2

主要步骤

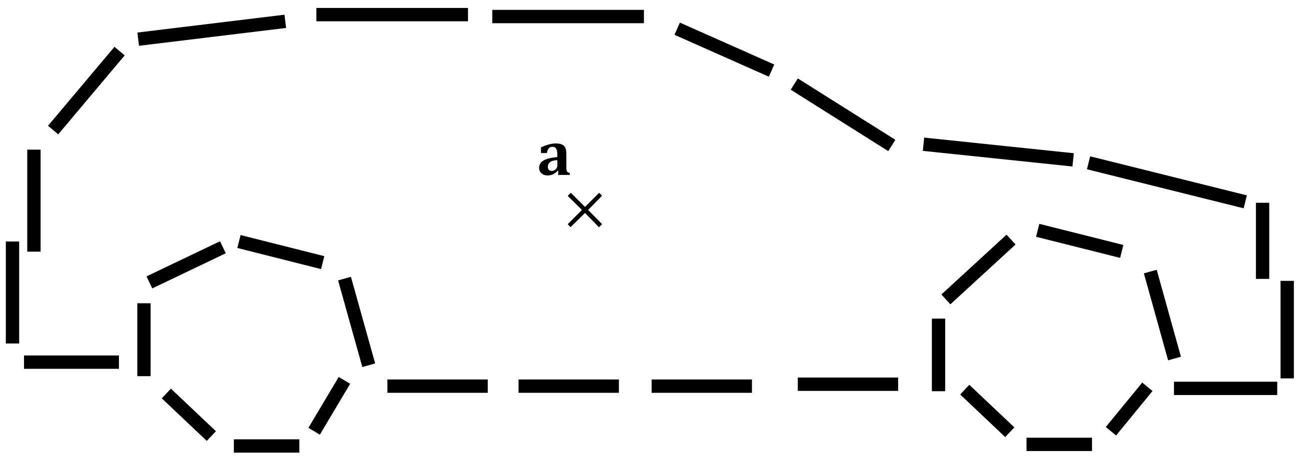
步骤1

构建形状模型

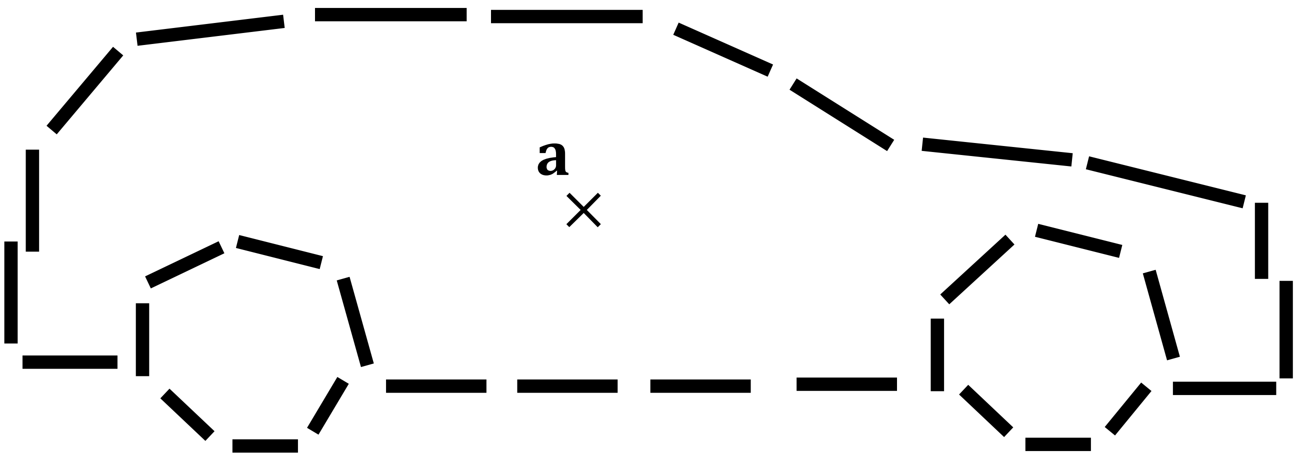




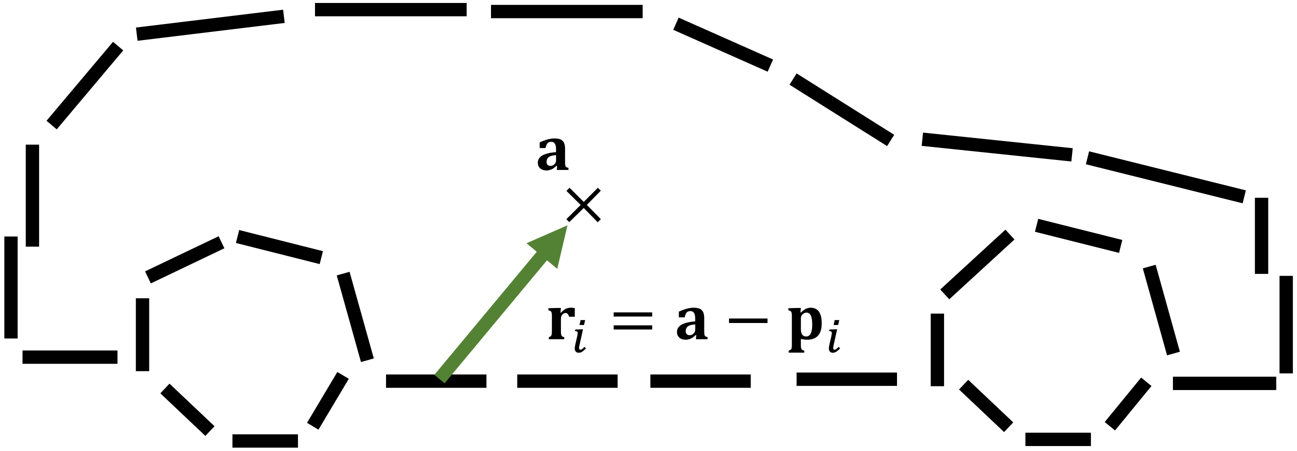
定义一个参考点，a



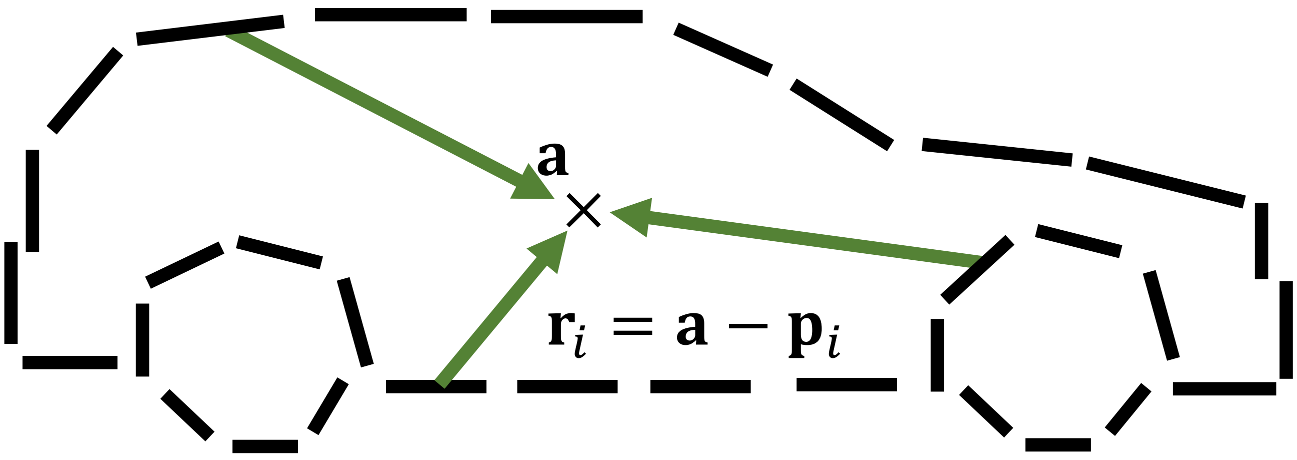
定义一个参考点，a



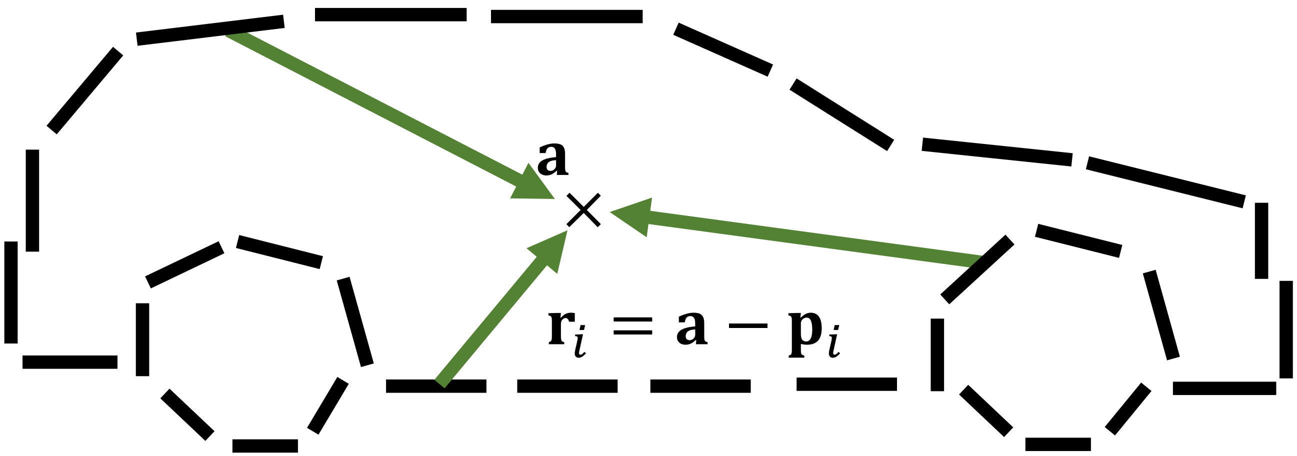
在每个边界点，计算位移向量$\mathbf{r}_i = \mathbf{a} - \mathbf{p}_i$



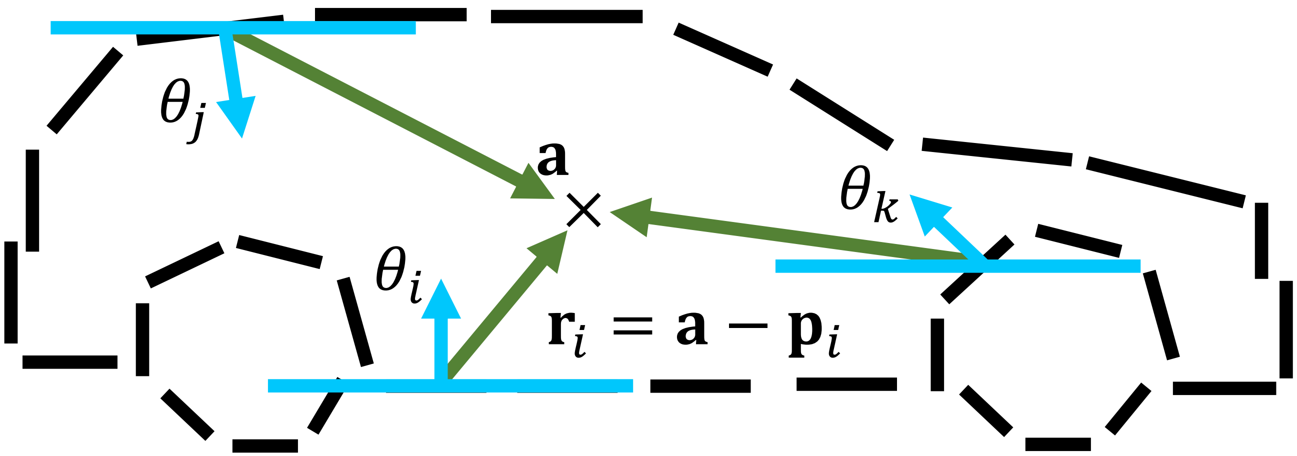
在每个边界点，计算位移向量$\mathbf{r}_i = \mathbf{a} - \mathbf{p}_i$



在每个边界点，计算位移向量$\mathbf{r}_i = \mathbf{a} - \mathbf{p}_i$



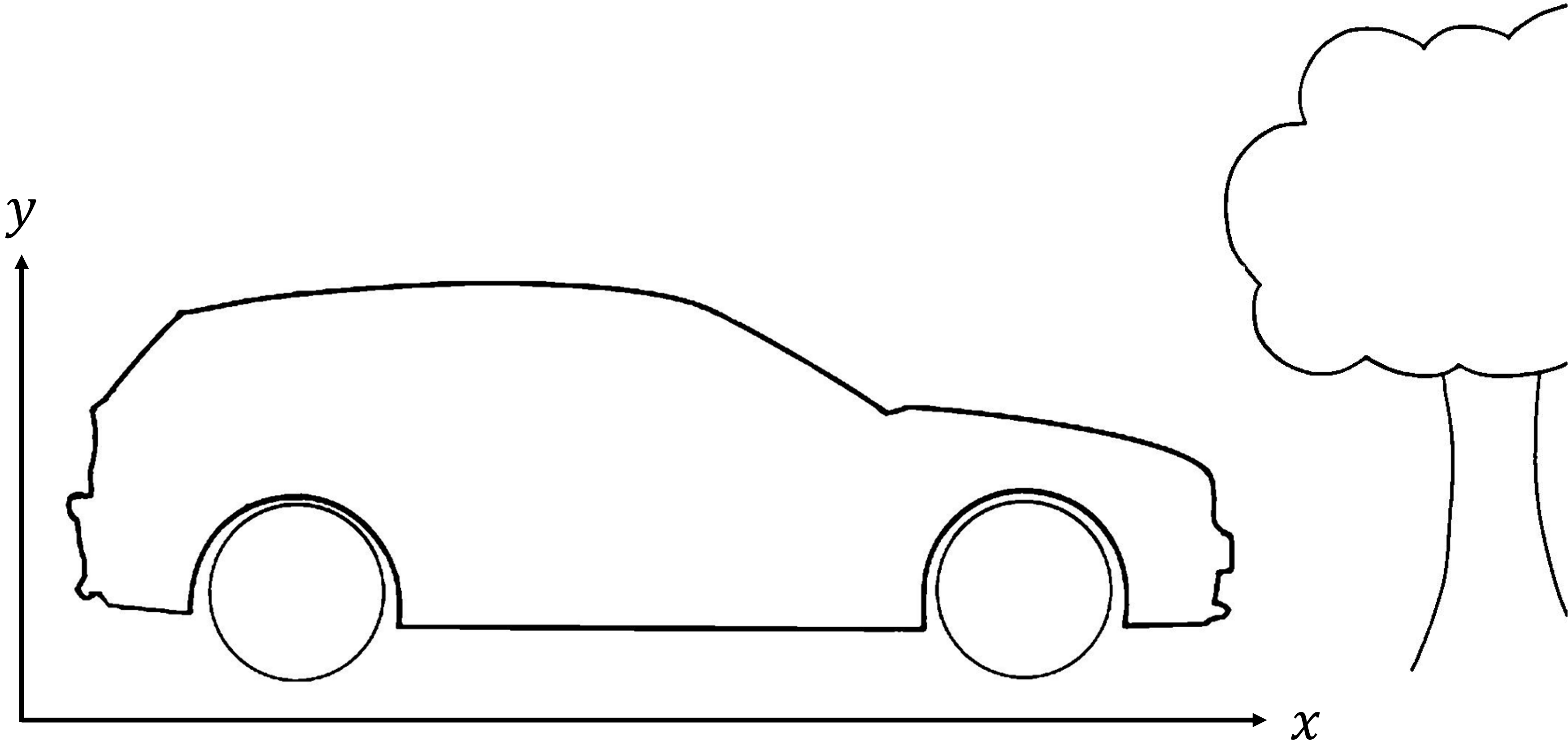
将位移矢量存储在由梯度方向θ索引的表中



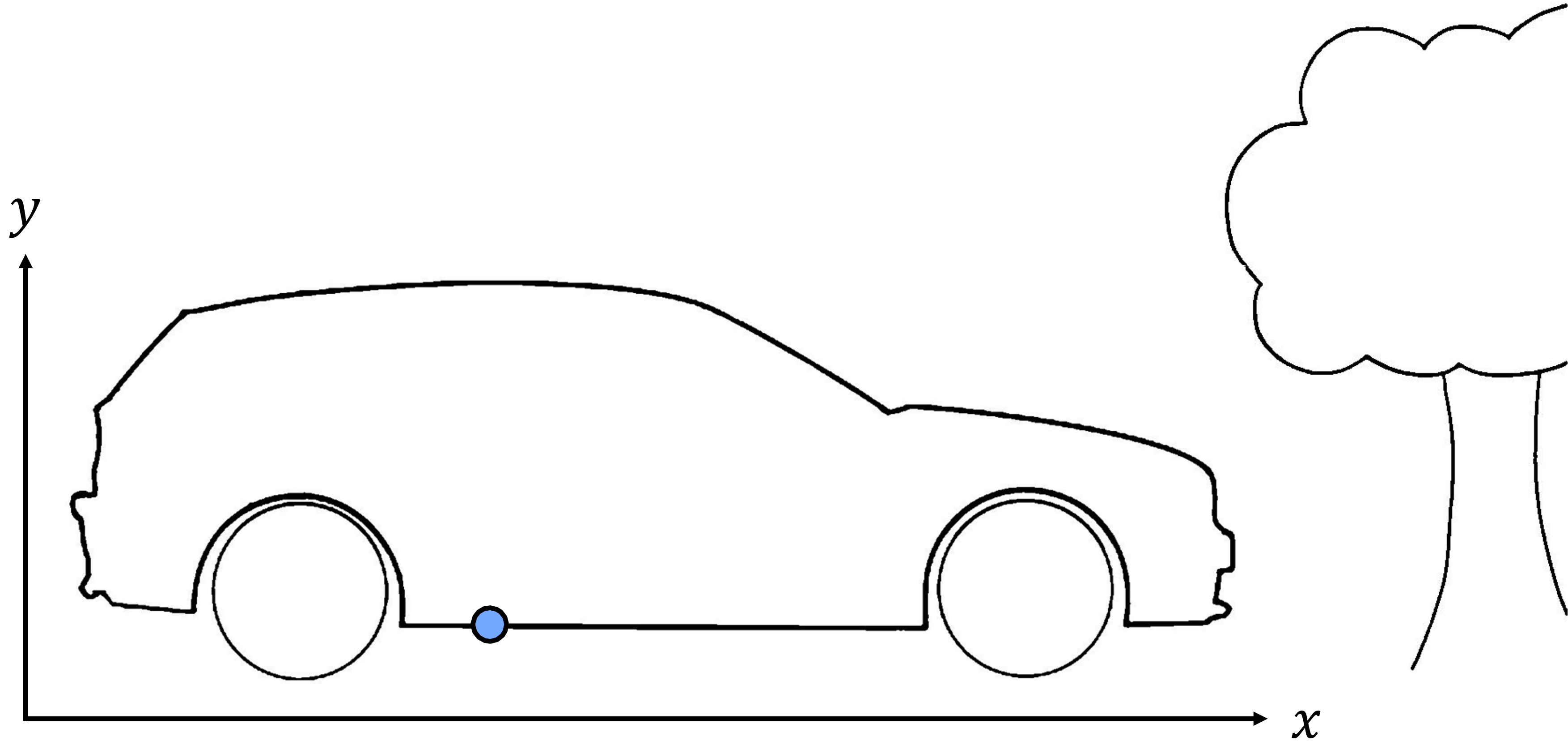
将位移矢量存储在由梯度方向θ索引的表中

步骤2

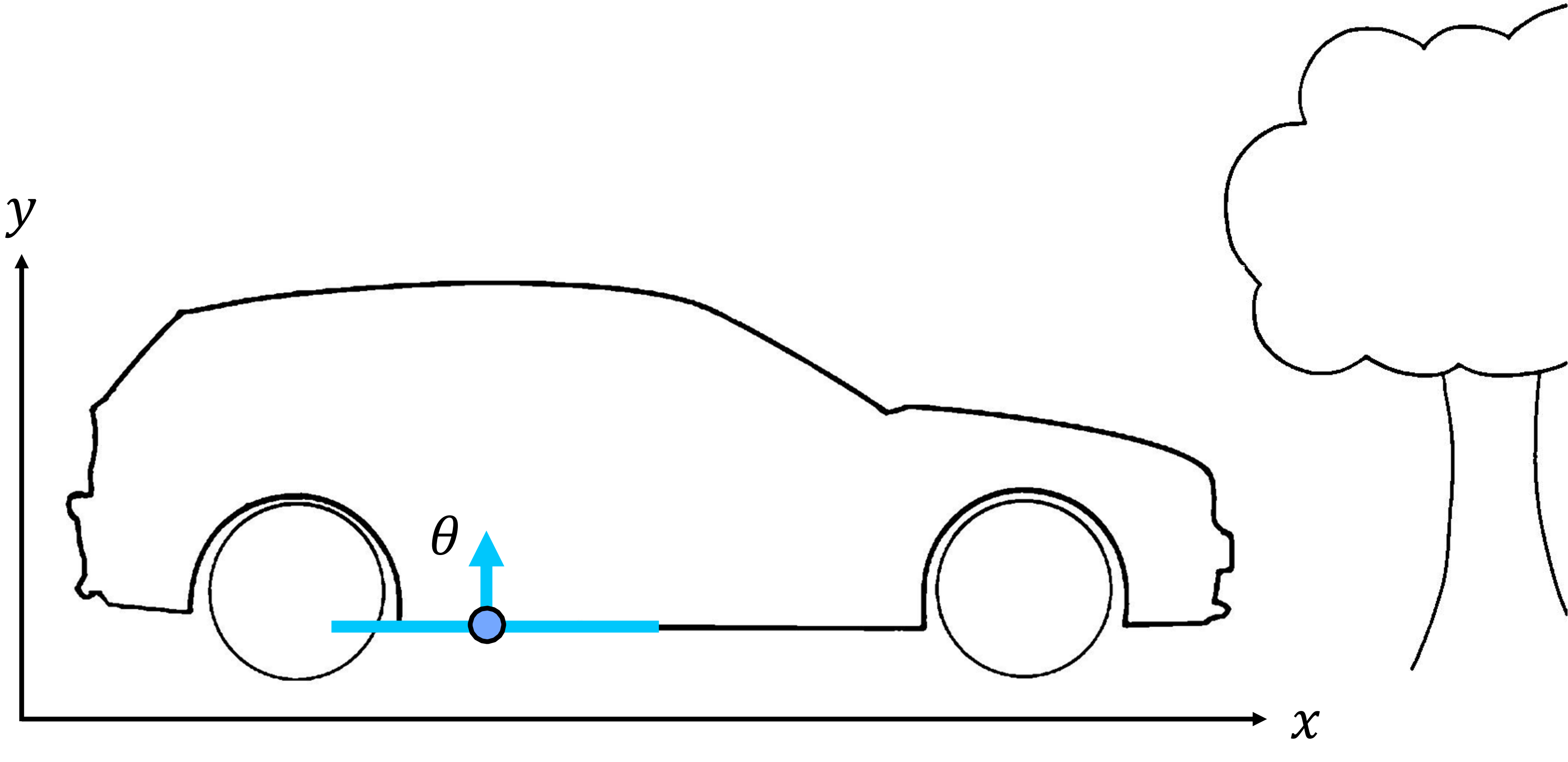
形状检测



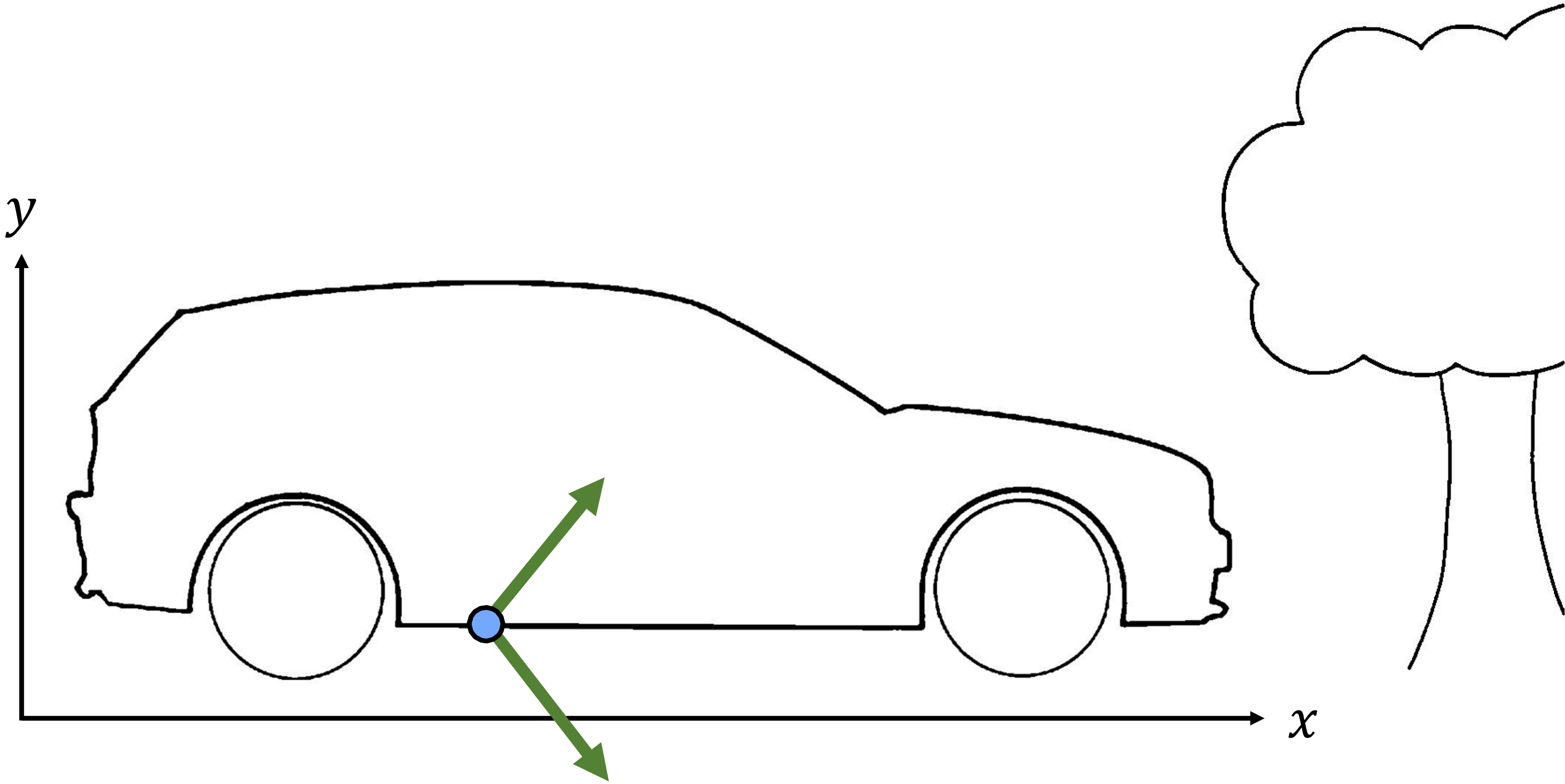
y
x



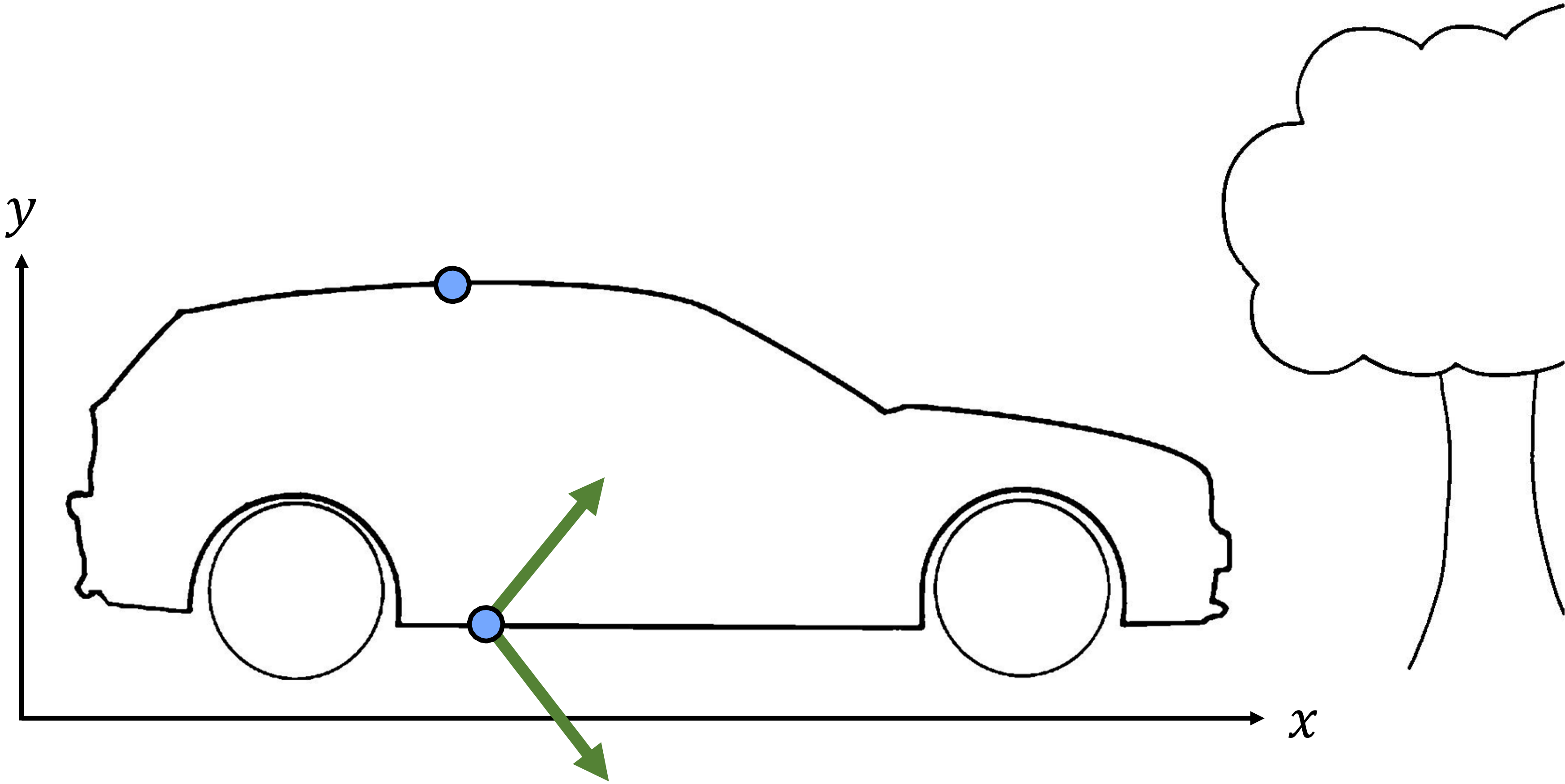
y
x



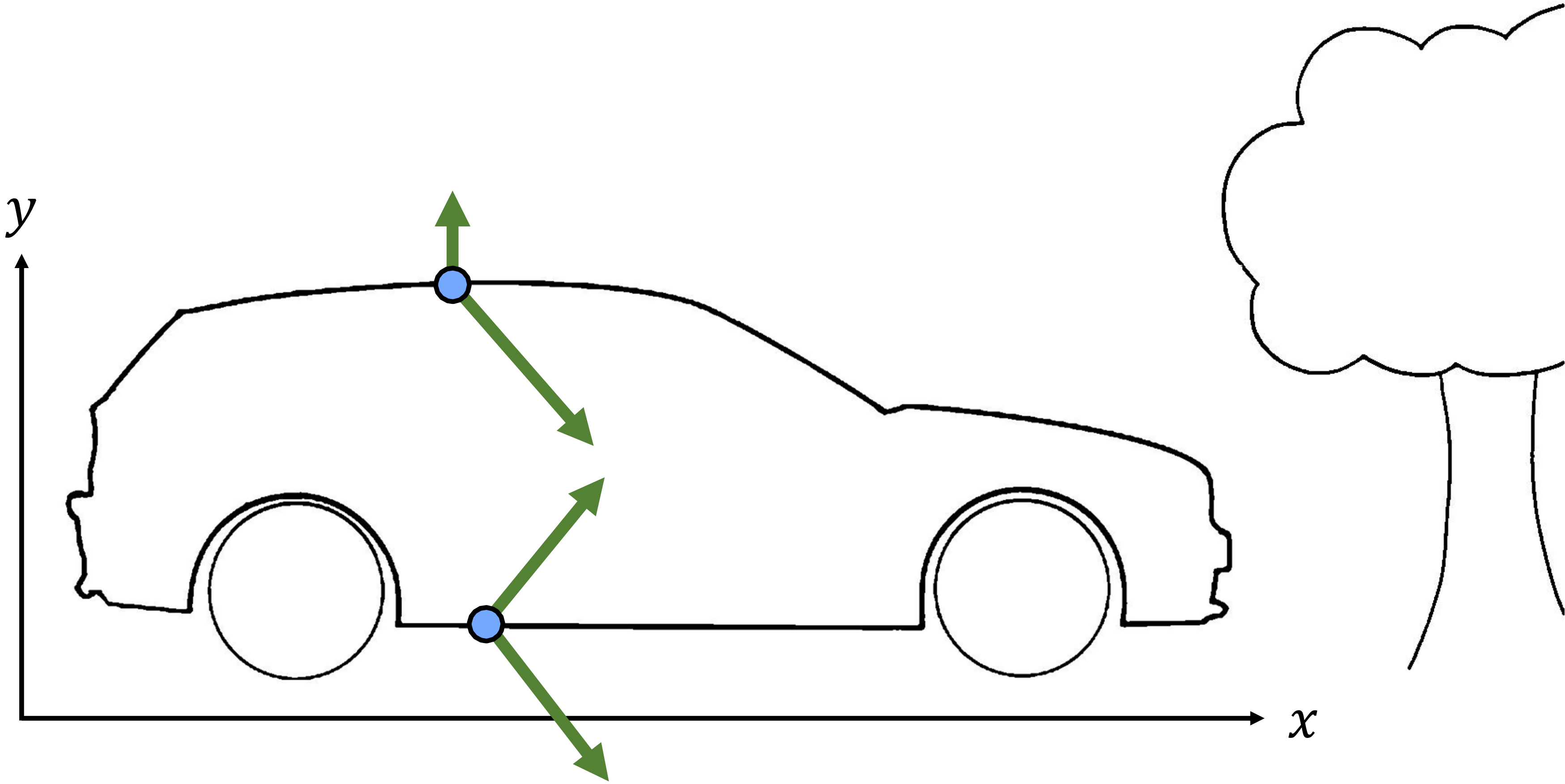
y
θ
x



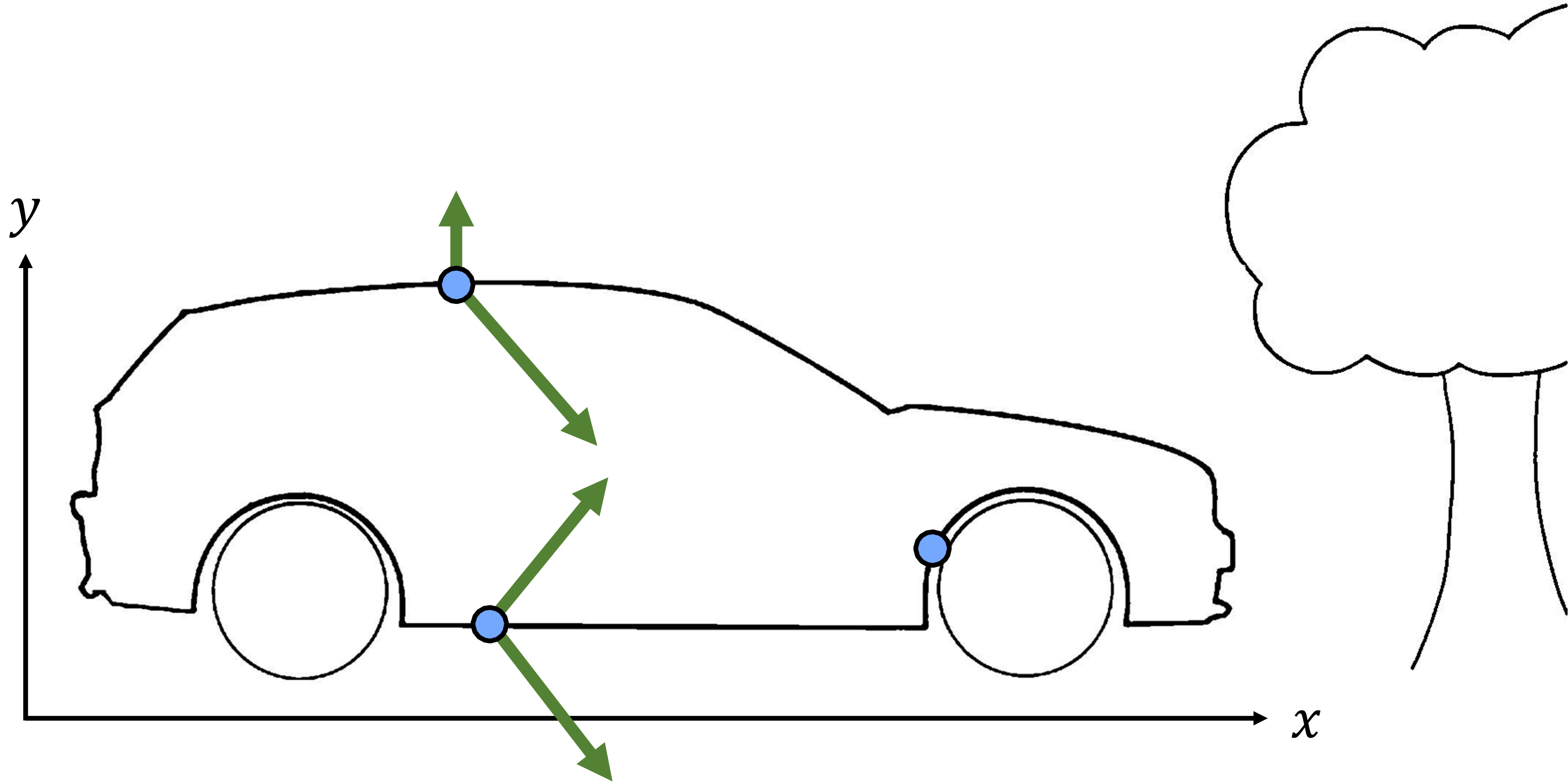
y
x

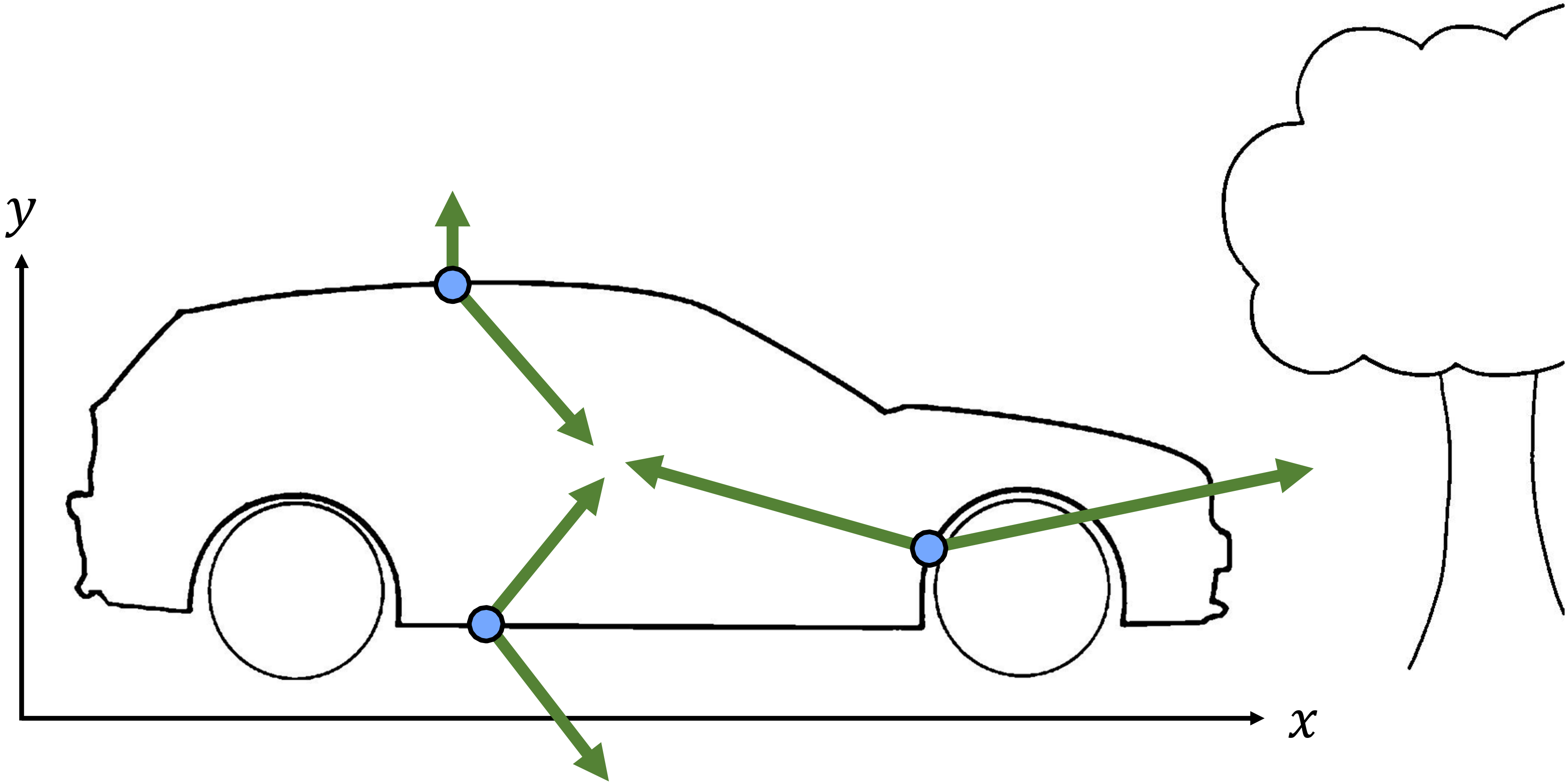


y
x

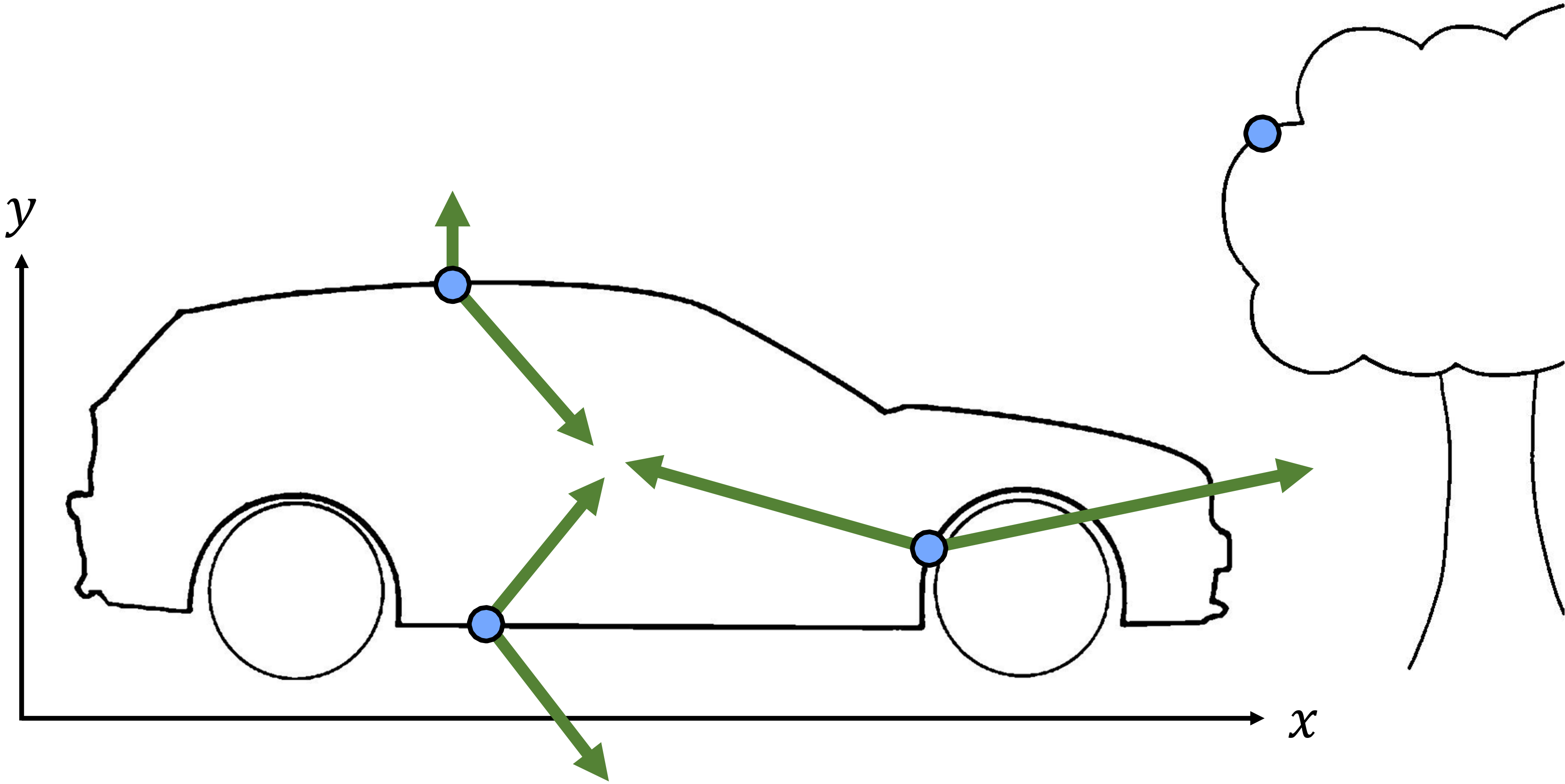


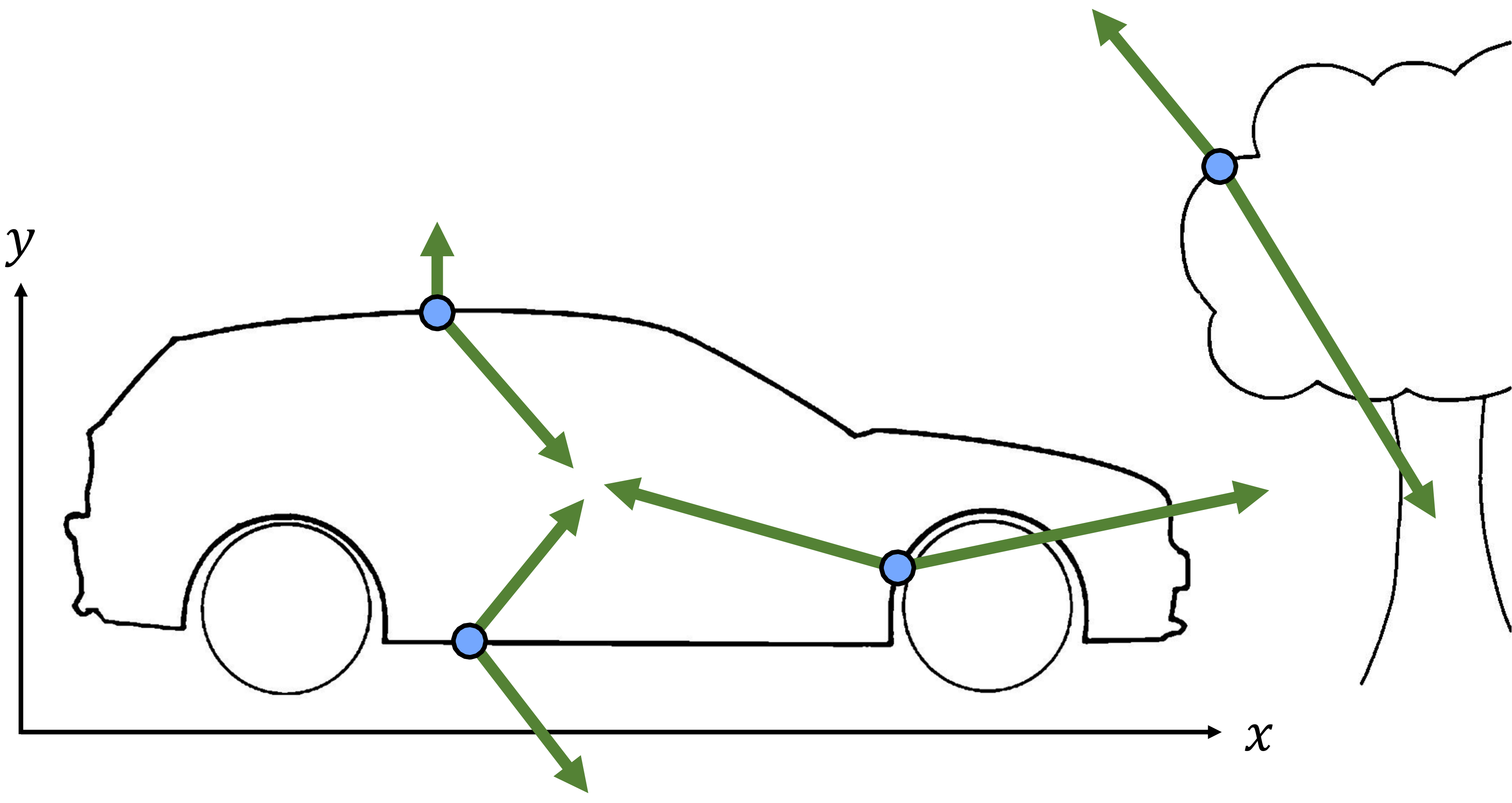
y
x

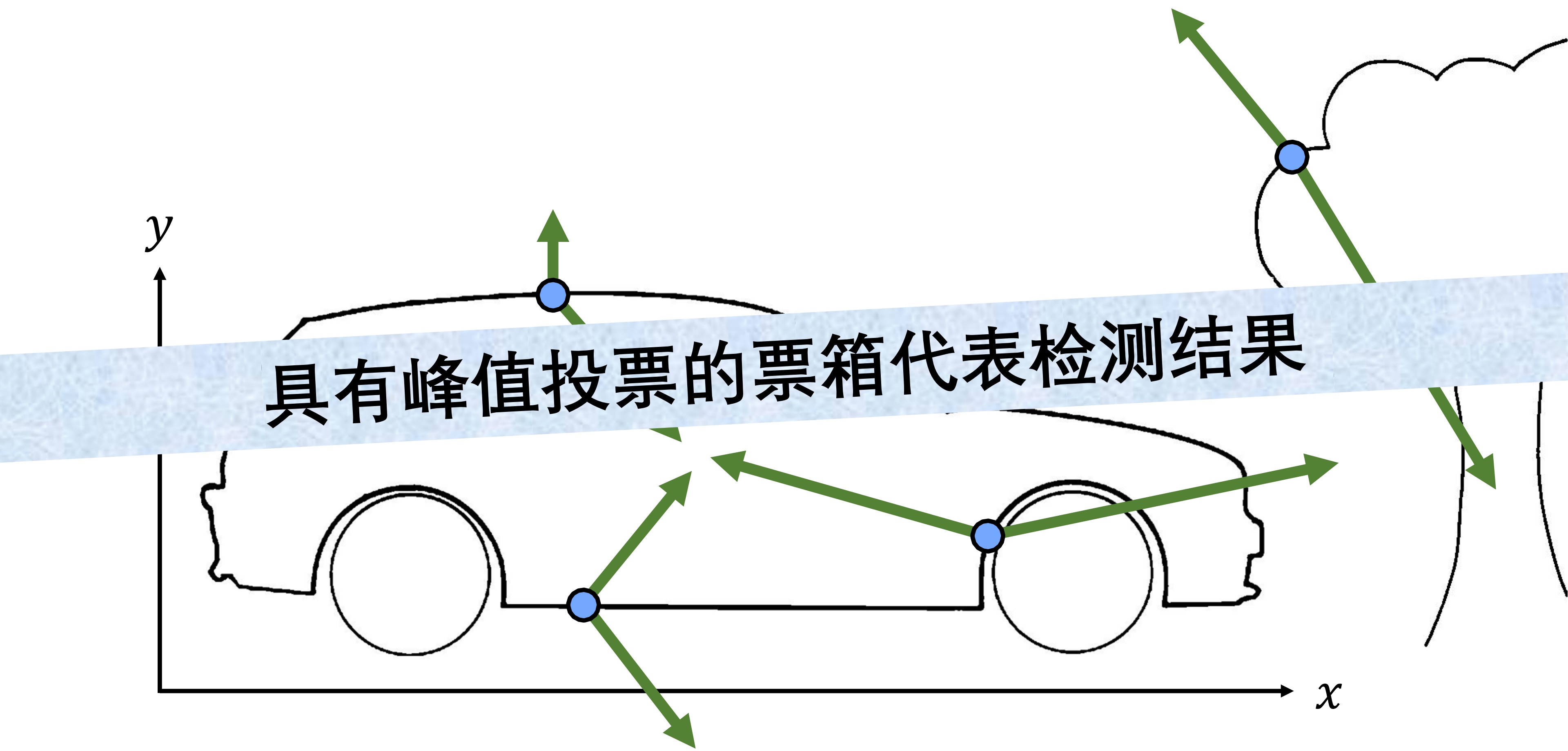

y
x

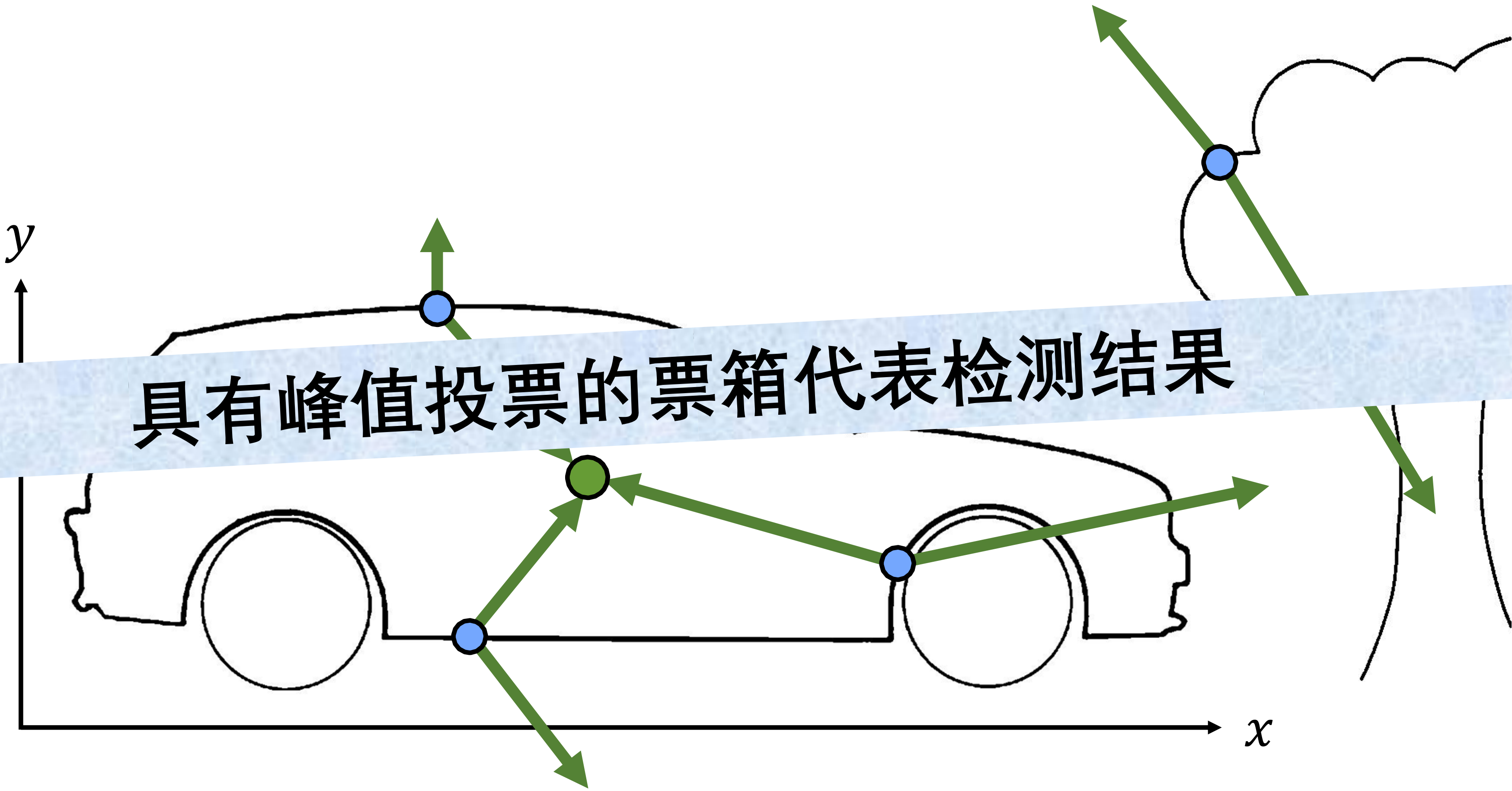


y
x


y
x


y
x


具有峰值投票的票箱代表检测结果
y
x


y
x
具有峰值投票的票箱代表检测结果

视觉码字

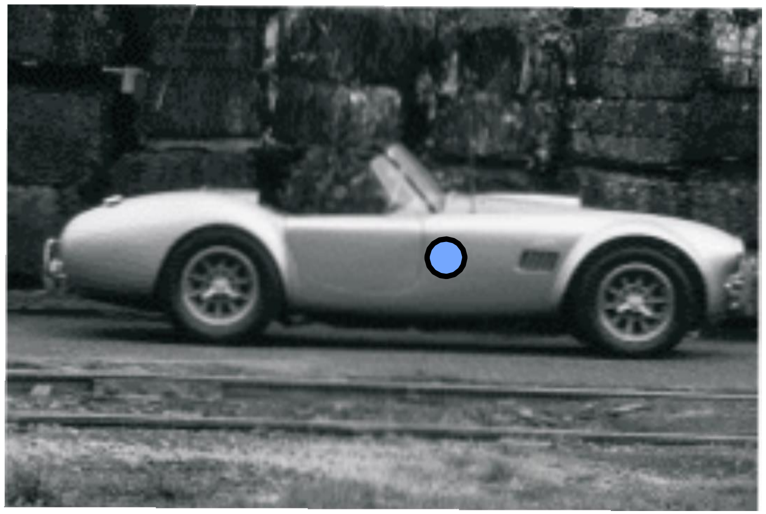
视觉码字

用“视觉码字”对位移进行索引，
而不是用梯度方向

训练图像

视觉码字

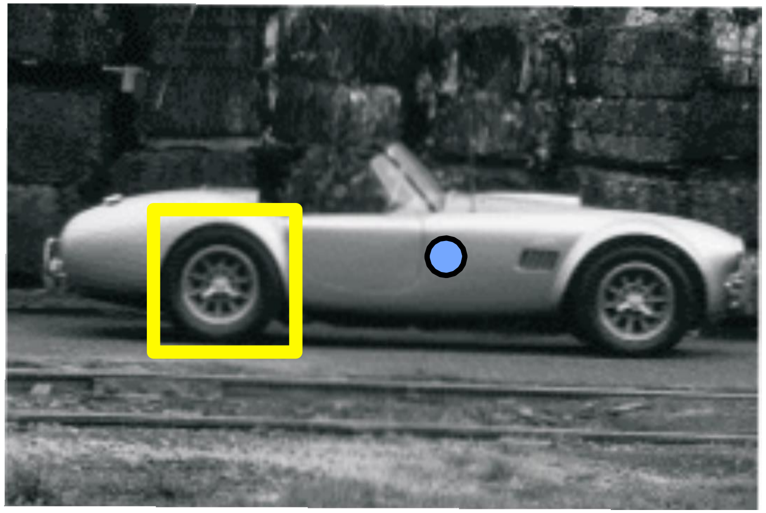
用“视觉码字”对位移进行索引，
而不是用梯度方向



训练图像

视觉码字

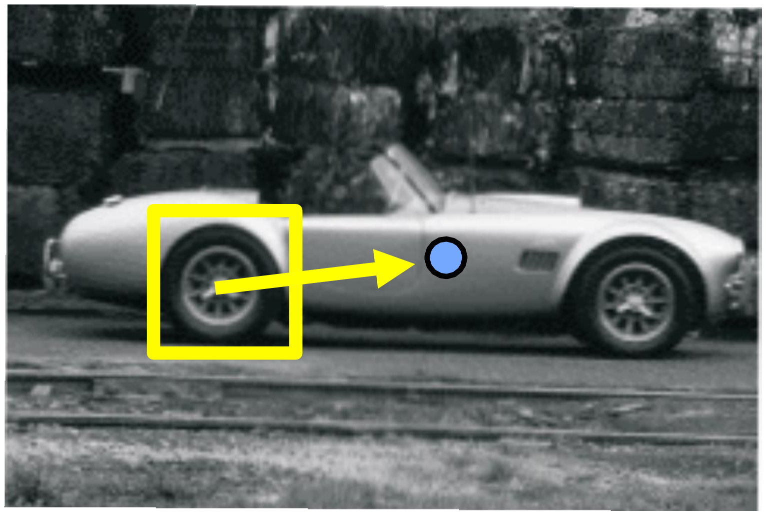
用“视觉码字”对位移进行索引，
而不是用梯度方向



训练图像

视觉码字

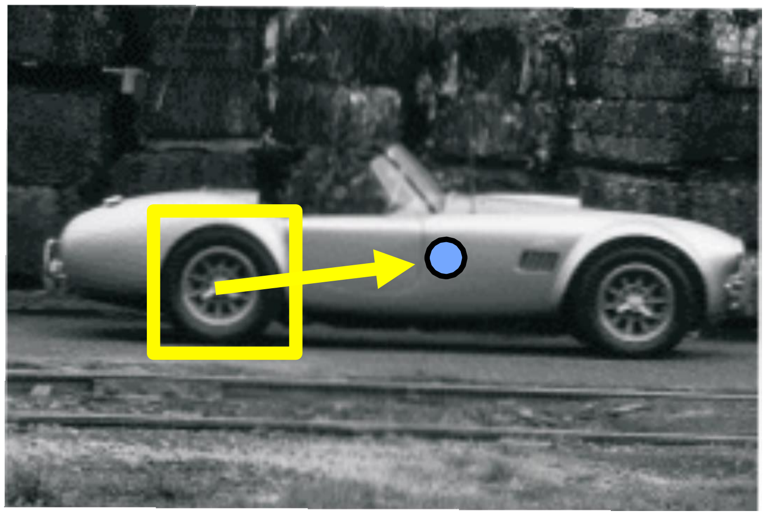
用“视觉码字”对位移进行索引，
而不是用梯度方向

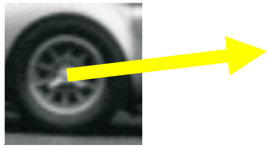


训练图像

视觉码字

用“视觉码字”对位移进行索引，
而不是用梯度方向





用“视觉码字”对位移进行索引，
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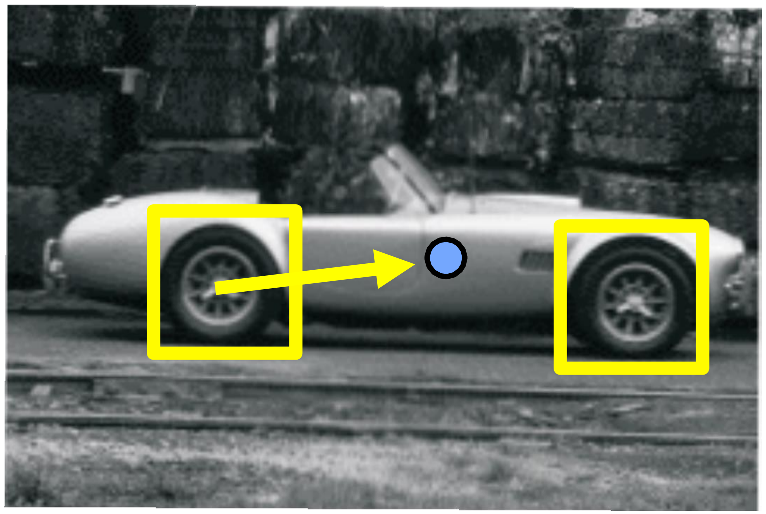
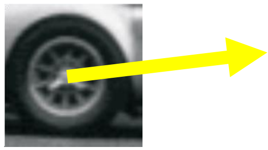
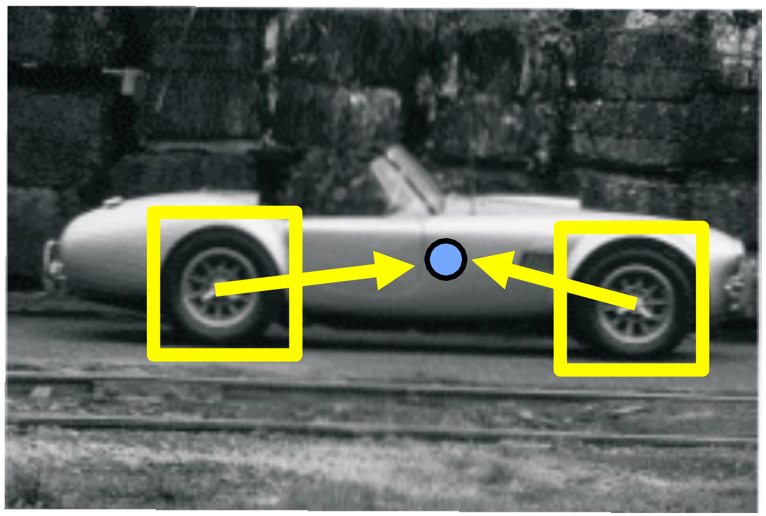


训练图像

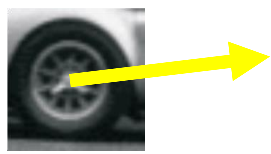


视觉码字

用“视觉码字”对位移进行索引，
而不是用梯度方向

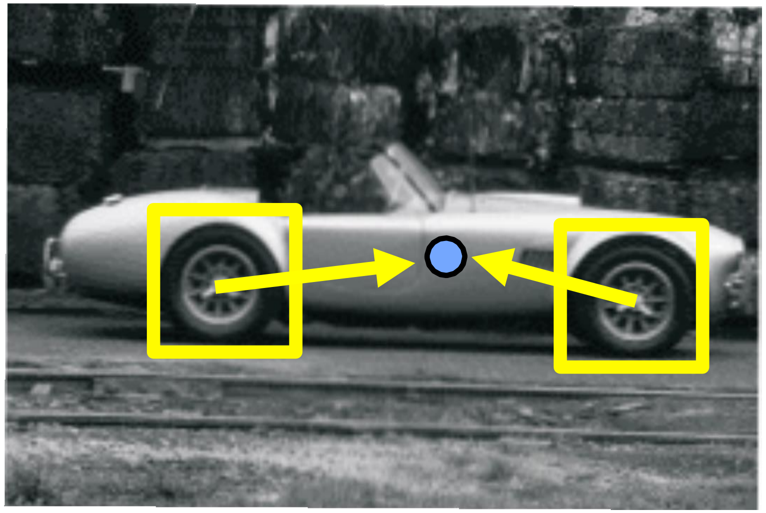


训练图像

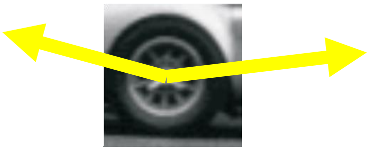


视觉码字

用“视觉码字”对位移进行索引，
而不是用梯度方向



训练图像

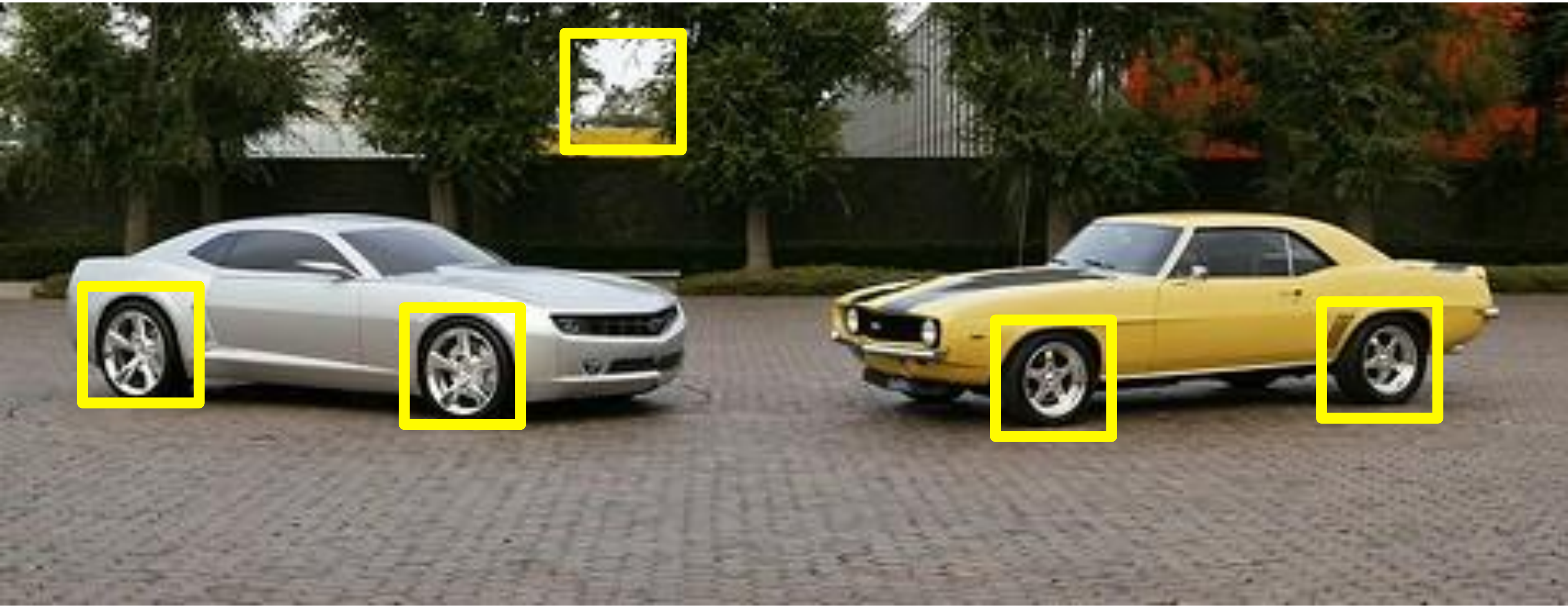


视觉码字

用“视觉码字”对位移进行索引，
而不是用梯度方向

测试图像

Leibe, Leonardis and Schiele, "Combined Object Categorization and Segmentation with Implicit Shape Model", 2004



测试图像

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测试图像

Leibe, Leonardis and Schiele, “Combined Object Categorization and Segmentation with Implicit Shape Model”, 2004

RANSAC

RANdom SAmple Consensus

随机抽样一致

Graphics and Image Processing

J. D. Foley
Editor

Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

Martin A. Fischler and Robert C. Bolles
SRI International

A new paradigm, Random Sample Consensus (RANSAC), for fitting a model to experimental data is introduced. RANSAC is capable of interpreting/smoothing data containing a significant percentage of gross errors, and is thus ideally suited for applications in automated image analysis where interpretation is based on the data provided by error-prone feature detectors. A major portion of this paper describes the application of RANSAC to the Location Determination Problem (LDP): Given an image depicting a set of landm[illegible] in space [illegible] response to a RANSAC requirement, new results are derived on the minimum number of landmarks needed to obtain a solution, and algorithms are presented for

I. Introduction

We introduce a new paradigm, Random Sample Consensus (RANSAC), for fitting a model to experimental data; and illustrate its use in scene analysis and automated cartography. The application discussed, the location determination problem (LDP), is treated at a level beyond that of a mere example of the use of the RANSAC paradigm; new basic findings concerning the conditions under which the LDP can be solved are presented and a comprehensive approach to the solution of this problem that we anticipate will have near-term practical applications is described.

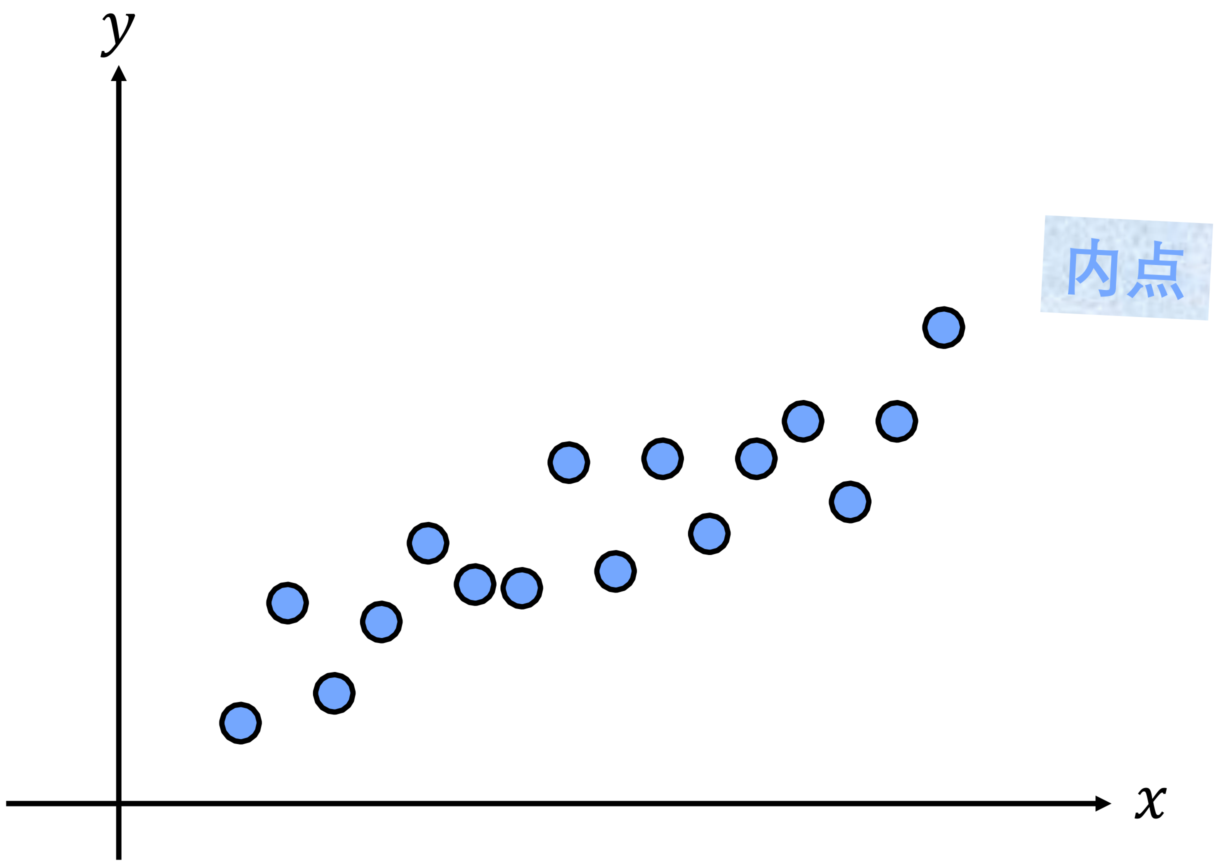
To a large extent, scene analysis (and, in fact, science in general) is concerned with the interpretation of sensed data in terms of a set of predefined models. Conceptually, interpretation involves two distinct activities: First, there is the problem of finding the best match between the data and one of the available models (the classification problem); Second, there is the problem of computing the best values for the free parameters of the selected model (the parameter estimation problem). In practice, these two problems are not independent—a solution to the parameter estimation problem is often required to solve the classification problem.

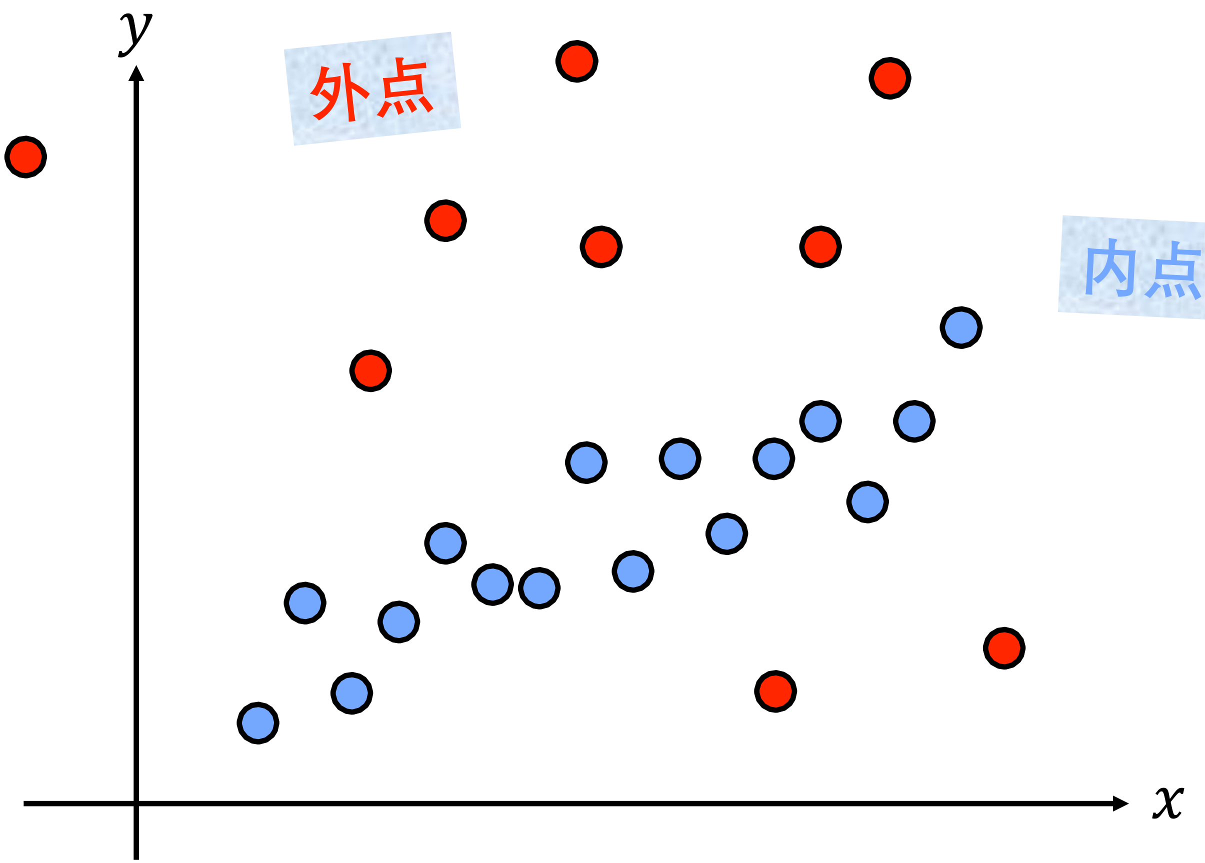
Classical techniques for parameter estimation, such as least squares, optimize (according to a specified objective function) the fit of a functional description (model) to *all* of the presented data. These techniques have no internal mechanisms for detecting and rejecting gross errors. They [illegible] that the maximum expected deviation of any datum from the assumed model is a direct function of the size of the data set, and thus regardless of the size of the data set, there

Communications of the ACM, 1981

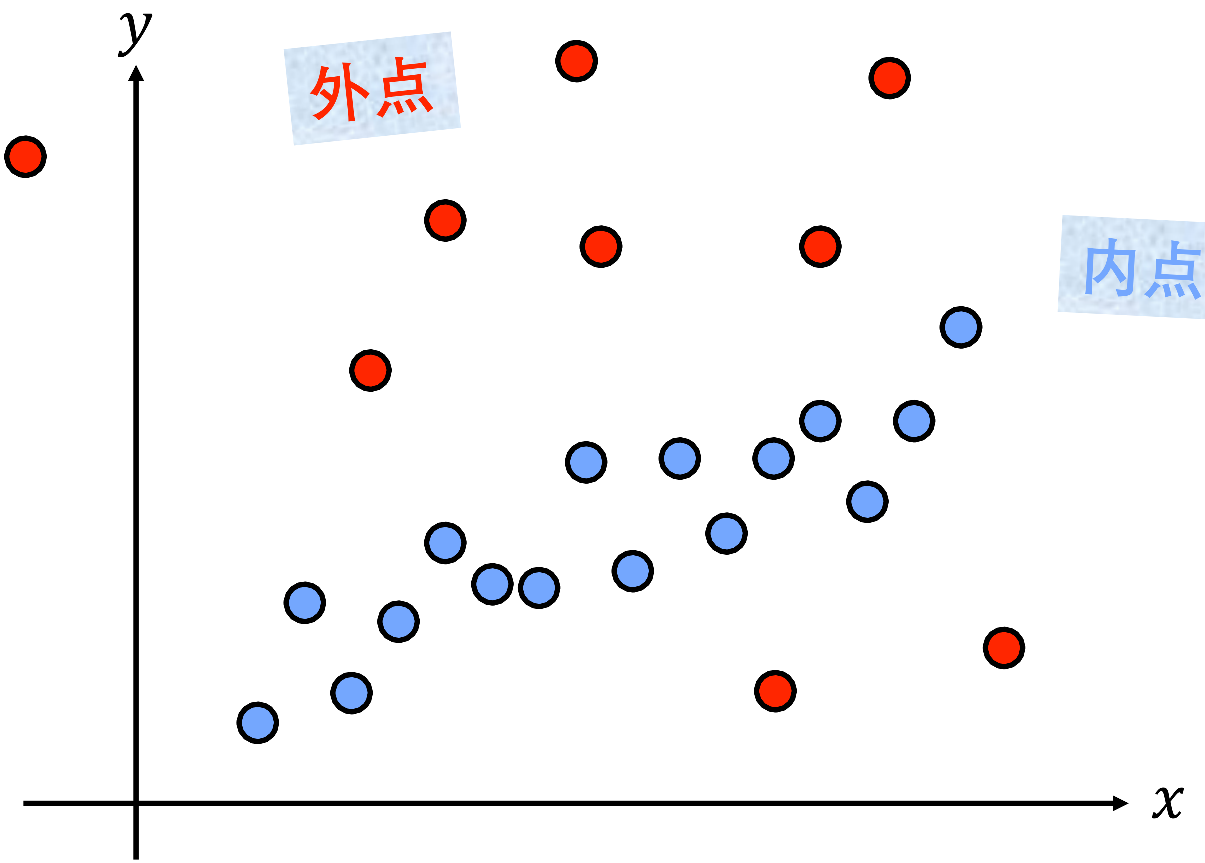
将参数模型拟合到包含外点的数据的一般过程


y
x

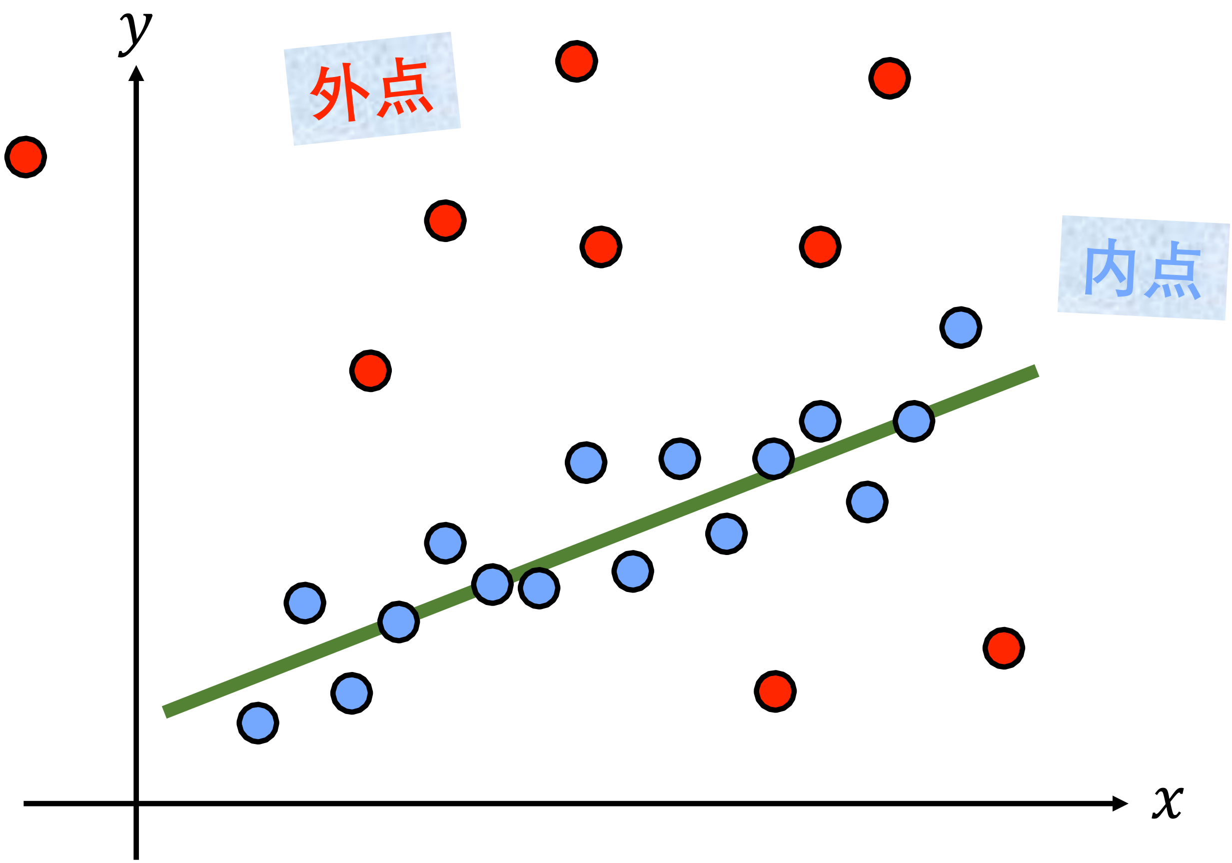

y
内点
x



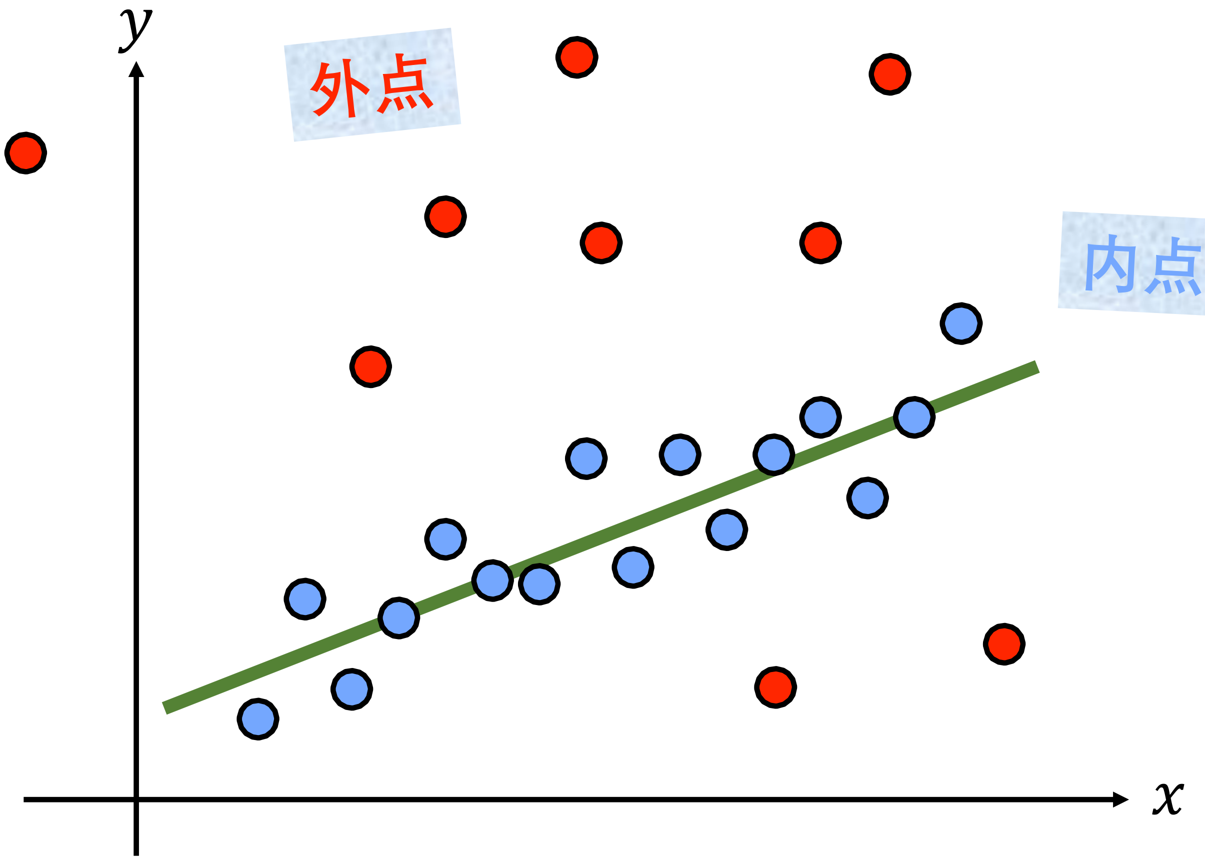
外点
内点
y
x



目标： 将模型拟合到带有外点的数据

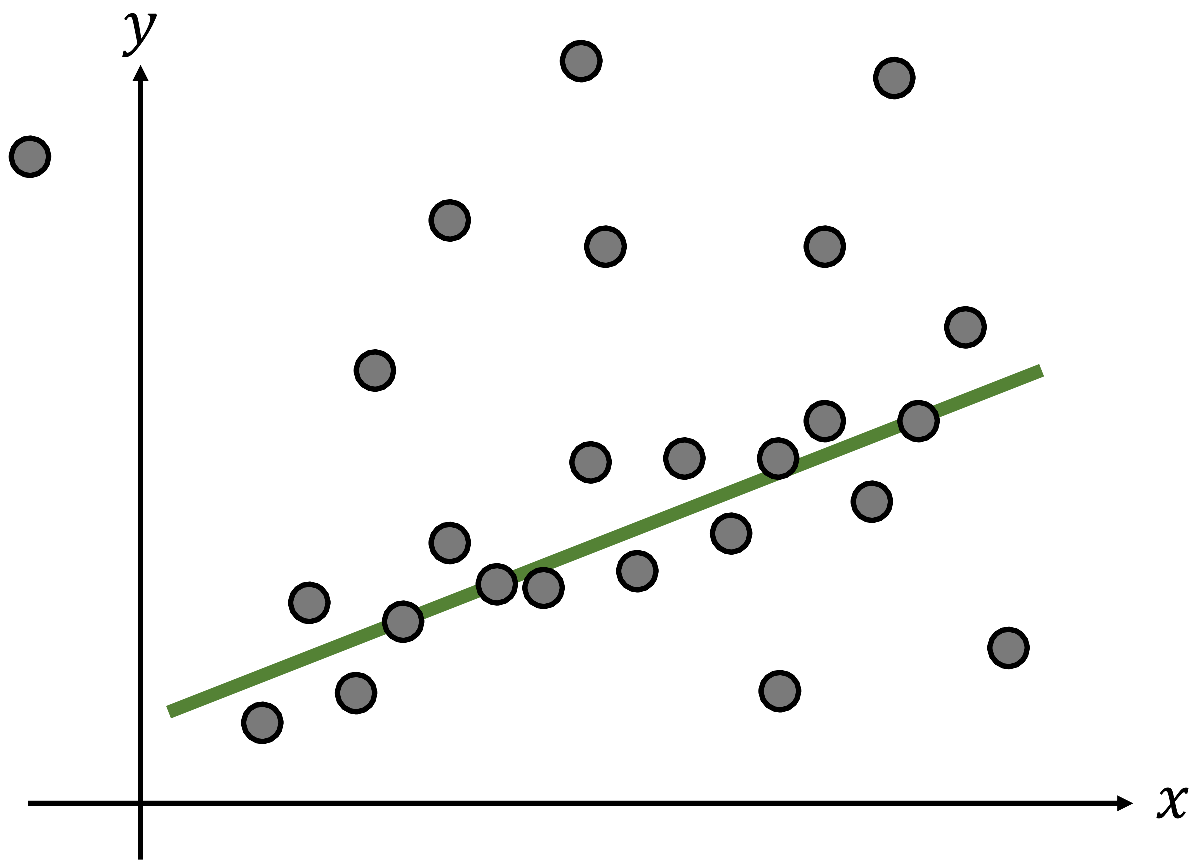


目标：将模型拟合到带有外点的数据



未知的

目标： 将模型拟合到带有外点的数据



未知的

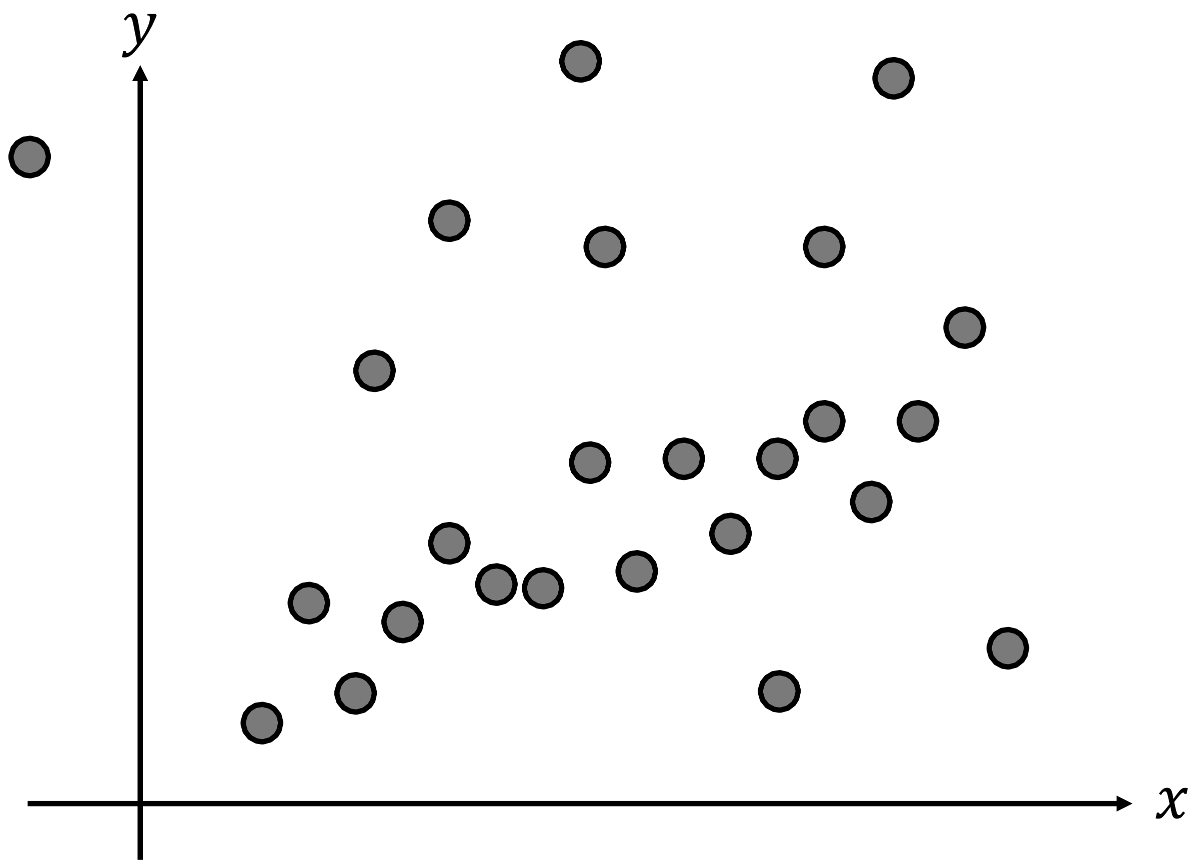
目标：将模型拟合到带有外点的数据

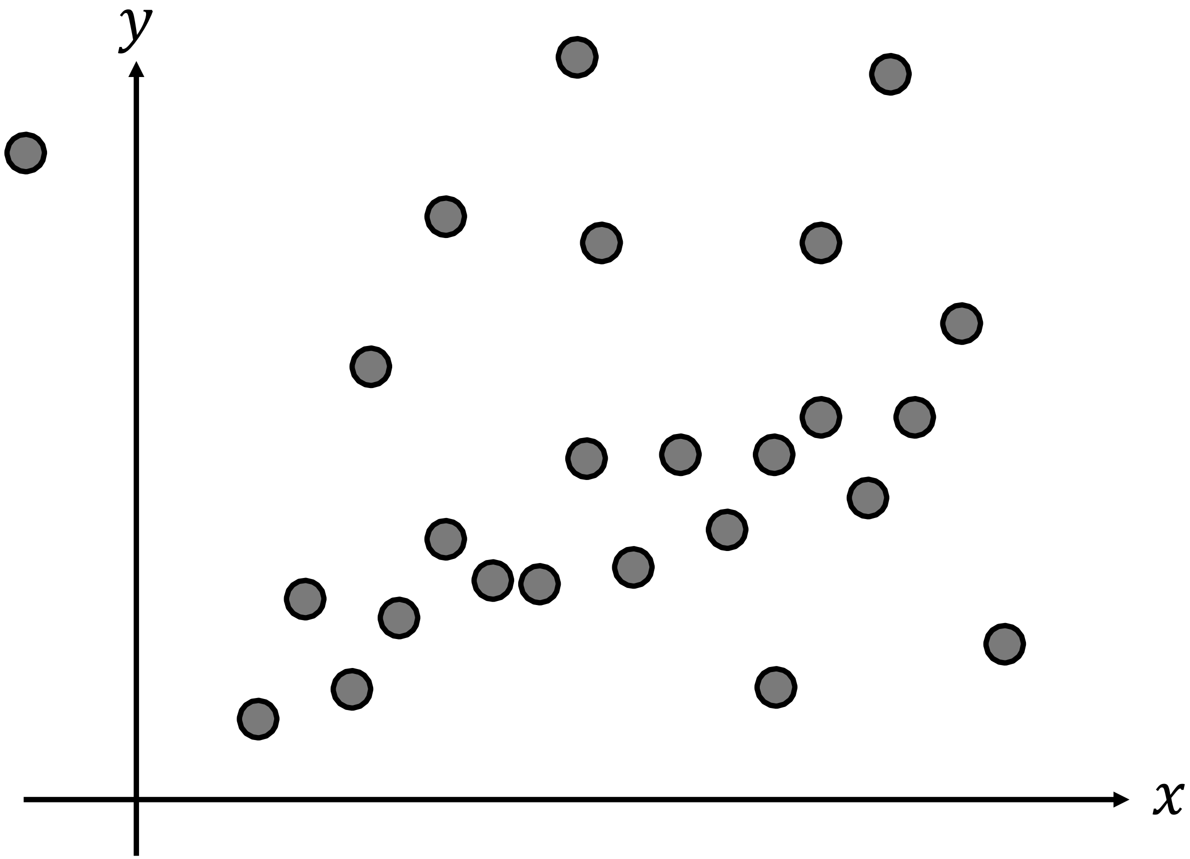
4

主要步骤

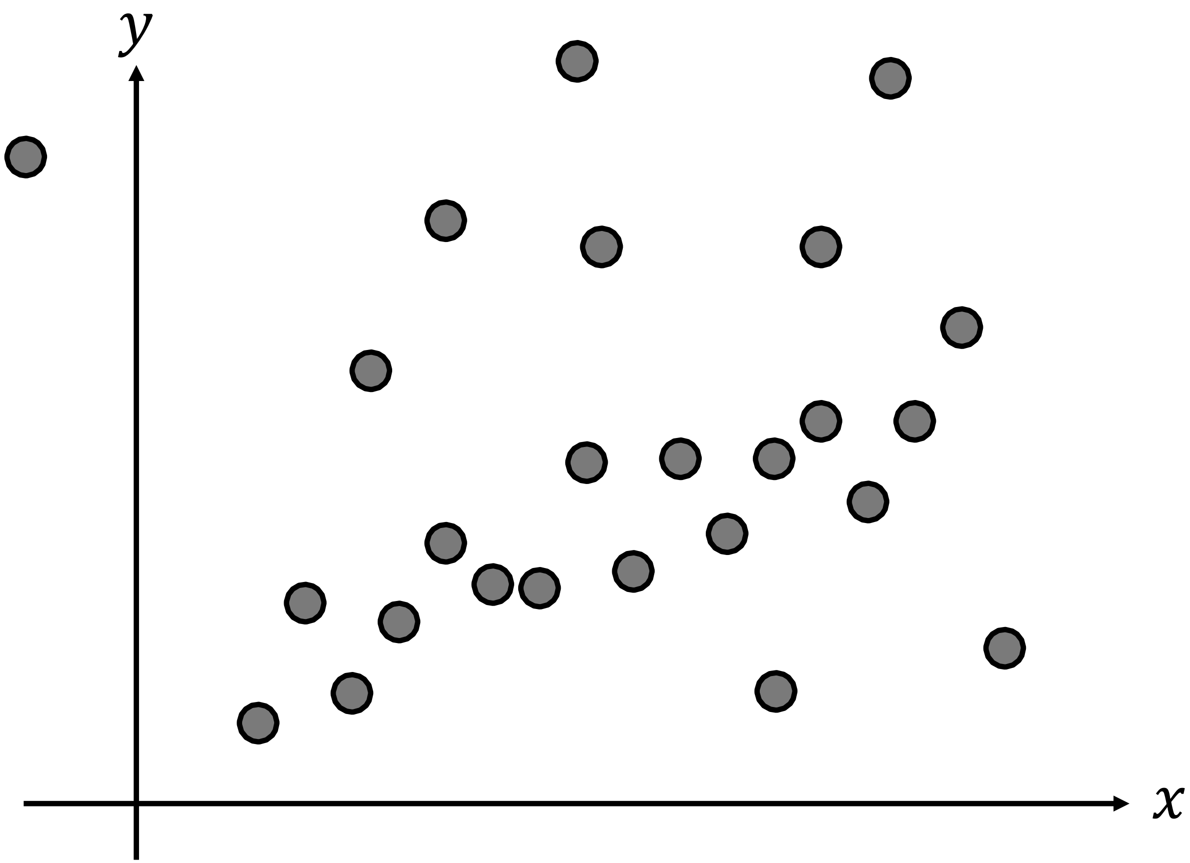
步骤1

选择最小点集

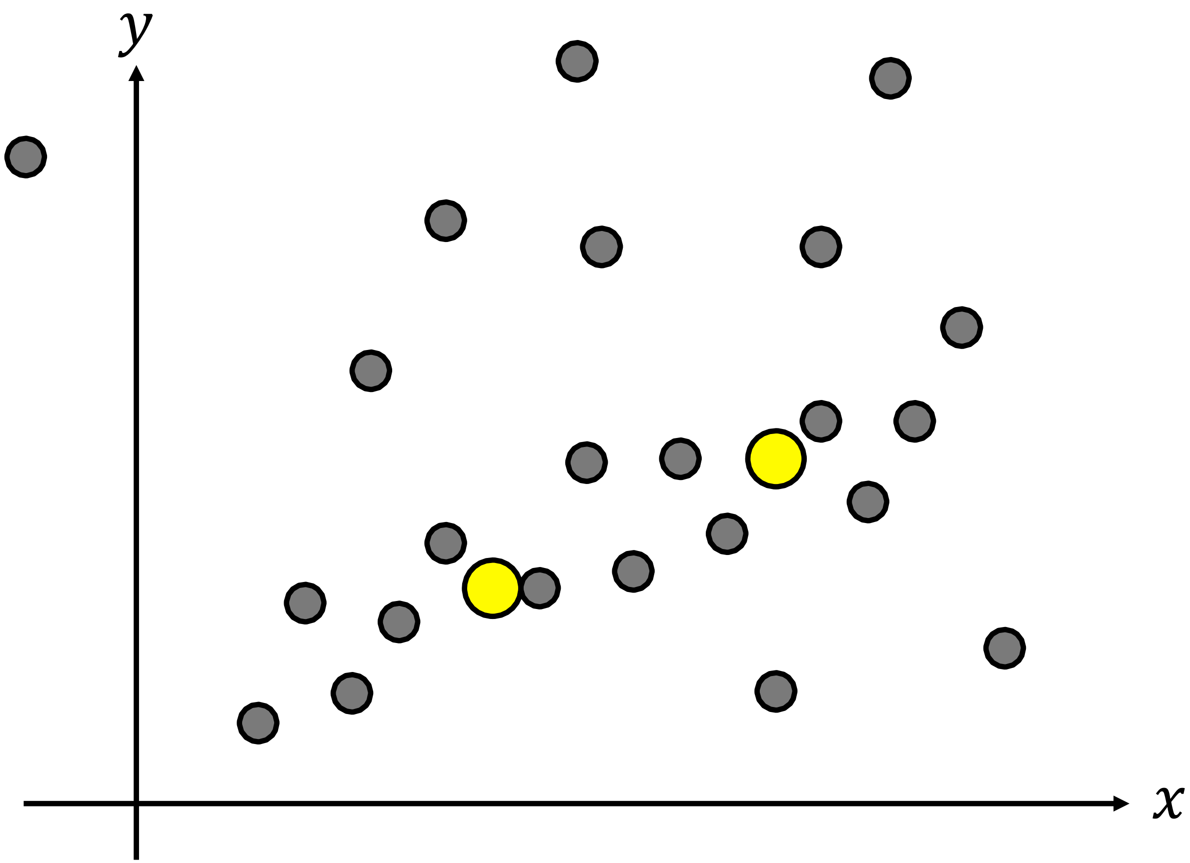

y
x



随机选择能够拟合模型的**最小**点集



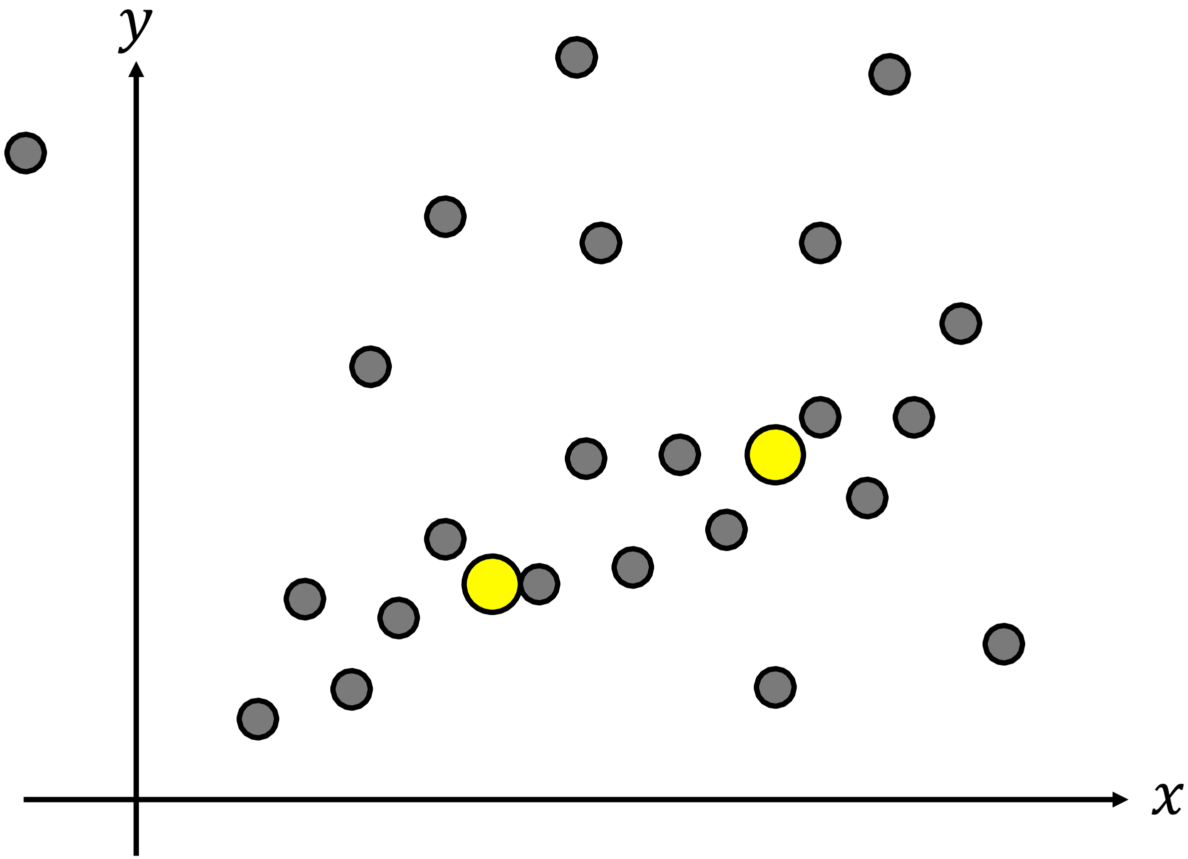
最小点集的大小是多少?

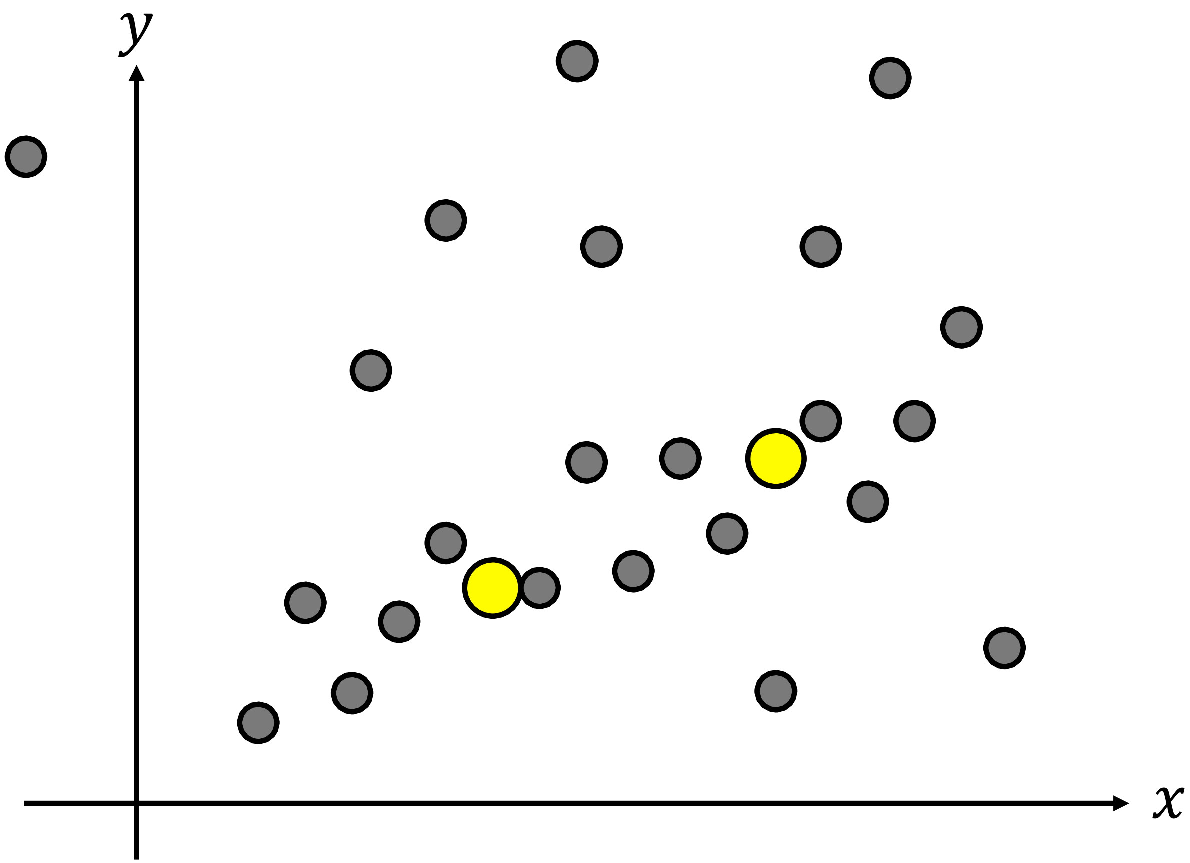


最小点集的大小是多少?

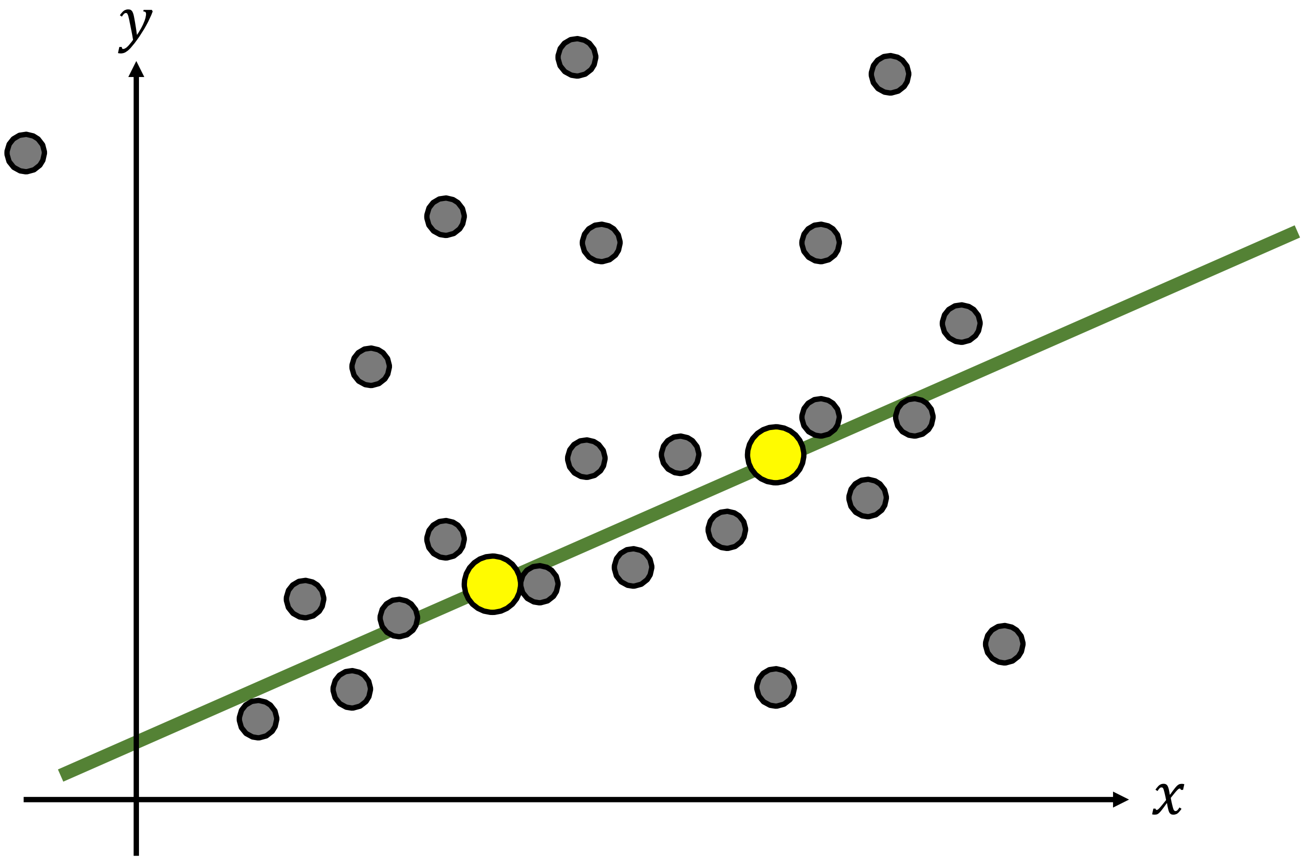
步骤2

找到最合适的直线


y
x



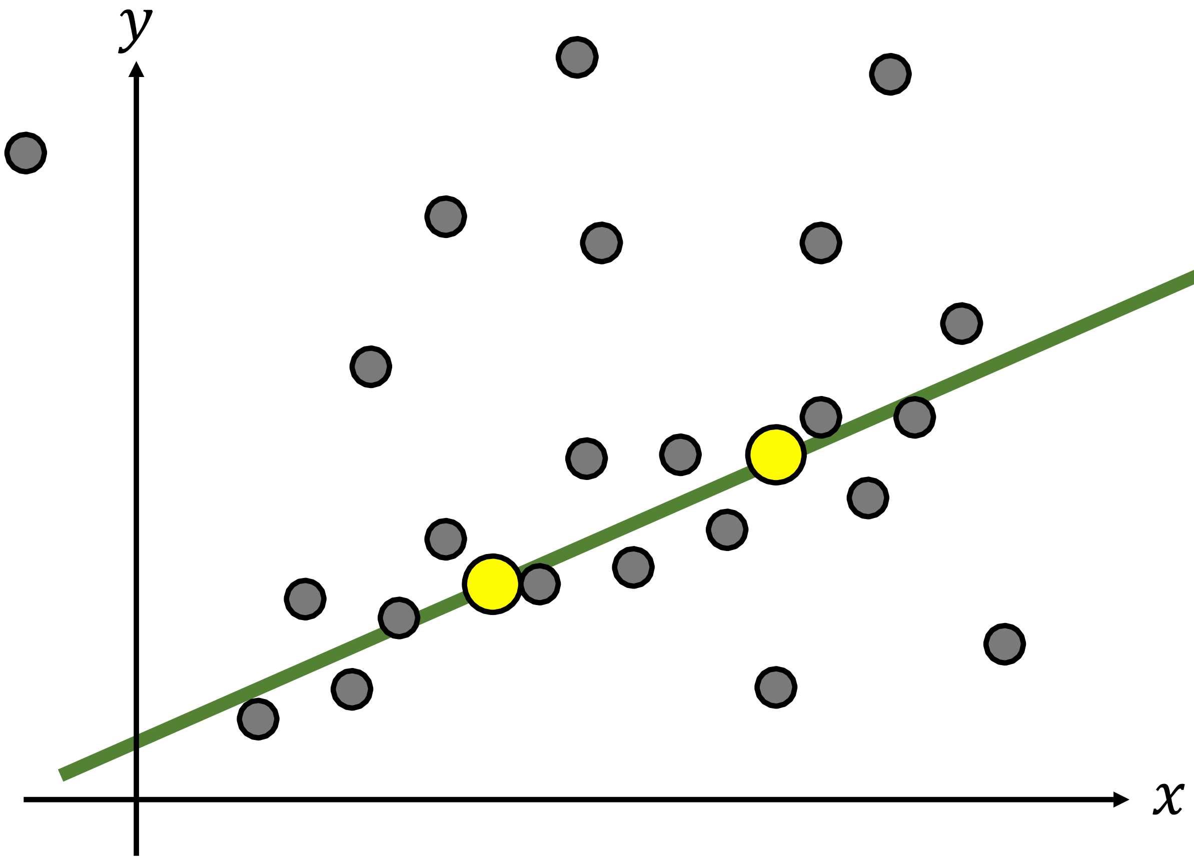
计算最合适的直线



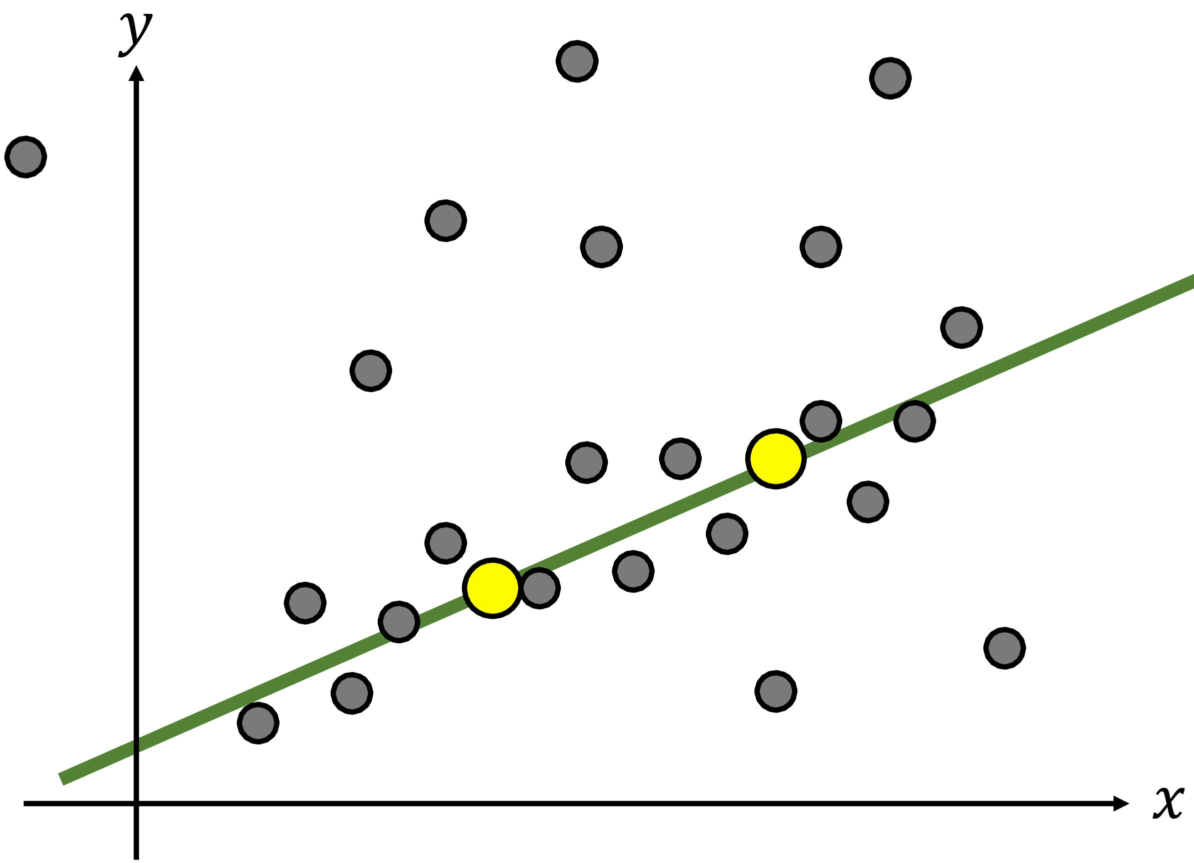
计算最合适的直线

步骤3

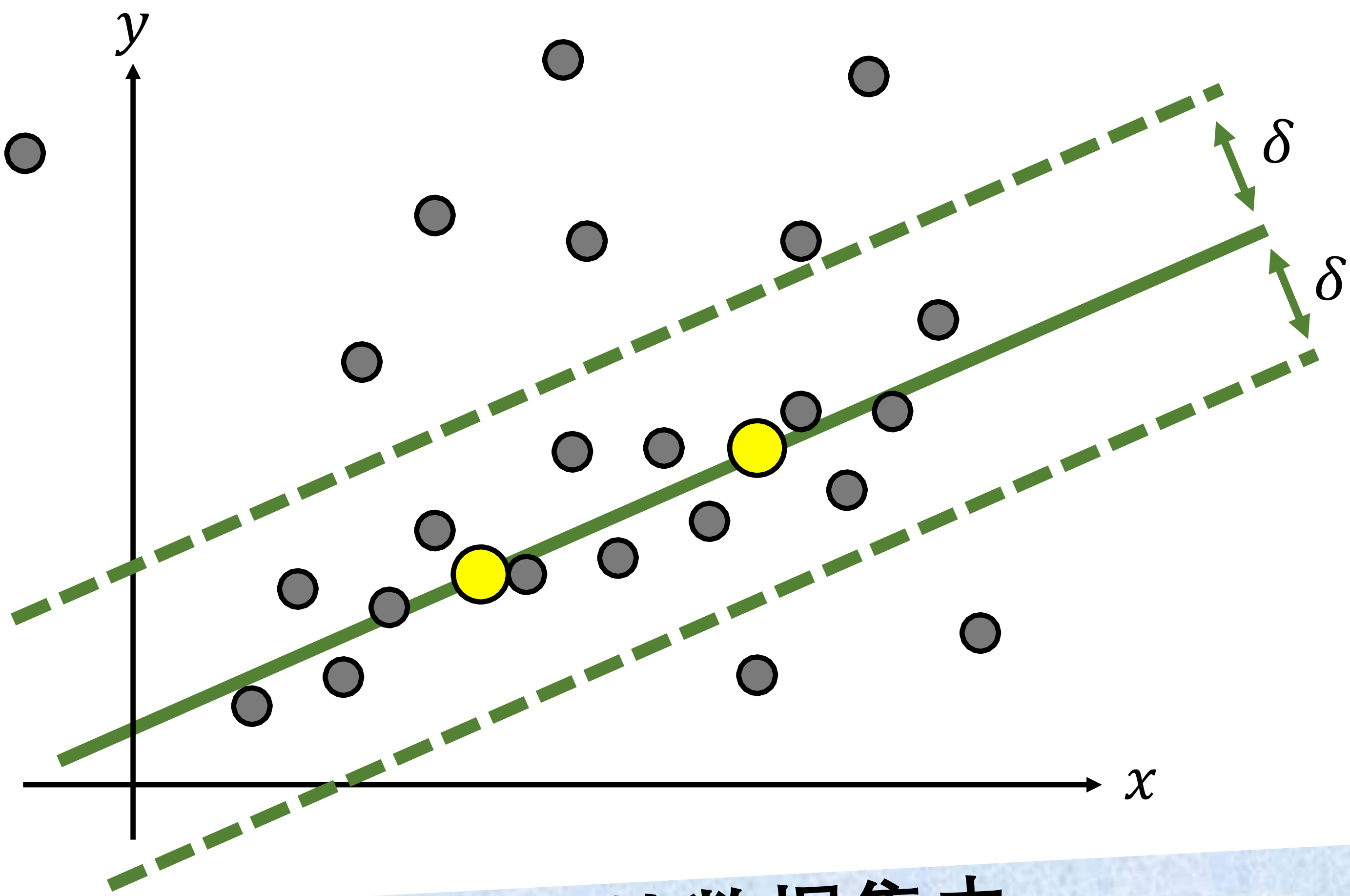
确定内点



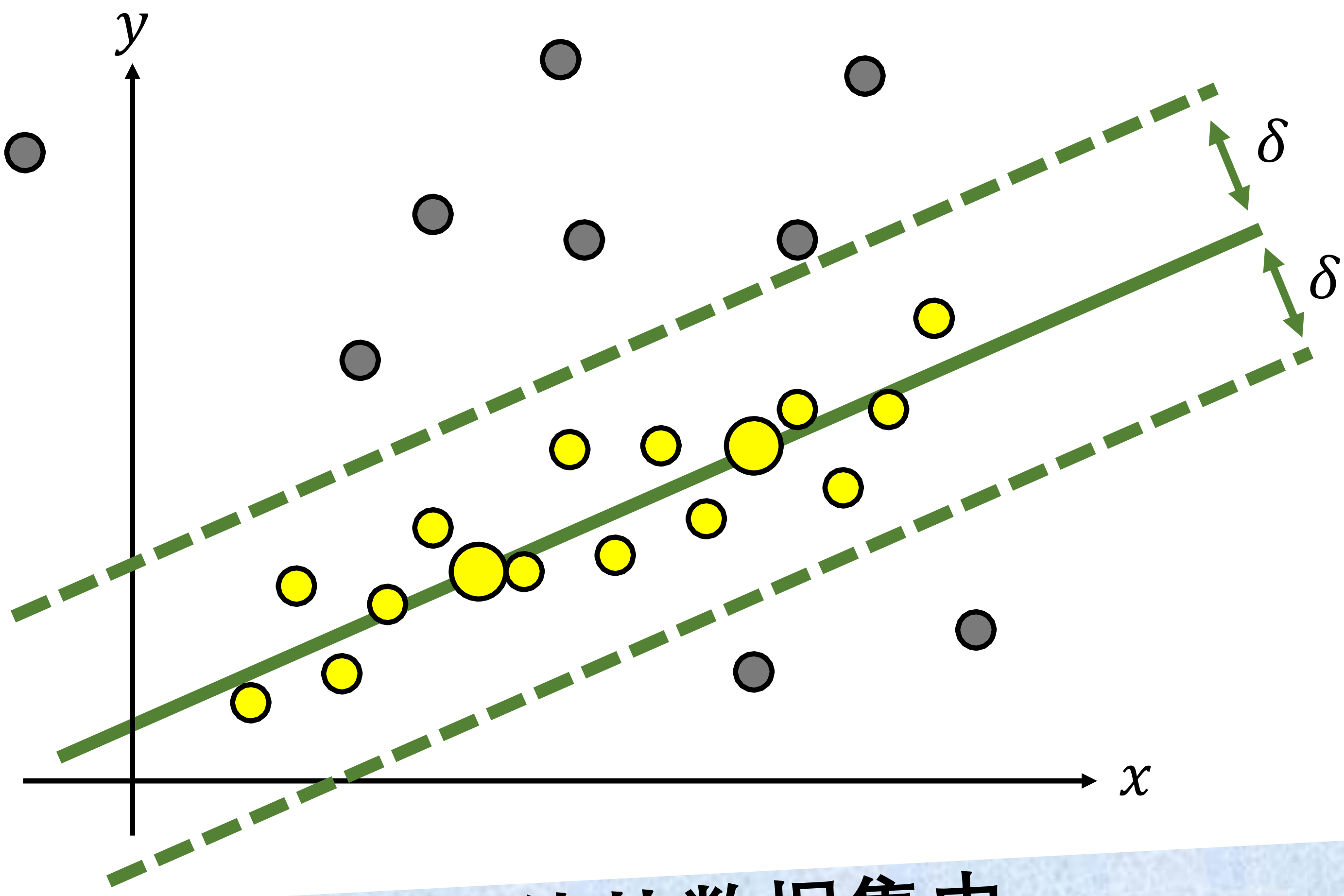
y
x



根据阈值从数据集中
计算用于建模的内点集



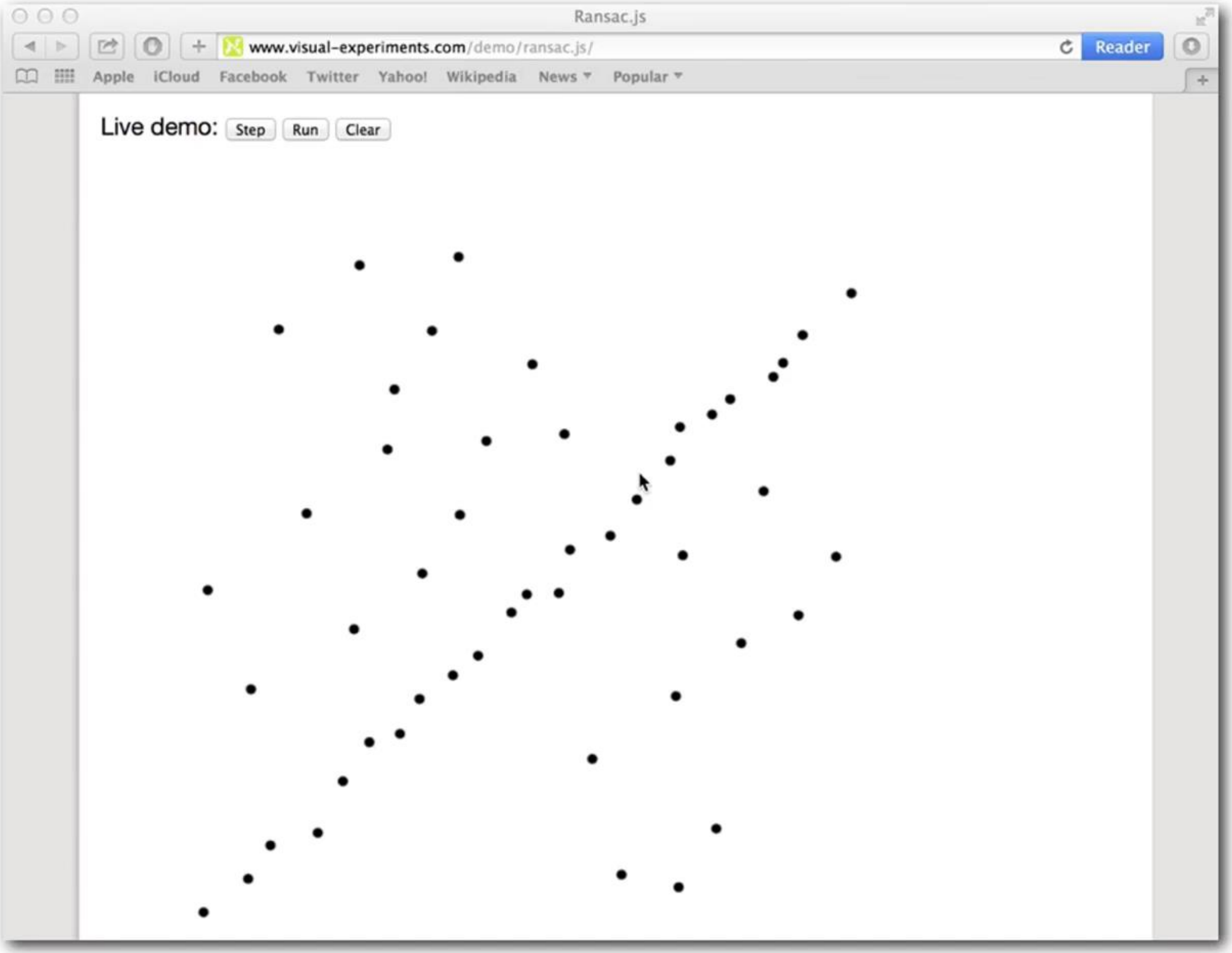
根据阈值从数据集中
计算用于建模的内点集



根据阈值从数据集中
计算用于建模的内点集

步骤4

将步骤1-3重复N次并使用具有最多内点的样本估计最终模型


Ransac.js
www.visual-experiments.com/demo/ransac.js/
Reader
Apple iCloud Facebook Twitter Yahoo! Wikipedia News Popular
Live demo:
Step
Run
Clear

需要多少次

采样？

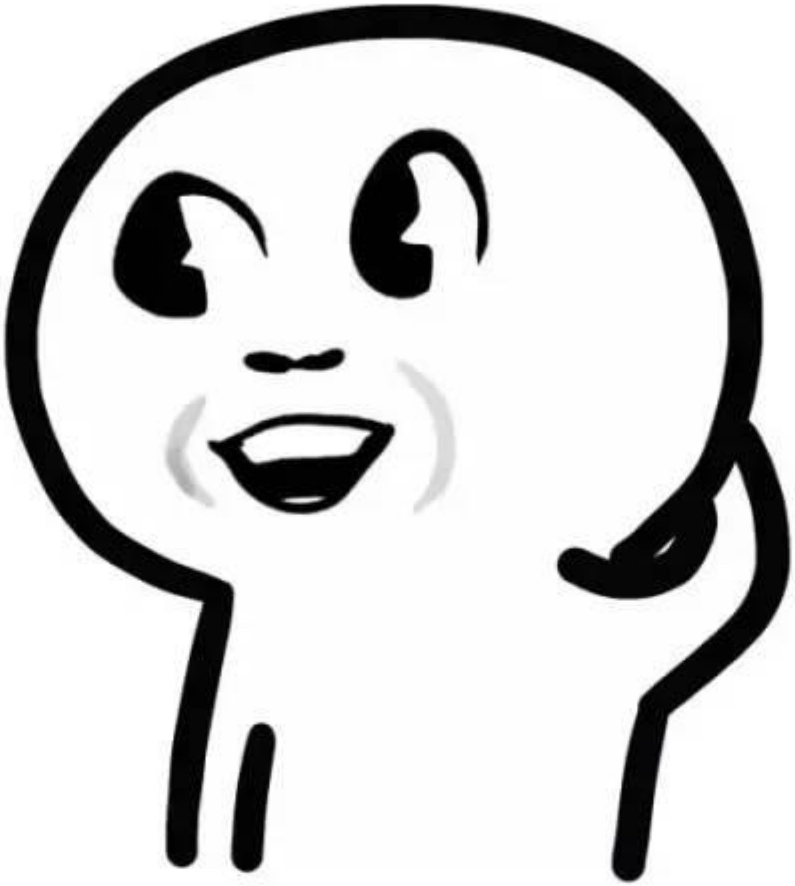
N = 采样的次数

e = 点是外点的概率

s = 一次采样的点数

p = 期望的得到一个好采样的概率

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$



看懵了？

N = 采样的次数

e = 点是外点的概率

s = 一次采样的点数

p = 期望的得到一个好采样的概率

e

N = 采样的次数

e = 点是外点的概率

s = 一次采样的点数

p = 期望的得到一个好采样的概率

$$1 - e$$

$N =$ 采样的次数

$e =$ 点是外点的概率

$s =$ 一次采样的点数

$p =$ 期望的得到一个好采样的概率

$$1 - e$$

选择一个点它是一个内点的概率

$N =$ 采样的次数

$e =$ 点是外点的概率

$s =$ 一次采样的点数

$p =$ 期望的得到一个好采样的概率

$$(1-e)^s$$

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$(1-e)^s$$

连续选择s个内点的概率

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$(1-e)^s$$

样本中只包含内点

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$1-(1-e)^{s}$$

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$1-(1-e)^s$$

一个或更多点是外点的概率

$N =$ 采样的次数

$e =$ 点是外点的概率

$s =$ 一次采样的点数

$p =$ 期望的得到一个好采样的概率

$$1-(1-e)^s$$

样本被污染了

$N =$ 采样的次数

$e =$ 点是外点的概率

$s =$ 一次采样的点数

$p =$ 期望的得到一个好采样的概率

$$(1-(1-e)^s)^N$$

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$(1-(1-e)^{s})^{N}$$

***N*次采样被污染的概率**

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$1-\left(1-\left(1-e\right)^{s}\right)^{N}$$

N = 采样的次数

e = 点是外点的概率

s = 一次采样的点数

p = 期望的得到一个好采样的概率

$$1-(1-(1-e)^{s})^{N}$$

至少一个s点样本只包含内点的概率

$N=$ 采样的次数

$e=$ 点是外点的概率

$s=$ 一次采样的点数

$p=$ 期望的得到一个好采样的概率

$$1-(1-(1-e)^{s})^{N}$$

至少一次采样没有被污染

$N =$ 采样的次数

$e =$ 点是外点的概率

$s =$ 一次采样的点数

$p =$ 期望的得到一个好采样的概率

$$p = 1 - (1 - (1 - e)^s)^N$$

N = 采样的次数

e = 点是外点的概率

s = 一次采样的点数

p = 期望的得到一个好采样的概率

$$p = 1 - (1 - (1 - e)^s)^N$$

如何求N?

N = 采样的次数

e = 点是外点的概率

s = 一次采样的点数

p = 期望的得到一个好采样的概率

$$p = 1 - (1 - (1 - e)^s)^N$$

$$N = \frac{\log(1 - p)}{\log(1 - (1 - e)^s)}$$

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$

选择N使得有$p = 0.99$的概率至少一次随机采样没有外点

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$

选择N使得有$p = 0.99$的概率
至少一次随机采样没有外点

	外点的比例，e (%)						
s	5	10	20	25	30	40	50
2							
3							
4							
5							
6							
7							
8							

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$

选择N使得有$p = 0.99$的概率至少一次随机采样没有外点

s	外点的比例，e (%) 5	10	20	25	30	40	50
2	2	3	5	6	7	11	17
3							
4							
5							
6							
7							
8							

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$

选择N使得有$p = 0.99$的概率至少一次随机采样没有外点

	外点的比例，e (%)						
s	5	10	20	25	30	40	50
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4							
5							
6							
7							
8							

$$N = \frac{\log(1-p)}{\log(1-(1-e)^s)}$$

选择N使得有$p = 0.99$的概率至少一次随机采样没有外点

	外点的比例，e (%)						
s	5	10	20	25	30	40	50
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

优点

优点

容易实现

优点

容易实现

适用于多种不同的情景

缺点

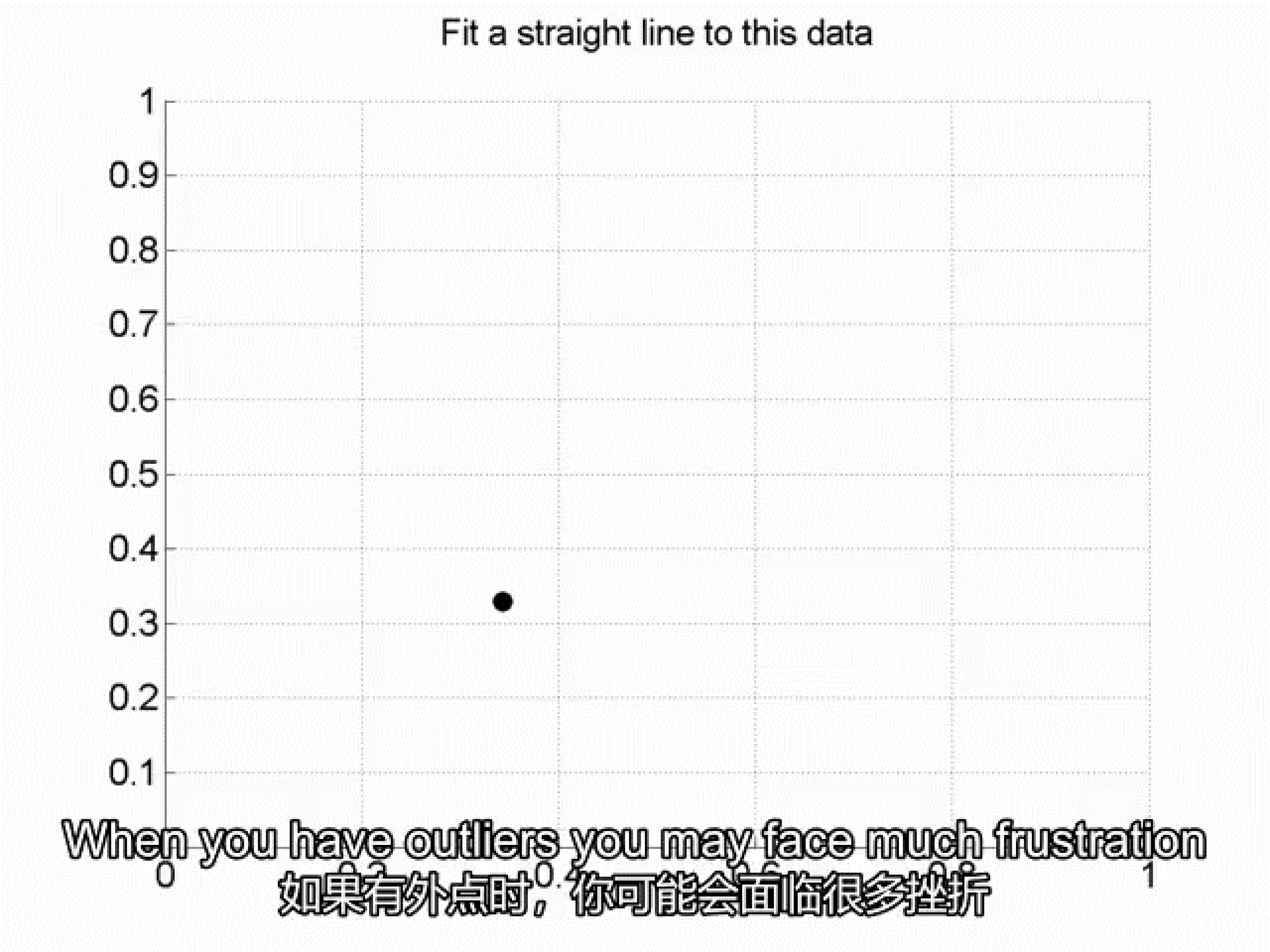
缺点

要调整的参数很多

缺点

要调整的参数很多

如果内点/外点的比例太小，则无法使用


Fit a straight line to this data
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
When you have outliers you may face much frustration
如果有外点时，你可能会面临很多挫折