计算机视觉

姿态估计

中国传媒大学
COMMUNICATION UNIVERSITY OF CHINA

本节主题：

相机标定

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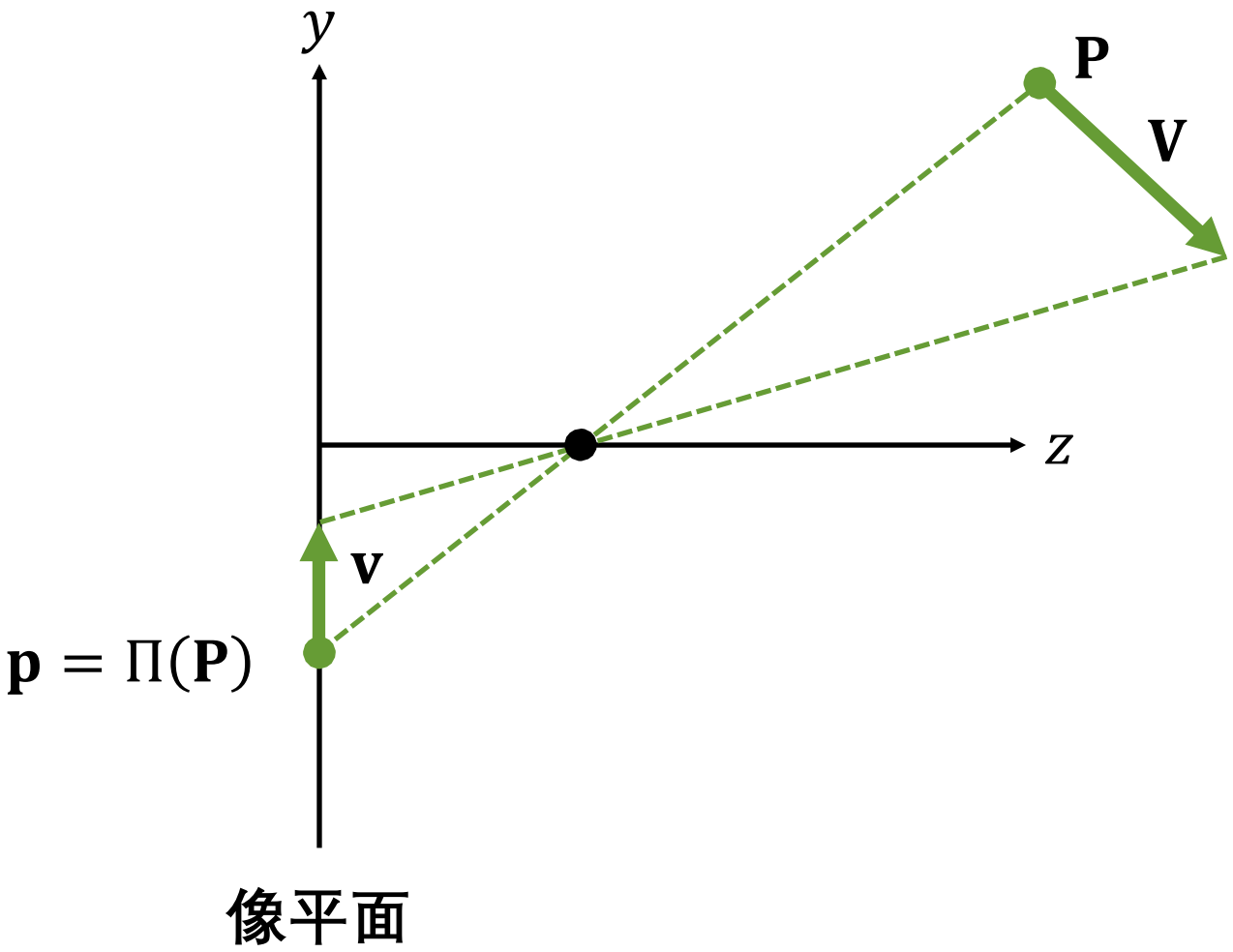
姿态估计



由运动恢复结构

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

如何估计相机的姿态?

如何估计相机的姿态?

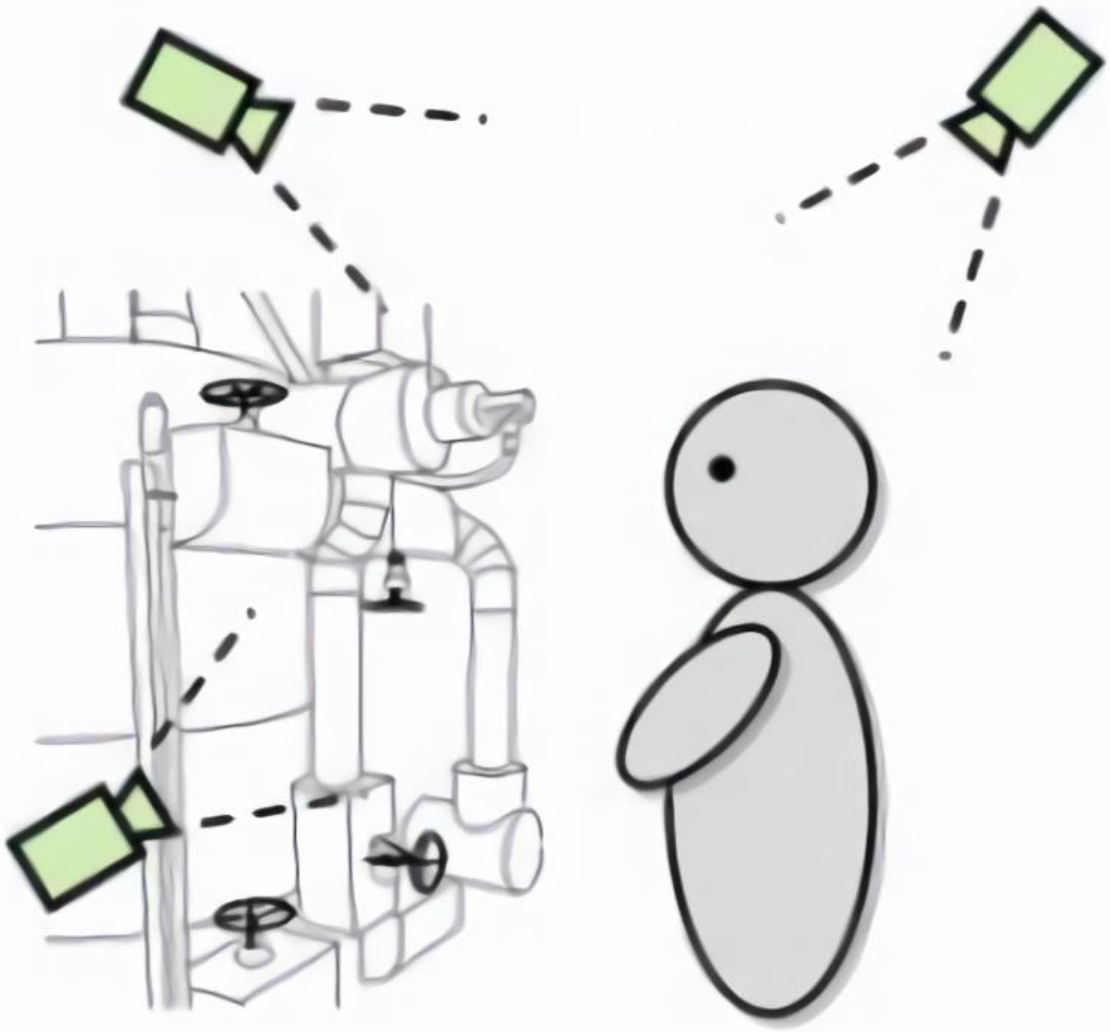
3D位置和朝向

如何估计相机的姿态？

3D位置和朝向

世界坐标下平移和旋转

从外到内
跟踪



鸣谢：Ryerson Multimedia Research Lab

HTC Vive


htc
htc
htc
EVEREST VR
VIVEPORT
SUBSCRIPTION
Tilt Brush
by Google

HTC灯塔

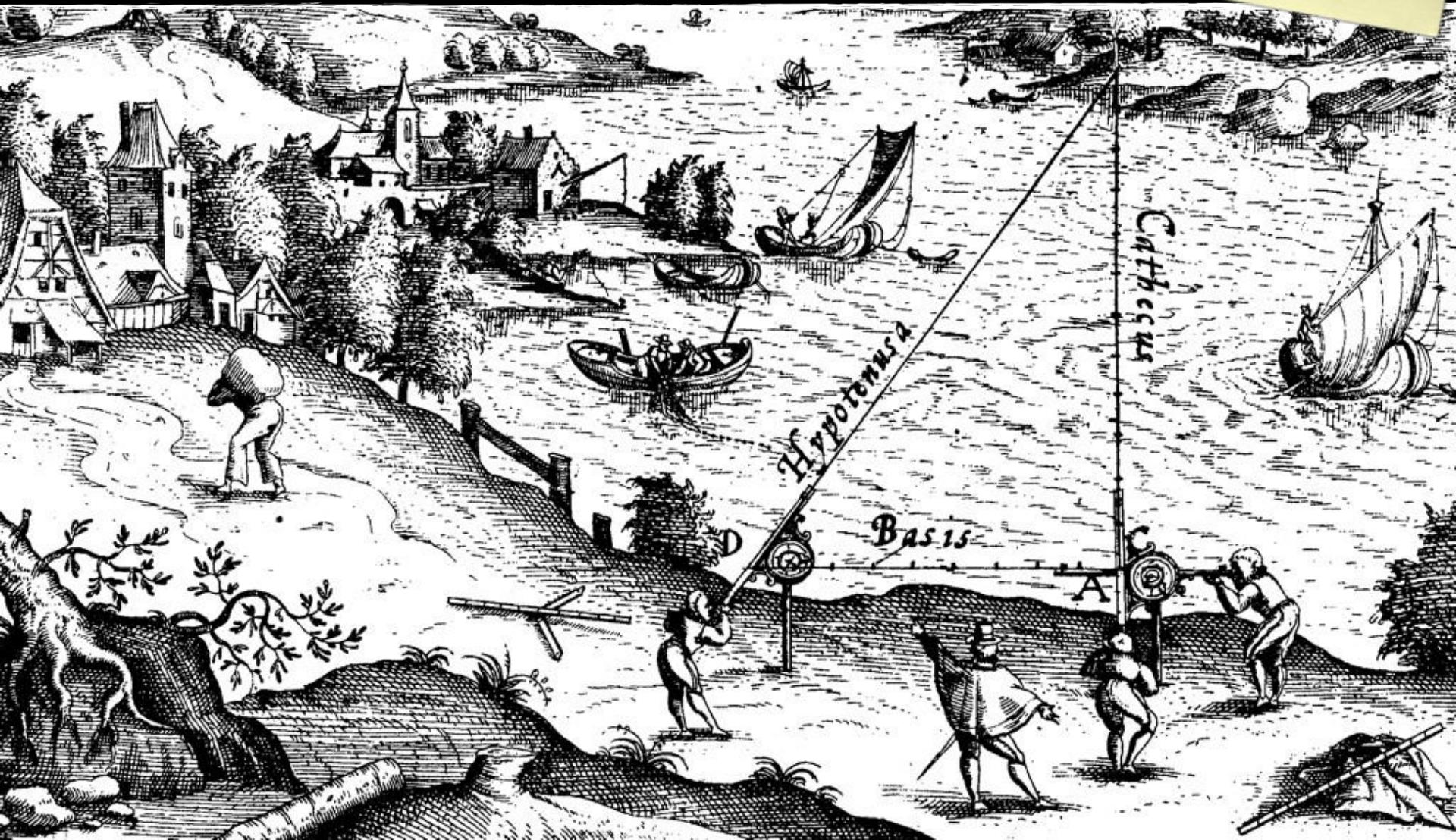
CAVE

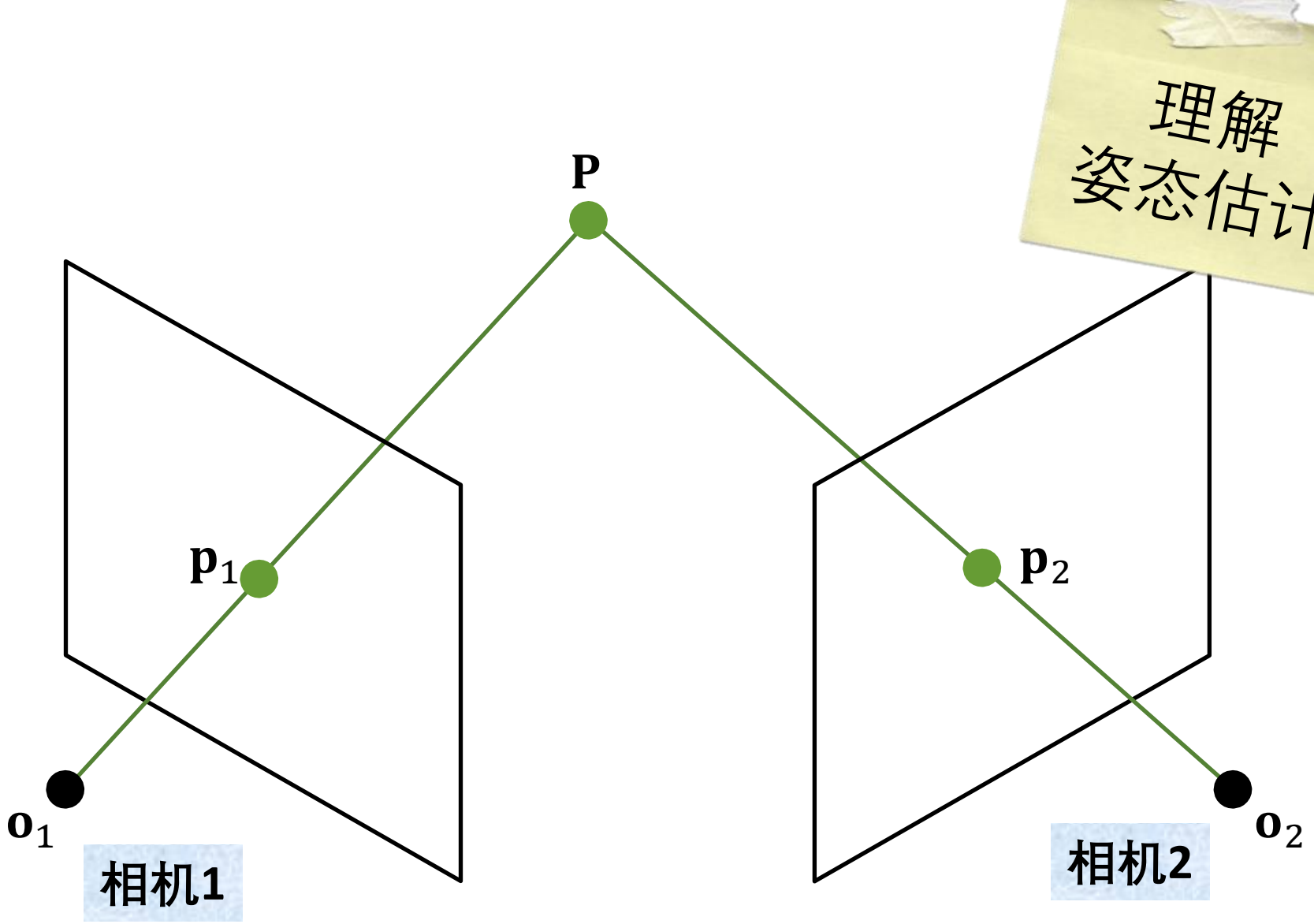
鸣谢： Ryerson Multimedia Research Lab

动作捕捉

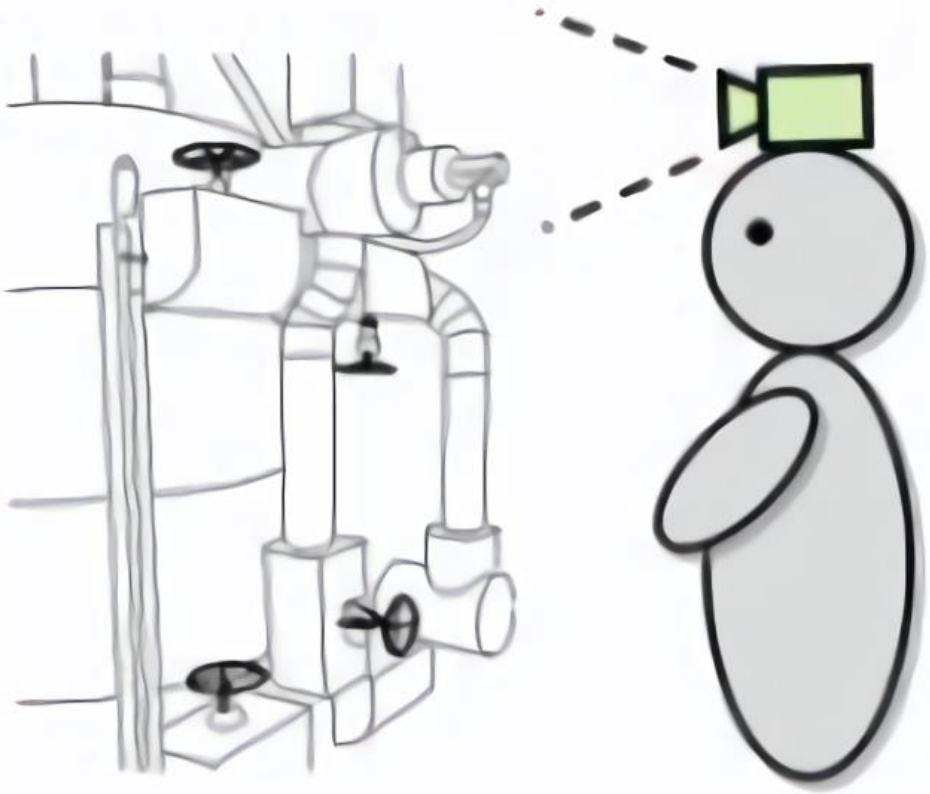
三角测量

理解
姿态估计




理解
姿态估计
P
p_1
p_2
o_1
o_2
相机1
相机2

从内到外
跟踪




Power Painter.EASM
POWERPAINTER
5GPH
AR

of coordinated covers
ook. Cover: 50% viscose/
4cm. Sågmyra grey/check.
chair £80 Hand made, gives soft
ms. Each piece of furniture is unique.
H100cm. 202.016.82

Scan page to see more

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

问题陈述

3D空间中的点

给定一组3D-2D对应点$\{\mathbf{P}_i \ \mathbf{p}_i\}$

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

图像中的点

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

和相机模型$\Pi = \mathbf{K}[\mathbf{R}|\mathbf{t}]$

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估计相机模型参数

问题陈述

给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

和相机模型$\Pi = \mathbf{K}[\mathbf{R}|\mathbf{t}]$

估计相机模型参数，即相机内参和外参

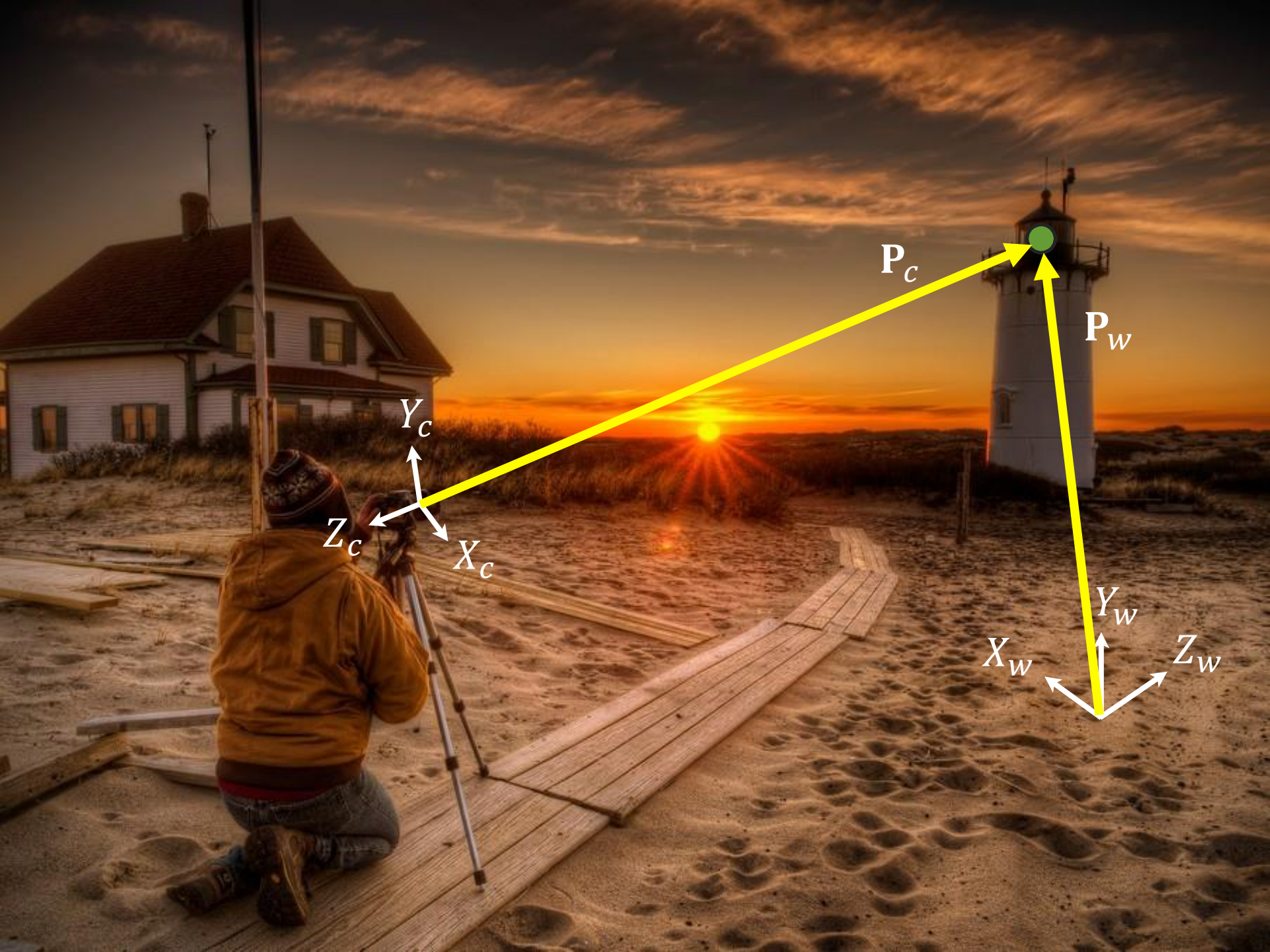
问题陈述

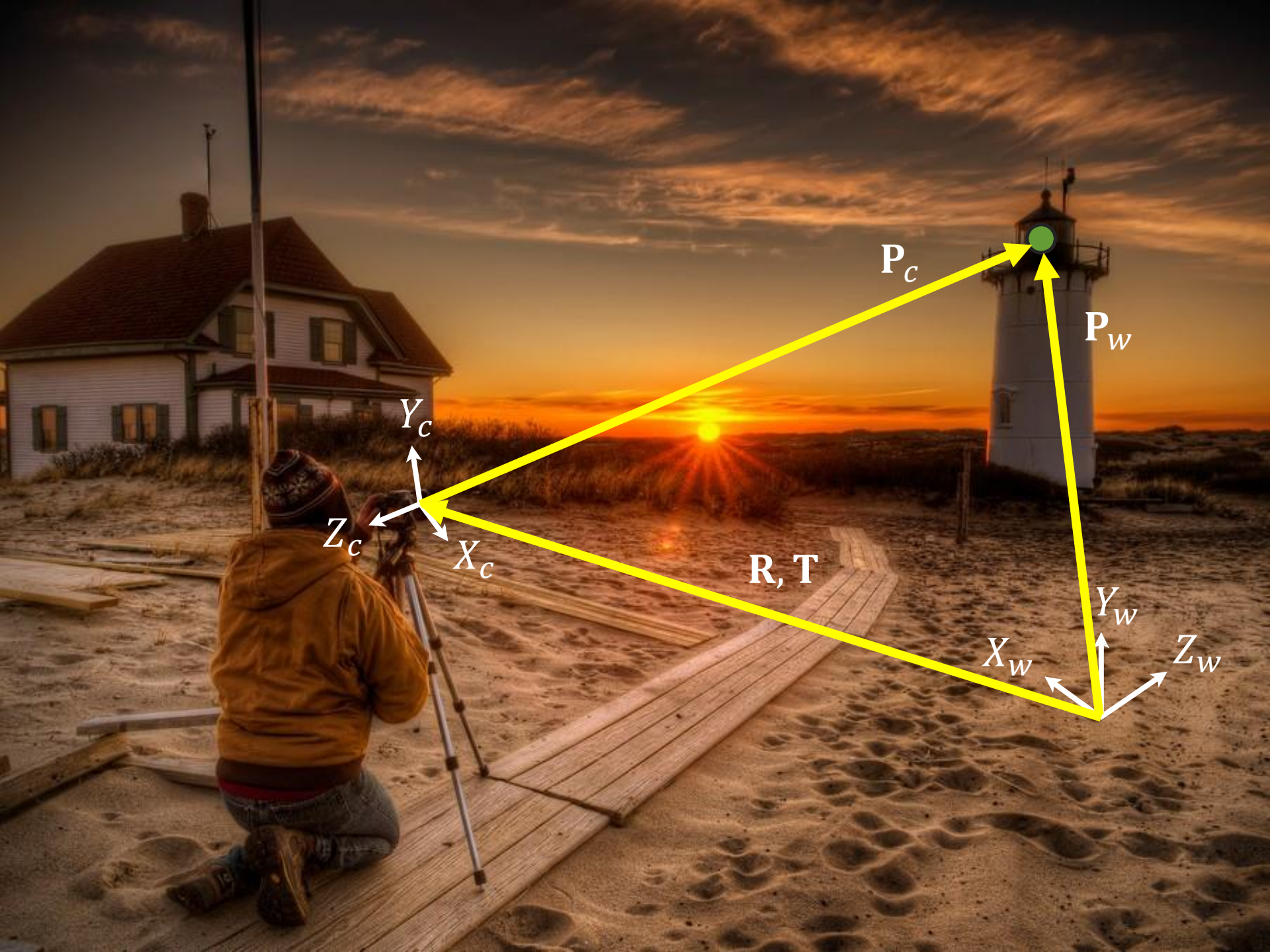
给定一组3D-2D对应点$\{\mathbf{P}_i, \mathbf{p}_i\}$

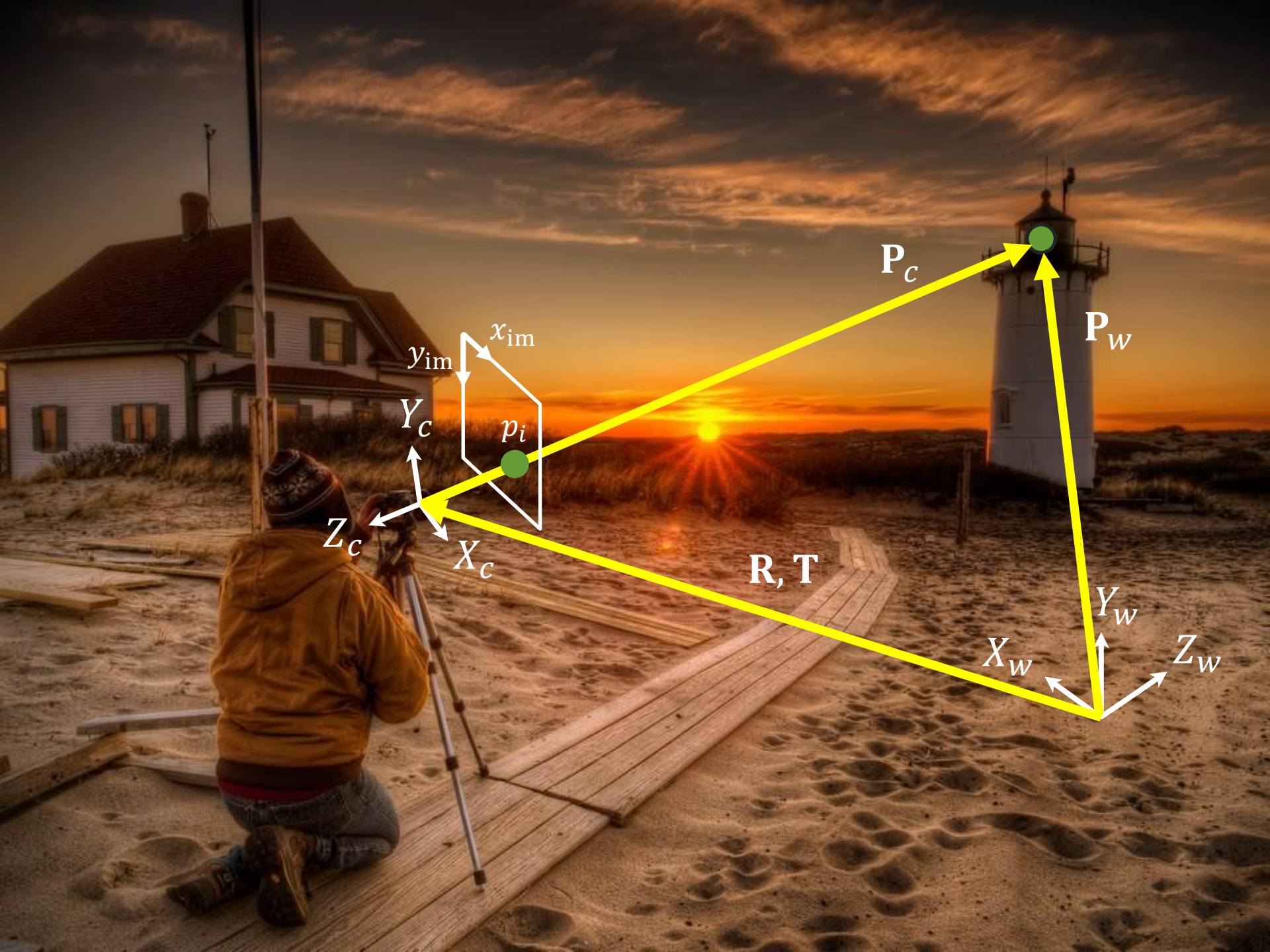
和相机模型$\Pi = \mathbf{K}[\mathbf{R}|\mathbf{t}]$

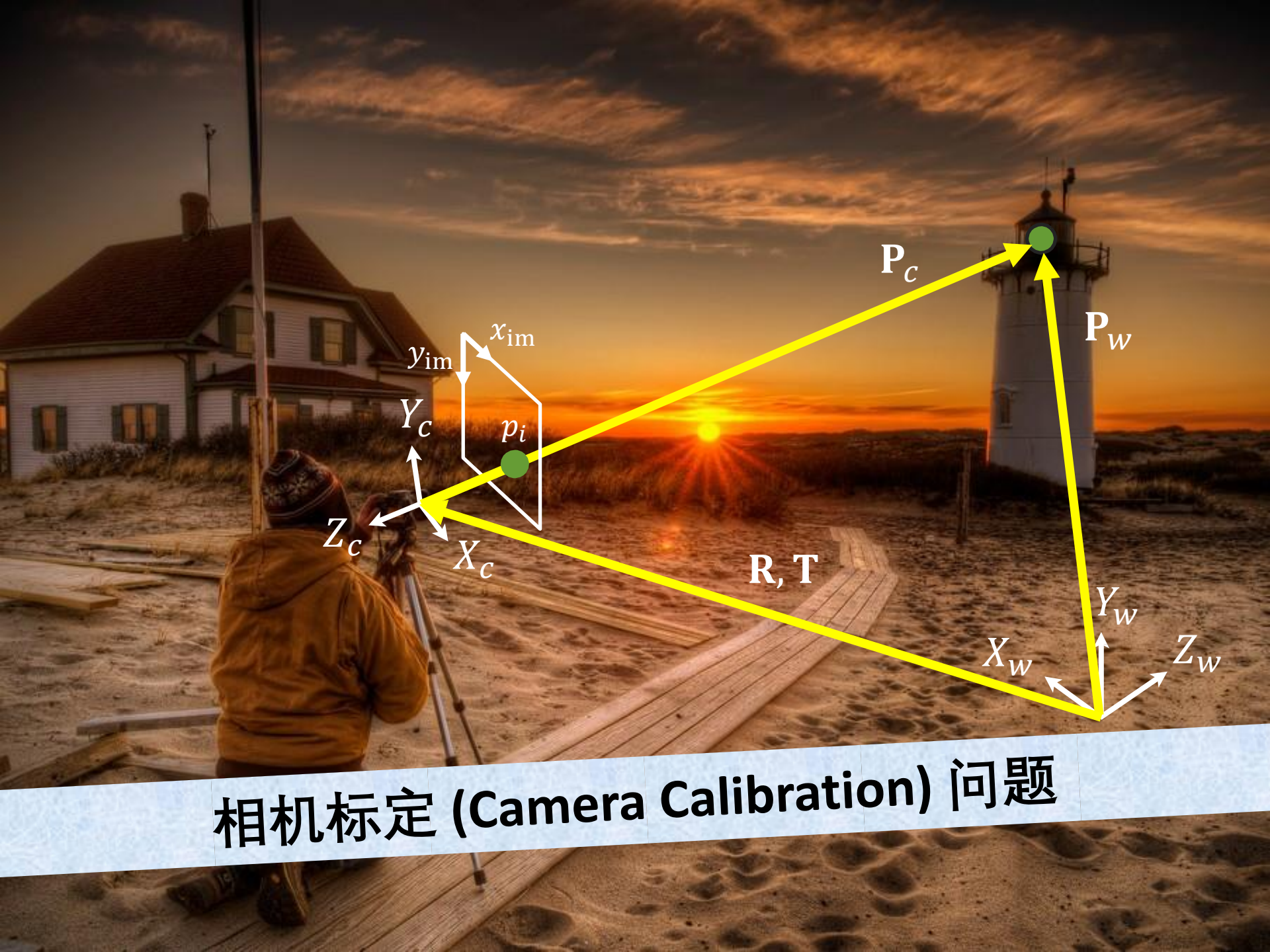
估计相机模型参数，即相机内参和外参

相机“姿态”


$\mathbf{P}_c$
$\mathbf{P}_w$
Y_c
Z_c
X_c
Y_w
X_w
Z_w


$\mathbf{P}_c$
$\mathbf{P}_w$
Y_c
Z_c
X_c
$\mathbf{R}, \mathbf{T}$
Y_w
X_w
Z_w


$\mathbf{P}_c$
$\mathbf{P}_w$
x_{im}
y_{im}
p_i
Y_c
Z_c
X_c
$\mathbf{R}, \mathbf{T}$
Y_w
X_w
Z_w



相机标定 (Camera Calibration) 问题

$$\begin{pmatrix} \alpha x_{im} \\ \alpha y_{im} \\ \alpha \end{pmatrix} = \underbrace{\begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{内参矩阵 } \mathbf{M}_{\text{int}}} \underbrace{\begin{pmatrix} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ \mathbf{R} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}}_{\text{外参矩阵 } \mathbf{M}_{\text{ext}}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

回顾

$$\begin{pmatrix} \alpha x_{im} \\ \alpha y_{im} \\ \alpha \end{pmatrix} = \underbrace{\begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{内参矩阵 } \mathbf{M}_{\text{int}}} \underbrace{\begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}}_{\text{外参矩阵 } \mathbf{M}_{\text{ext}}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

内参矩阵是可逆的

回顾

$$\begin{pmatrix} \alpha x_{im} \\ \alpha y_{im} \\ \alpha \end{pmatrix} = \underbrace{\begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{内参矩阵 } \mathbf{M}_{\text{int}}} \underbrace{\begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}}_{\text{外参矩阵 } \mathbf{M}_{\text{ext}}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

除以α

回顾

相机模型

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]$$

$$= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

如何求解相机模型参数?

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

展开

$$\begin{pmatrix} \alpha x_{\text{im}} \\ \alpha y_{\text{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\text{w}} \\ Y_{\text{w}} \\ Z_{\text{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\text{im}} \\ \alpha y_{\text{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\text{T}} \\ \boldsymbol{\pi}_2^{\text{T}} \\ \boldsymbol{\pi}_3^{\text{T}} \end{pmatrix} \mathbf{X}$$

展开

$$x_{\text{im}} = \frac{\boldsymbol{\pi}_1^{\text{T}}\mathbf{X}}{\boldsymbol{\pi}_3^{\text{T}}\mathbf{X}} \qquad y_{\text{im}} = \frac{\boldsymbol{\pi}_2^{\text{T}}\mathbf{X}}{\boldsymbol{\pi}_3^{\text{T}}\mathbf{X}}$$

$$\begin{pmatrix} \alpha x_{\text{im}} \\ \alpha y_{\text{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\text{w}} \\ Y_{\text{w}} \\ Z_{\text{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\text{im}} \\ \alpha y_{\text{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\text{T}} \\ \boldsymbol{\pi}_2^{\text{T}} \\ \boldsymbol{\pi}_3^{\text{T}} \end{pmatrix} \mathbf{X}$$

展开

$$x_{\text{im}} = \frac{\boldsymbol{\pi}_1^{\text{T}}\mathbf{X}}{\boldsymbol{\pi}_3^{\text{T}}\mathbf{X}} \qquad y_{\text{im}} = \frac{\boldsymbol{\pi}_2^{\text{T}}\mathbf{X}}{\boldsymbol{\pi}_3^{\text{T}}\mathbf{X}}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix} \begin{pmatrix} X_{\mathrm{w}} \\ Y_{\mathrm{w}} \\ Z_{\mathrm{w}} \\ 1 \end{pmatrix}$$

改写

$$\begin{pmatrix} \alpha x_{\mathrm{im}} \\ \alpha y_{\mathrm{im}} \\ \alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\pi}_1^{\mathrm{T}} \\ \boldsymbol{\pi}_2^{\mathrm{T}} \\ \boldsymbol{\pi}_3^{\mathrm{T}} \end{pmatrix} \mathbf{X}$$

展开

$$x_{\mathrm{im}} = \frac{\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}} \qquad y_{\mathrm{im}} = \frac{\boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X}}{\boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}}$$

改写

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

3D空间和图像中的一对对应点

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

N对对应点

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\mathbf{A}_{2N\times 12}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\mathbf{A}_{2N\times 12} \qquad \mathbf{x}_{12\times 1}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\mathbf{A}_{2N\times 12} \qquad \mathbf{x}_{12\times 1}$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

$$\boldsymbol{\pi}_1^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}x_{\mathrm{im}} = 0 \qquad \boldsymbol{\pi}_2^{\mathrm{T}}\mathbf{X} - \boldsymbol{\pi}_3^{\mathrm{T}}\mathbf{X}y_{\mathrm{im}} = 0$$

改写成矩阵

$$\begin{pmatrix} \mathbf{X}_1^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_1^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_1^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^{\mathrm{T}} & \mathbf{0} & -x_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \\ \mathbf{0} & \mathbf{X}_N^{\mathrm{T}} & -y_{\mathrm{im}}\mathbf{X}_N^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi}_1 \\ \boldsymbol{\pi}_2 \\ \boldsymbol{\pi}_3 \end{pmatrix} = \mathbf{0}$$

$$\mathbf{A}_{2N\times 12} \qquad \mathbf{x}_{12\times 1}$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \ \text{ subject to } \|\mathbf{x}\| = 1$$

$$\mathbf{Ax} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

$$\mathbf{Ax} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

计算A的SVD，$\mathbf{A} = \mathbf{UDV}^{\mathrm{T}}$，解就是V的最后一列

$$\mathbf{Ax} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

计算A的SVD，$\mathbf{A} = \mathbf{UDV}^{\mathrm{T}}$，解就是V的最后一列

$$\mathbf{\Pi} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$\mathbf{Ax} = \mathbf{0}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{Ax}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

齐次最小二乘问题

计算A的SVD，$\mathbf{A} = \mathbf{UDV}^{\mathrm{T}}$，解就是V的最后一列

$$\mathbf{\Pi} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

如何得到内参和外参?

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]$$

$$= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\text{int}}[\mathbf{R}|-\mathbf{RT}]$$

$$= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\text{int}}[\mathbf{R}|-\mathbf{RT}]$$

$$= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$[\mathbf{R}|-\mathbf{RT}]\begin{pmatrix} \mathbf{T} \\ 1 \end{pmatrix} = \mathbf{RT} - \mathbf{RT} = \mathbf{0}$$

$$\boldsymbol{\Pi}\begin{pmatrix}\mathbf{T}\\1\end{pmatrix}=\mathbf{0}$$

$$\Pi\binom{\mathbf{T}}{1} = \mathbf{0}$$

是不是很眼熟?

$$\Pi\begin{pmatrix}\mathbf{T}\\1\end{pmatrix}=\mathbf{0}$$

是不是很眼熟?

齐次最小二乘问题

$$\Pi \begin{pmatrix} \mathbf{T} \\ 1 \end{pmatrix} = \mathbf{0}$$

是不是很眼熟？

齐次最小二乘问题

计算Π的SVD，$\Pi = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathrm{T}}$，解就是$\mathbf{V}$的最后一列

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}| - \mathbf{RT}]$$

$$= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= [\mathbf{M}_{\mathrm{int}}\mathbf{R} | -\mathbf{M}_{\mathrm{int}}\mathbf{R}\mathbf{T}]$$

$$= \left(\begin{array}{ccc|c} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{array}\right)$$

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$
$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}\mathbf{R}$$

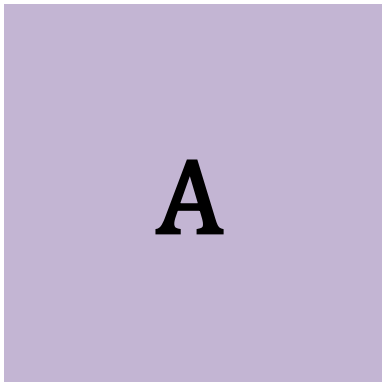
如何求解?

QR分解

(QR Decomposition)

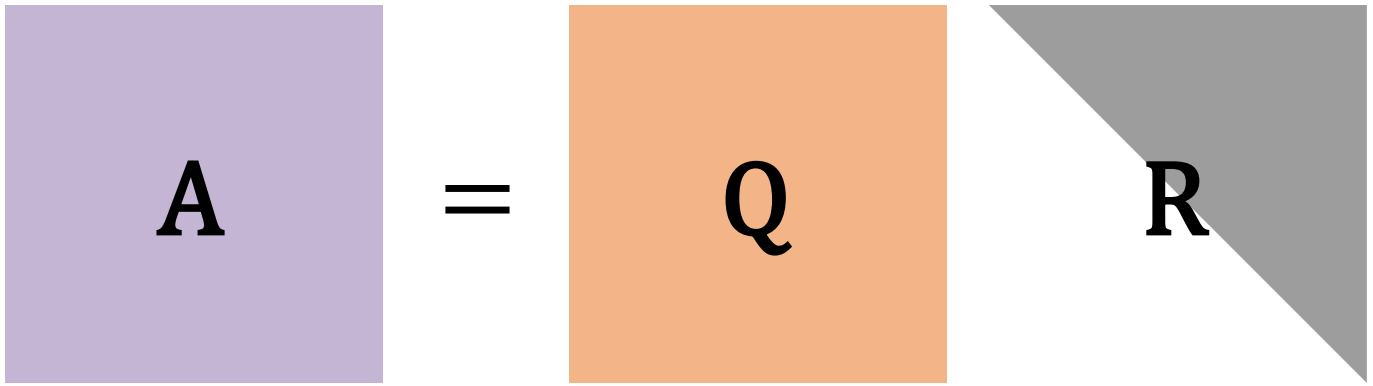
回顾：QR分解

QR分解



$n \times n$

QR分解



$n \times n$ $n \times n$ $n \times n$

定义：对于任意给定的实数方阵$\mathbf{A} \in \mathbb{R}^{n\times n}$，它的QR分解（QR Decomposition）定义为

$$\mathbf{A} = \mathbf{QR}$$

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$$\mathbf{A} = \mathbf{QR}$$

使得

定义：对于任意给定的实数方阵$\mathbf{A} \in \mathbb{R}^{n\times n}$，它的QR分解（QR Decomposition）定义为

$$\mathbf{A} = \mathbf{QR}$$

使得

Q是具有正交列向量的$n \times n$的正交矩阵

定义：对于任意给定的实数方阵$\mathbf{A} \in \mathbb{R}^{n\times n}$，它的QR分解（QR Decomposition）定义为

$$\mathbf{A} = \mathbf{QR}$$

使得

Q是具有正交列向量的$n \times n$的正交矩阵

R是$n \times n$的上三角矩阵

由QR到
RQ分解

由QR到RQ分解

$$E = \begin{pmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{pmatrix}$$

由QR到RQ分解

$$\mathbf{E}^{-1} = \mathbf{E} = \mathbf{E}^{\mathrm{T}}$$

由QR到RQ分解

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

由QR到RQ分解

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{pmatrix}$$

由QR到RQ分解

$$\begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{pmatrix}$$

上下反转

由QR到RQ分解

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$$

由QR到RQ分解

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix} = \begin{pmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{pmatrix}$$

由QR到RQ分解

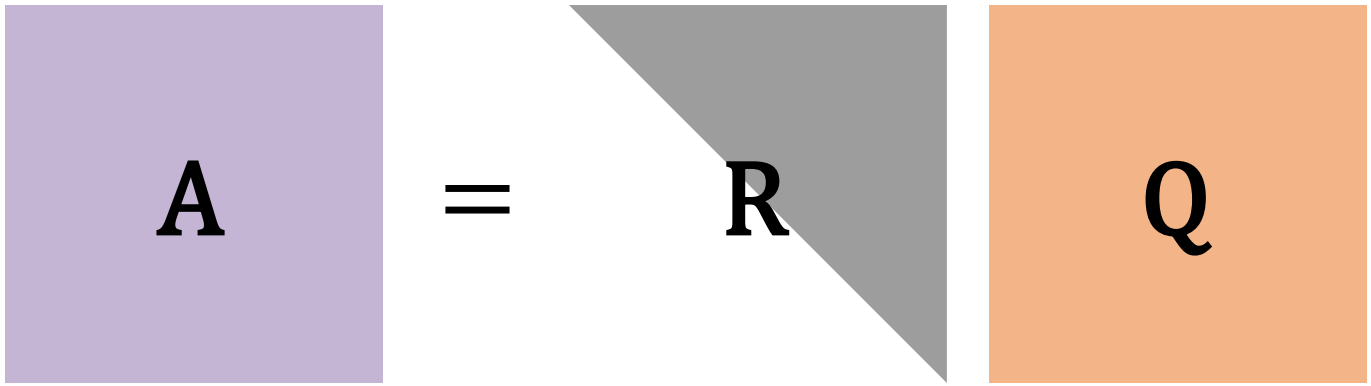
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix} = \begin{pmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{pmatrix}$$

左右反转

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

由QR到RQ分解



由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

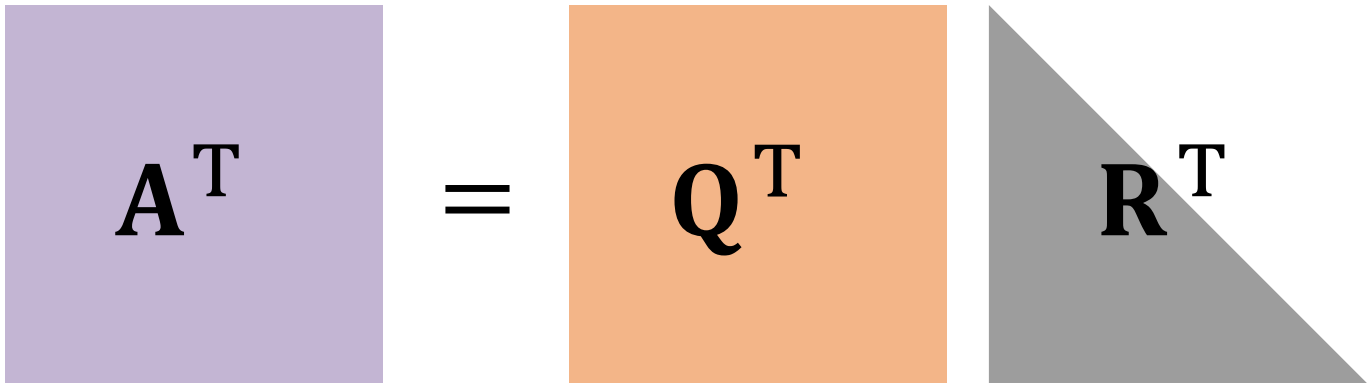
由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

由QR到RQ分解



由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

$$\mathbf{E}\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{EQ}\quad \mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

$$\mathbf{E}\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

由QR到RQ分解

$$\mathbf{A} = \mathbf{RQ}$$

转置

$$\mathbf{A}^{\mathrm{T}} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}$$

左右反转

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{R}^{\mathrm{T}}\mathbf{E}$$

上下反转

$$\mathbf{E}\mathbf{Q}\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

整理

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \left(\mathbf{Q}^{\mathrm{T}}\mathbf{E}\right)\left(\mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}\right)$$

由QR到RQ分解

$$\mathbf{A}^{\mathrm{T}}\mathbf{E} = \mathbf{E}\mathbf{Q}^{\mathrm{T}}\mathbf{E}\mathbf{R}^{\mathrm{T}}\mathbf{E}$$

$n \times n$ $n \times n$ $n \times n$

3

主要步骤

步骤1

转置并左右反转

$$\tilde{\mathbf{A}} = \mathbf{A}^{\mathrm{T}}\mathbf{E}$$

步骤2

计算$\tilde{A}$的QR分解

$$\tilde{\mathbf{A}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}$$

步骤3

计算R和Q

$$\mathbf{R} = \mathbf{E}\tilde{\mathbf{R}}^{\mathrm{T}}\mathbf{E}$$

$$\mathbf{Q} = \mathbf{E}\tilde{\mathbf{Q}}^{\mathrm{T}}$$

回顾：QR分解

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}\mathbf{R}$$

如何求解?

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

上三角矩阵

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix}$$

$$= \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

正交矩阵

上三角矩阵

$$\mathbf{M}_{\text{int}}\mathbf{R} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_5 & \pi_6 & \pi_7 \\ \pi_9 & \pi_{10} & \pi_{11} \end{pmatrix} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \mathbf{R}$$

正交矩阵

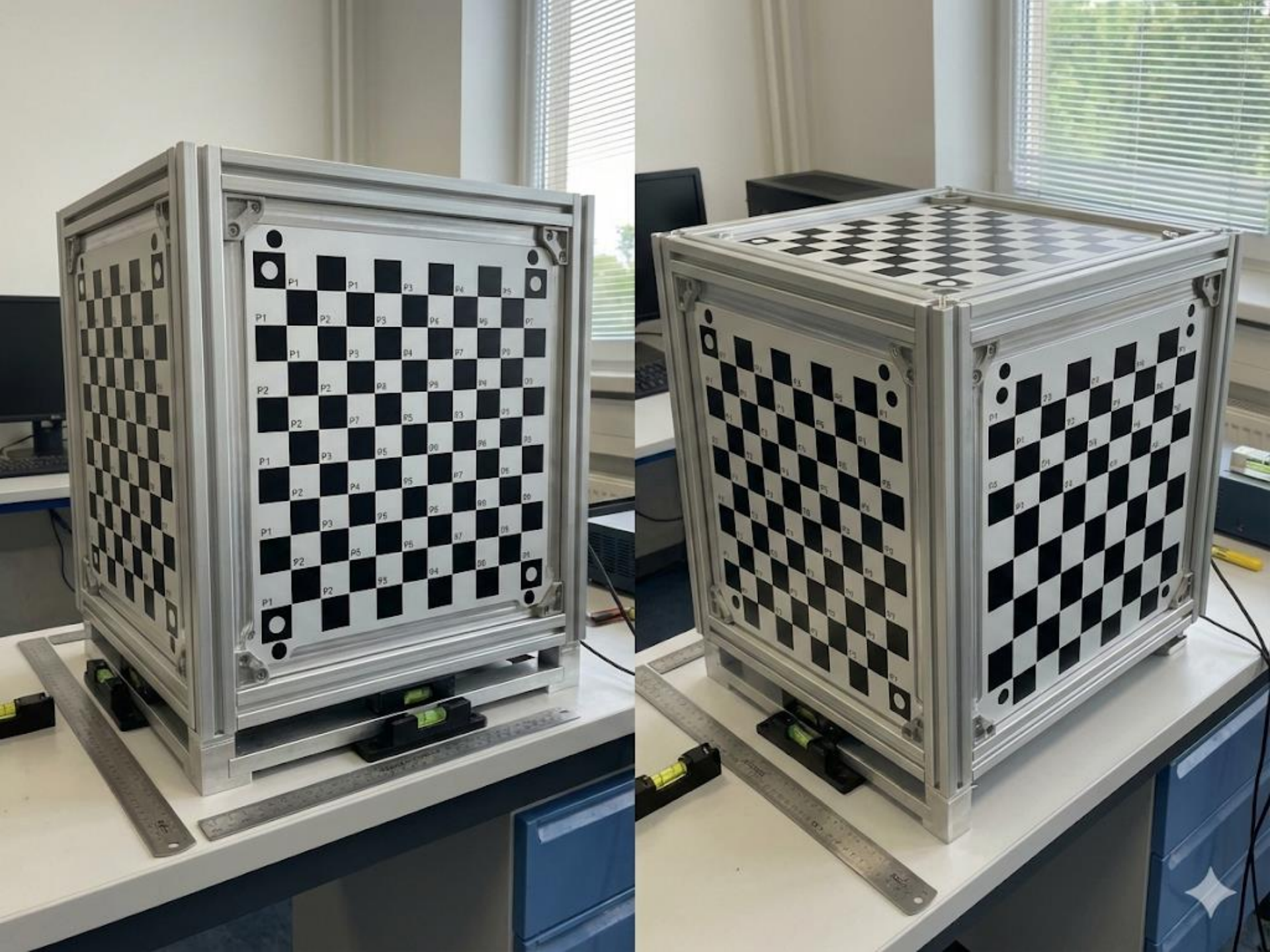
上三角矩阵

计算RQ分解，R是内参矩阵，Q是旋转矩阵

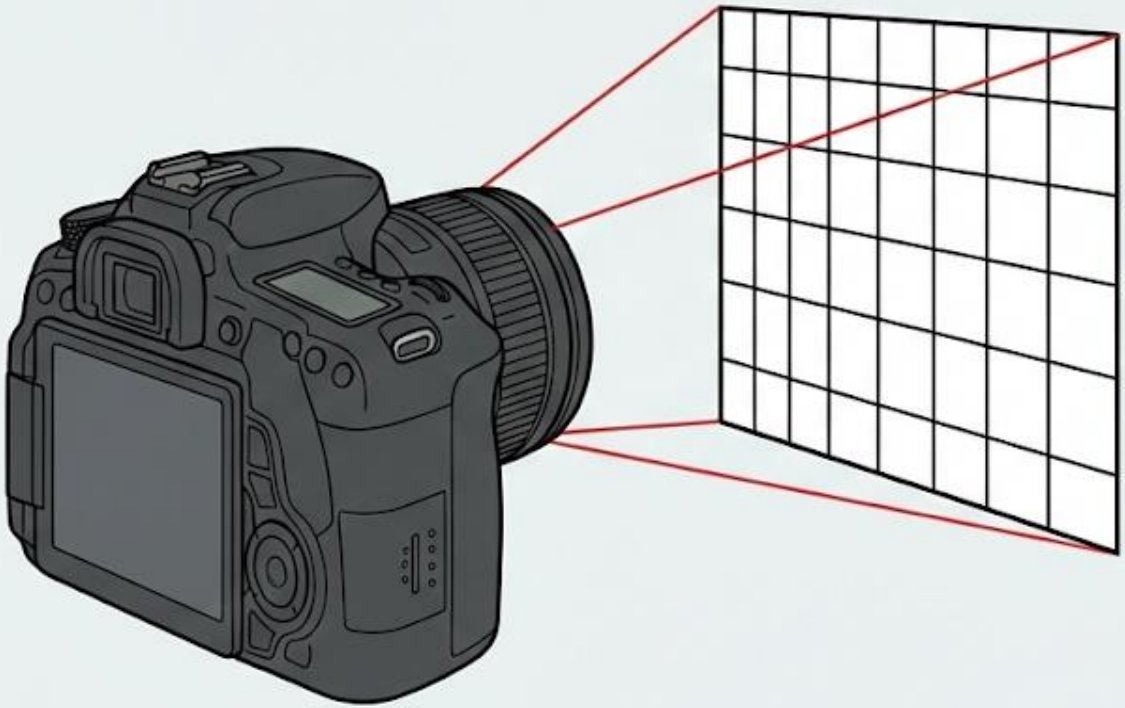
DLT

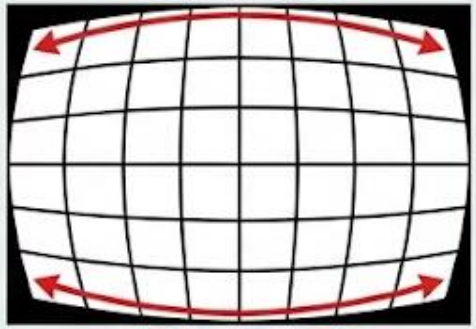
Direct Linear Transform

直接线性变换法

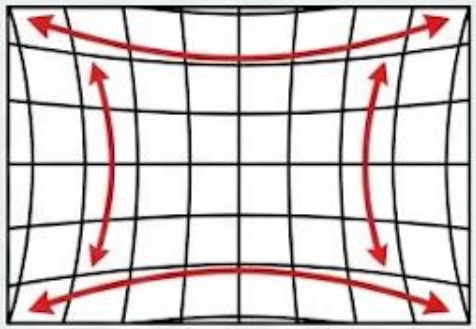


相机镜头畸变

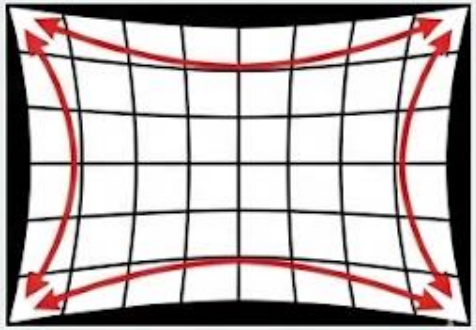




桶形畸变



枕形畸变



胡须形畸变

张正友法

A Flexible New Technique for Camera Calibration

Zhengyou Zhang, *Senior Member, IEEE*

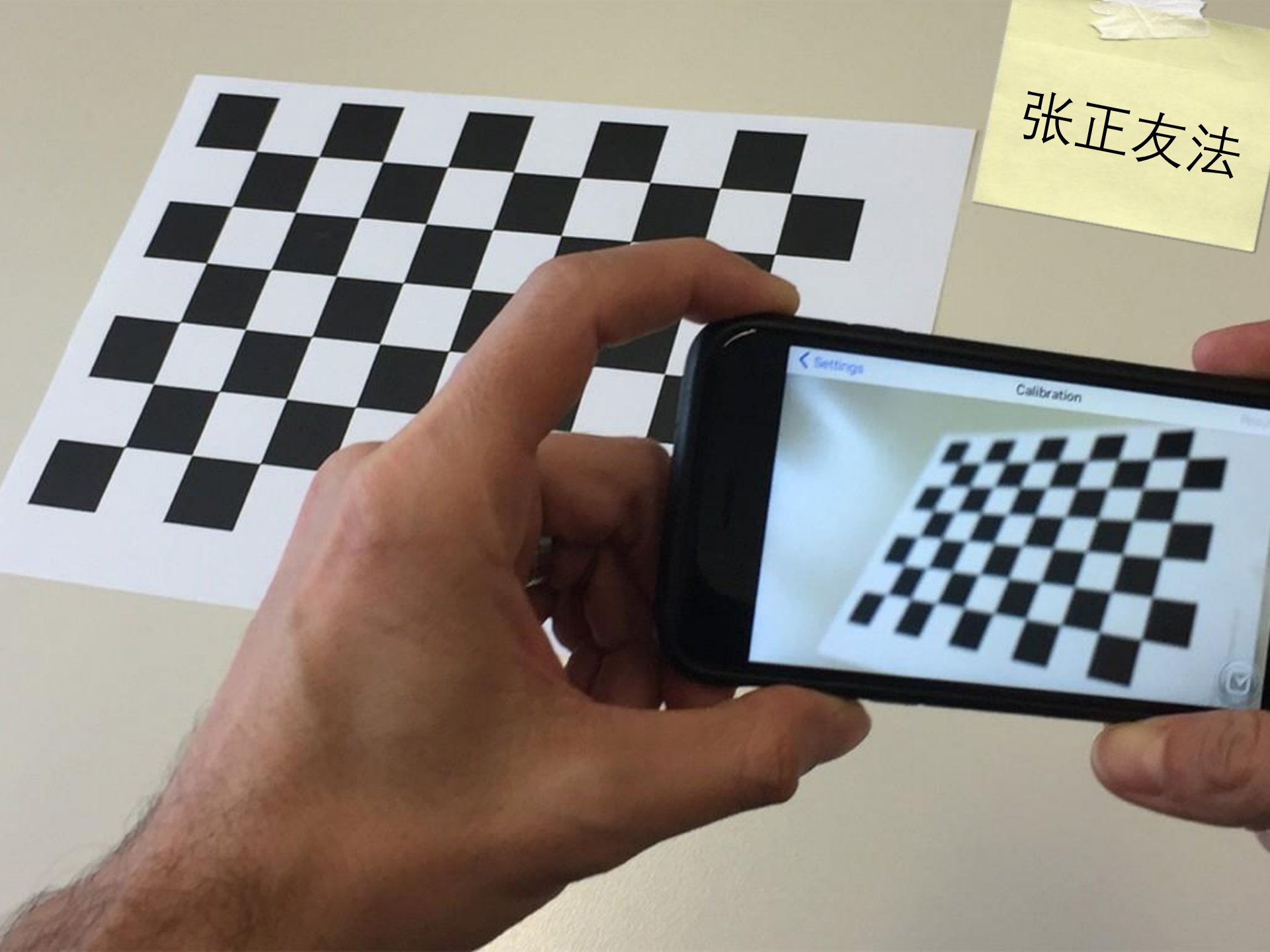
Abstract—We propose a flexible new technique to easily calibrate a camera. It only requires the camera to observe a planar pattern shown at a few (at least two) different orientations. Either the camera or the planar pattern can be freely moved. The motion need not be known. Radial lens distortion is modeled. The proposed procedure consists of a closed-form solution, followed by a nonlinear refinement based on the maximum likelihood criterion. Both computer simulation and real data have been used to test the proposed technique and very good results have been obtained. Compared with classical techniques which use expensive equipment such as two or three orthogonal planes, the proposed technique is easy to use and flexible. It advances 3D computer vision one more step from laboratory environments to real world use. The corresponding software is available from the author's Web page.

Index Terms—Camera calibration, calibration from planes, 2D pattern, flexible plane-based calibration, absolute conic, projective mapping, lens distortion, closed-form solution, maximum likelihood estimation, flexible setup.

Our current research is focused on a desktop vision system (DVS) since the potential for using DVSs is large. Cameras are becoming inexpensive and ubiquitous. A DVS aims at the general public who are not experts in computer vision. A typical computer user will perform vision tasks only from time to time, so they will not be willing to invest money for expensive equipment. Therefore, flexibility, robustness, and low cost are important. The camera calibration technique described in this paper was developed with these considerations in mind.

The proposed technique only requires the camera to observe a planar pattern shown at a few (at least two) different orientations. The pattern can be printed on a laser printer and attached to a "reasonable" planar surface (e.g., a hard book cover). Either the camera or the planar pattern can be moved by hand. The motion need not be known. The proposed approach, which uses 2D metric information, lies between the photogrammetric calibration, which uses explicit 3D model, and self-calibration, which uses motion rigidity or equivalently implicit 3D information. Both computer simulation and real data have been used to test the proposed technique and very good results have been obtained. Compared with classical techniques, the proposed technique is considerably more flexible: Anyone can make a calibration pattern by him/herself and the setup is very easy. Compared with self-calibration, it

TPAMI, 2000

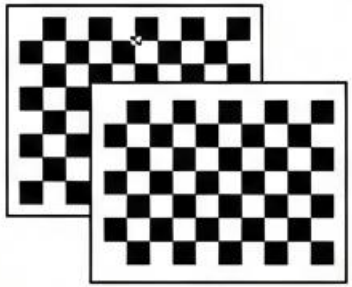

张正友法
Settings
Calibration

张正友法

初始估计

内参 $\mathbf{K}$，外参 $[\mathbf{R}_i|\mathbf{t}_i]$

观测 2D 点

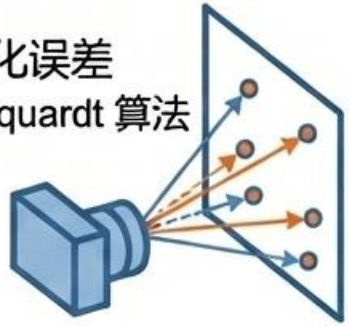


非线性优化

重投影误差函数

$$\sum \left\| \mathbf{p}_{ij} - \pi(\mathbf{K}, \mathbf{D}, \mathbf{R}_i, \mathbf{t}_i, \mathbf{P}_j) \right\|^2$$

选代最小化误差
Levenberg-Marquardt 算法



精确内参矩阵 (K)

精确外参矩阵 ($[\mathbf{R}_i|\mathbf{t}_i]$)

精确畸变系数 (D)

k1, k2, p1, p2…

Camera Calibration

Goal

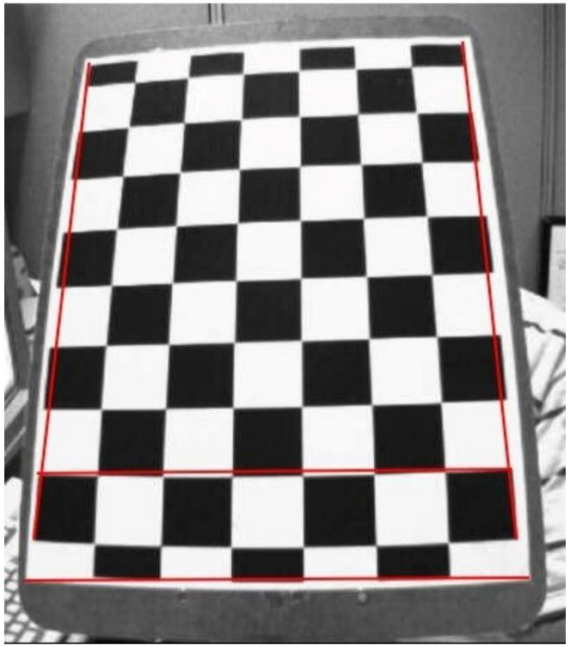
In this section, we will learn about

- types of distortion caused by cameras
- how to find the intrinsic and extrinsic properties of a camera
- how to undistort images based off these properties

Basics

Some pinhole cameras introduce significant distortion to images. Two major kinds of distortion are radial distortion and tangential distortion.

Radial distortion causes straight lines to appear curved. Radial distortion becomes larger the farther points are from the center of the image. For example, one image is shown below in which two edges of a chess board are marked with red lines. But, you can see that the border of the chess board is not a straight line and doesn't match with the red line. All the expected straight lines are bulged out. Visit Distortion (optics) for more details.



image

Python时间

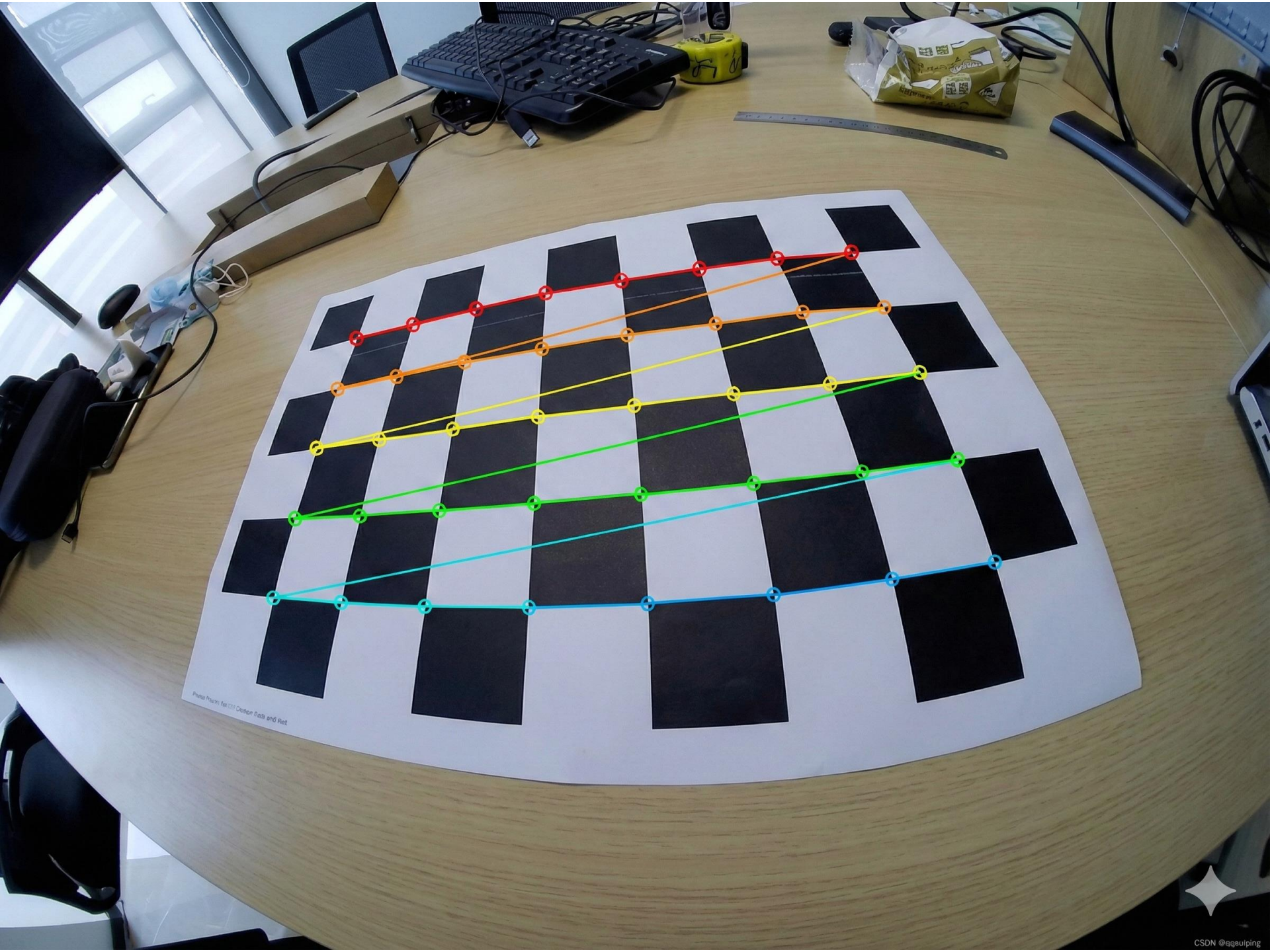
张正友法

```
import numpy as np
import cv2
import glob
import os

# 1. Configuration Parameters
# Number of inner corners on the chessboard (rows (height), columns (width))
# Note: This is not the number of squares, but the number of intersection points where corners meet.
# E.g., for a 9x6 grid of squares, the inner corners are usually (8, 5).
CHECKERBOARD_SIZE = (8, 5)

# Actual physical side length of each square on the chessboard (any unit, here using mm)
SQUARE_SIZE = 25.0  # e.g., 25mm

# Image path matching pattern
IMAGE_PATH_PATTERN = './calib_images/*.jpg'
```



```
# 2. Prepare 3D World Points
# Establish ideal coordinates for chessboard corners in the world coordinate system (Z=0).
# Format like: (0,0,0), (1,0,0), (2,0,0) ...., (8,5,0)
objp = np.zeros((CHECKERBOARD_SIZE[0] * CHECKERBOARD_SIZE[1], 3), np.float32)

# Generate grid, using 'ij' indexing to match matrix order (row, col)
grid_rows, grid_cols = np.meshgrid(
        np.arange(0, CHECKERBOARD_SIZE[0]),
        np.arange(0, CHECKERBOARD_SIZE[1]),
        indexing='ij'
)

# Combine into an (H, W, 2) array
grid = np.stack((grid_rows, grid_cols), axis=-1)
objp[:, :2] = grid_combined.reshape(-1, 2) * SQUARE_SIZE
```

```
# 3. Read Images and Detect Corners
images = glob.glob(IMAGE_PATH_PATTERN)

valid_images_count = 0
image_size = None

for fname in images:
    gray = cv2.imread(fname, cv2.IMREAD_COLOR)

    if image_size is None:
        image_size = gray.shape[::-1]

    # Find chessboard corners
    ret, corners = cv2.findChessboardCorners(
        gray, CHECKERBOARD_SIZE,
        cv2.CALIB_CB_ADAPTIVE_THRESH +
        cv2.CALIB_CB_NORMALIZE_IMAGE +
        cv2.CALIB_CB_FAST_CHECK
    )
```

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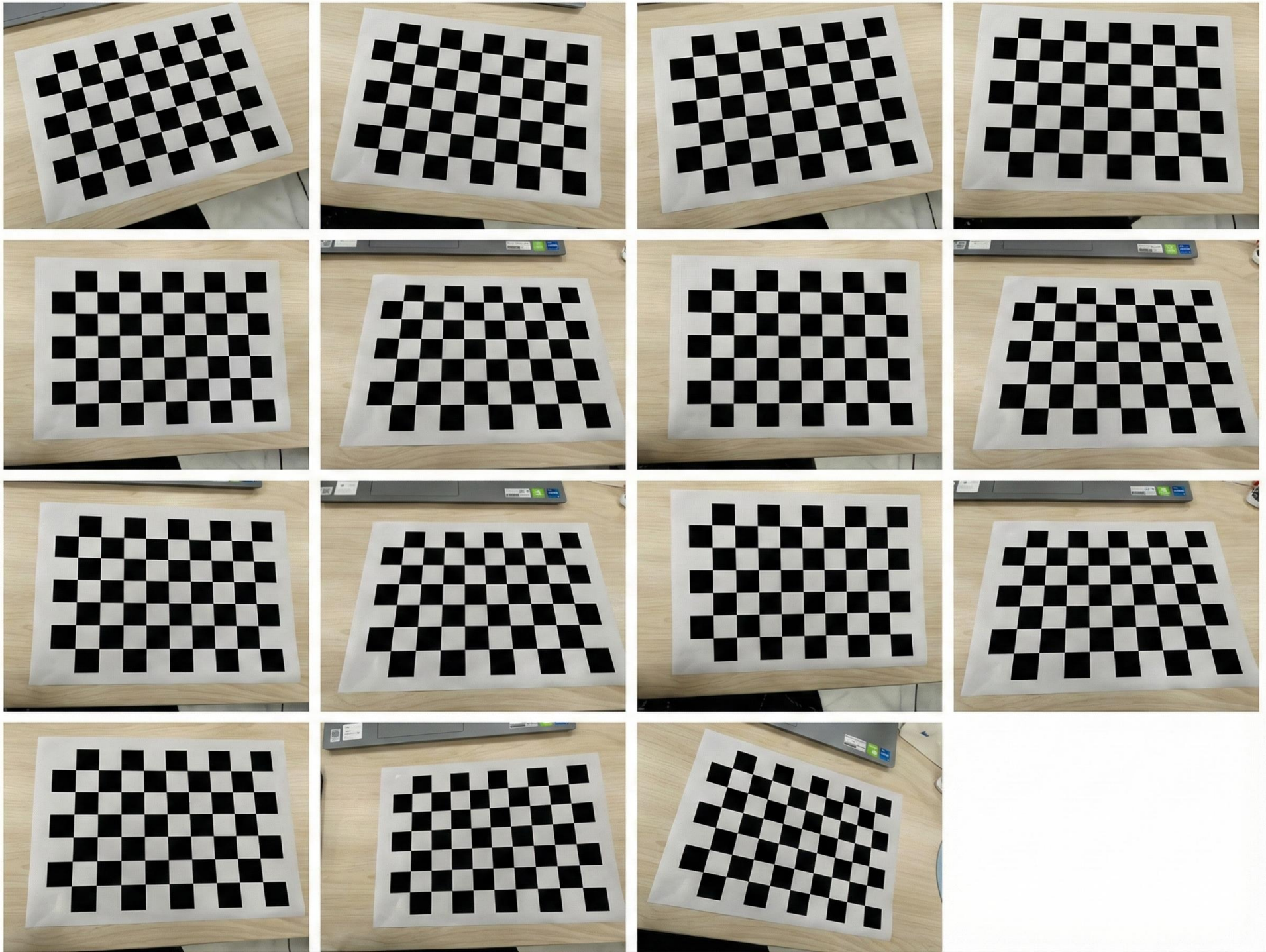
        cv2.CALIB_CB_NORMALIZE_IMAGE +
        cv2.CALIB_CB_FAST_CHECK
    )
```



```
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        cv2.CALIB_CB_ADAPTIVE_THRESH +
        cv2.CALIB_C                  AGE +
        cv2.CALIB_C
    )
```

自适应阈值

```
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        gray, CHECKERBOARD_SIZE,
        cv2.CALIB_CB_ADAPTIVE_THRESH +
        cv2.CALIB_CB_NORMALIZE_IMAGE +
        cv2.CALIB
    )
```

图像归一化

```
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        gray, CHECKERBOARD_SIZE,
        cv2.CALIB_CB_ADAPTIVE_THRESH +
        cv2.CALIB_CB_NORMALIZE_IMAGE +
        cv2.CALIB_CB_FAST_CHECK
    )
```

快速检查

```
        if ret == True:
            valid_images_count += 1
            objpoints.append(objp)

            # Sub-pixel
            refinement criteria = (cv2.TERM_CRITERIA_EPS +
                cv2.TERM_CRITERIA_MAX_ITER, 30, 0.001)
            corners_subpix =
                cv2.cornerSubPix(gray, corners, (11, 11), (-1, -1), criteria)
            imgpoints.append(corners_subpix)
            print(f"Processed: {os.path.basename(fname)}")
        else:
            print(f"Failed to detect corners: {os.path.basename(fname)}")

# 4. Execute Camera Calibration
ret_rms, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(
    objpoints, imgpoints, image_size, None, None
)
```

```
    if ret == True:
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```
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    # 4. Execute Camera Calibration
    ret_rms, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(
        objpoints, imgpoints, image_size, None, None
```

均方根重投影误差

```
        if ret == True:
            valid_images_count += 1
            objpoints.append(objp)

            # Sub-pixel
            refinement criteria = (cv2.TERM_CRITERIA_EPS +
                cv2.TERM_CRITERIA_MAX_ITER, 30, 0.001)
            corners_subpix =
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            imgpoints.append(corners_subpix)
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        else:
            print(f"Failed to detect corners: {os.path.basename(fname)}")

# 4. Execute Camera Calibration
ret_rms, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(
    c                                  e_size, None, None
)
```

相机内参矩阵

```
        if ret == True:
            valid_images_count += 1
            objpoints.append(objp)

            # Sub-pixel
            refinement criteria = (cv2.TERM_CRITERIA_EPS +
                cv2.TERM_CRITERIA_MAX_ITER, 30, 0.001)
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ret_rms, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(
)
```

畸变系数：[k1, k2, p1, p2, k3]

```
        if ret == True:
            valid_images_count += 1
            objpoints.append(objp)

            # Sub-pixel
            refinement criteria = (cv2.TERM_CRITERIA_EPS +
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# 4. Execute Camera Calibration
ret_rms, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(
    objpoin                                    None, None
)
```
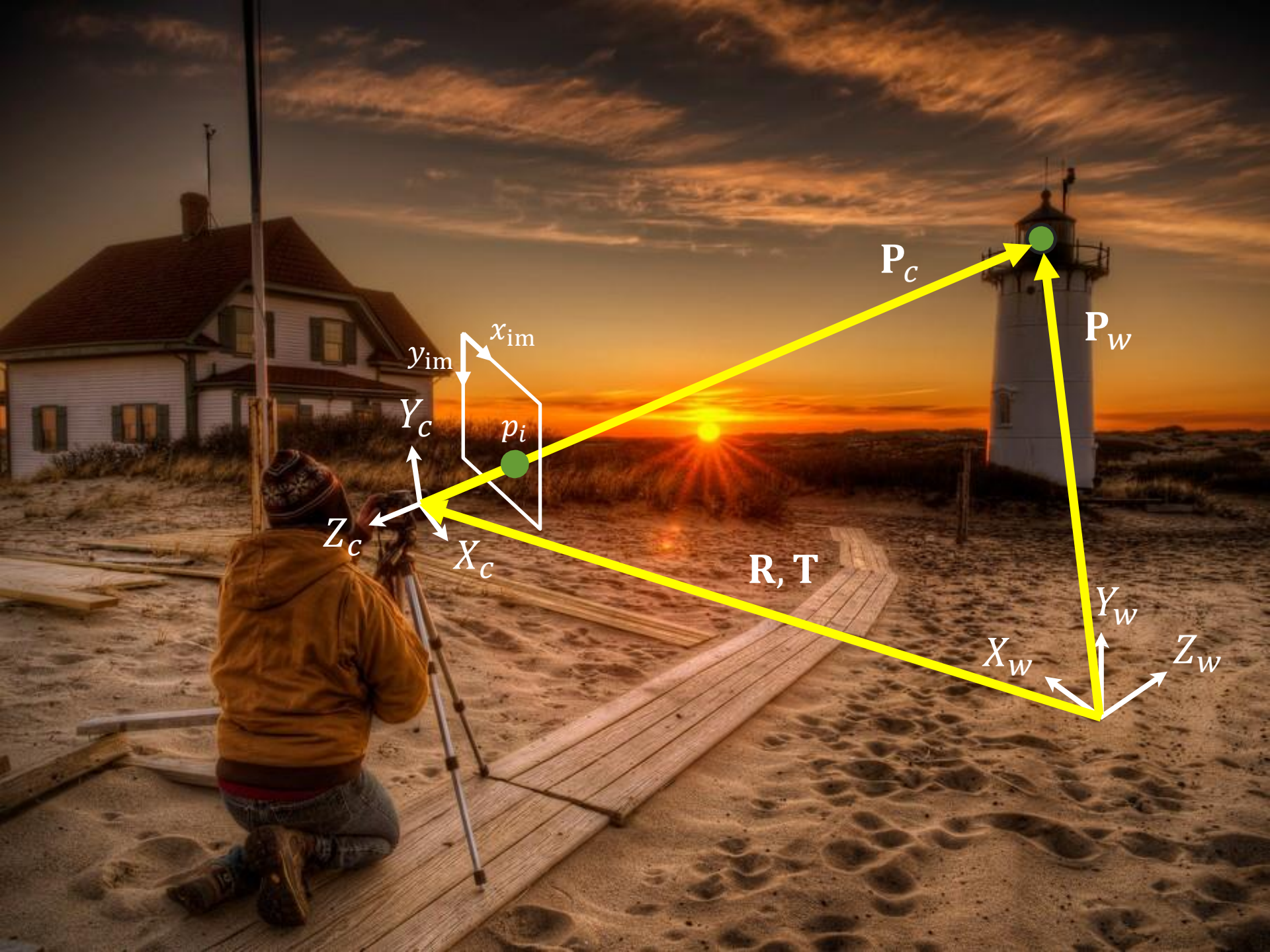

Rodrigues向量

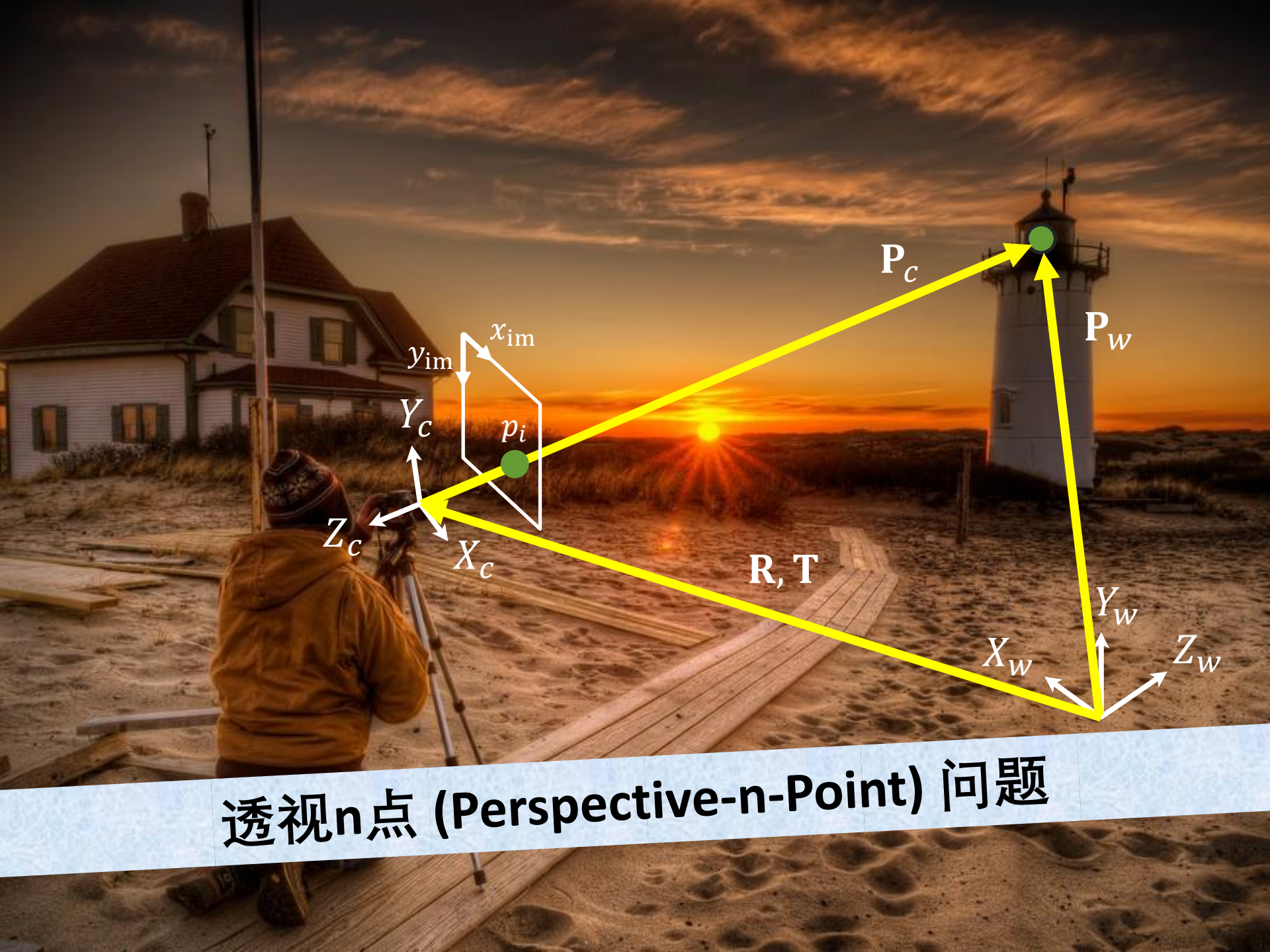
```
        if ret == True:
            valid_images_count += 1
            objpoints.append(objp)

            # Sub-pixel
            refinement criteria = (cv2.TERM_CRITERIA_EPS +
                cv2.TERM_CRITERIA_MAX_ITER, 30, 0.001)
            corners_subpix =
                cv2.cornerSubPix(gray, corners, (11, 11), (-1, -1), criteria)
            imgpoints.append(corners_subpix)
            print(f"Processed: {os.path.basename(fname)}")
        else:
            print(f"Failed to detect corners: {os.path.basename(fname)}")

# 4. Execute Camera Calibration
ret_rms, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(
    objpoints, imgpoi                                  one, None
)
```

平移向量

Python时间

鸣谢： Pepsi Max


$\mathbf{P}_c$
$\mathbf{P}_w$
x_{im}
y_{im}
p_i
Y_c
Z_c
X_c
$\mathbf{R}, \mathbf{T}$
Y_w
X_w
Z_w



透视n点 (Perspective-n-Point) 问题

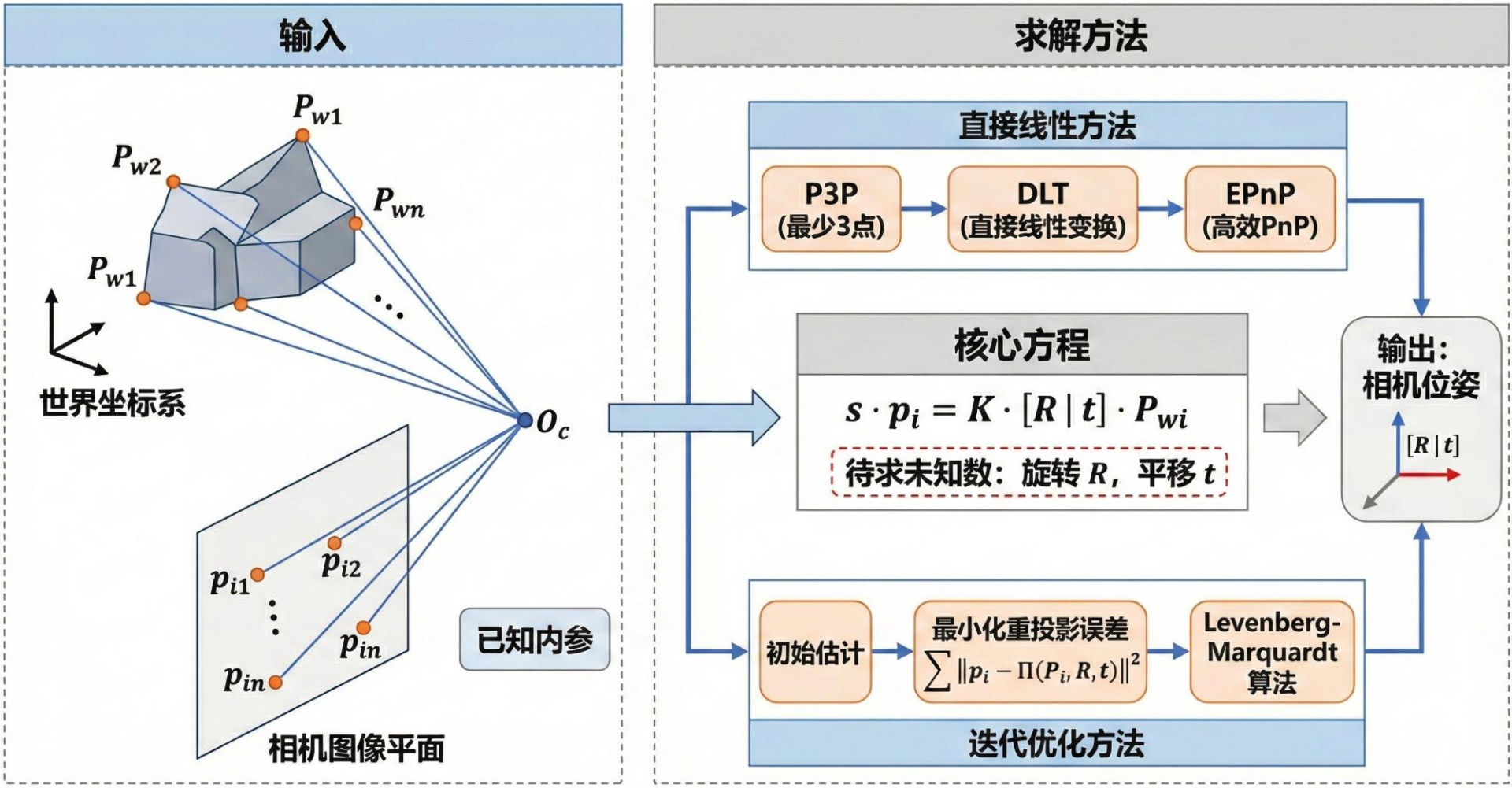
求解
PnP问题

$$\mathbf{\Pi} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{R} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{pmatrix}$$

$$= \mathbf{M}_{\mathrm{int}}[\mathbf{R}|-\mathbf{RT}]$$

$$= \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \\ \pi_5 & \pi_6 & \pi_7 & \pi_8 \\ \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} \end{pmatrix}$$

$$[\mathbf{R}|-\mathbf{RT}]\begin{pmatrix} \mathbf{T} \\ 1 \end{pmatrix} = \mathbf{RT} - \mathbf{RT} = \mathbf{0}$$

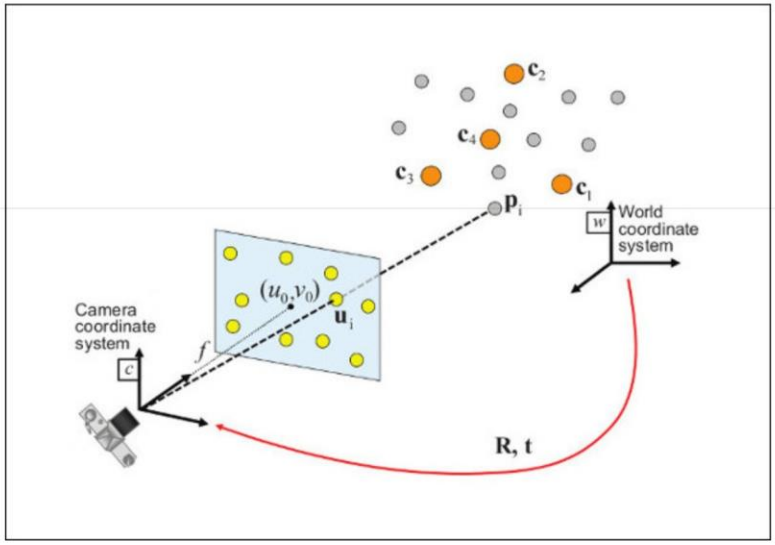

输入
P_{w1}
P_{w2}
P_{wn}
P_{w1}
世界坐标系
O_c
p_{i1}
p_{i2}
p_{in}
p_{in}
相机图像平面
已知内参
求解方法
直接线性方法
P3P
(最少3点)
DLT
(直接线性变换)
EPnP
(高效PnP)
核心方程
$s \cdot p_i = K \cdot [R \mid t] \cdot P_{wi}$
待求未知数：旋转 R，平移 t
输出：
相机位姿
$[R \mid t]$
初始估计
最小化重投影误差
$\sum \|p_i - \Pi(P_i, R, t)\|^2$
Levenberg-
Marquardt
算法
迭代优化方法

Perspective-n-Point (PnP) pose computation

Pose computation overview

The pose computation problem [148] consists in solving for the rotation and translation that minimizes the reprojection error from 3D-2D point correspondences.

The `solvePnP` and related functions estimate the object pose given a set of object points, their corresponding image projections, as well as the camera intrinsic matrix and the distortion coefficients, see the figure below (more precisely, the X-axis of the camera frame is pointing to the right, the Y-axis downward and the Z-axis forward).



Points expressed in the world frame $\mathbf{X_w}$ are projected into the image plane $[u, v]$ using the perspective projection model Π and the camera intrinsic parameters matrix $\mathbf{A}$ (also denoted $\mathbf{K}$ in the literature):

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{A} \, \Pi \, {}^{c}\mathbf{T}_{\mathbf{w}} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

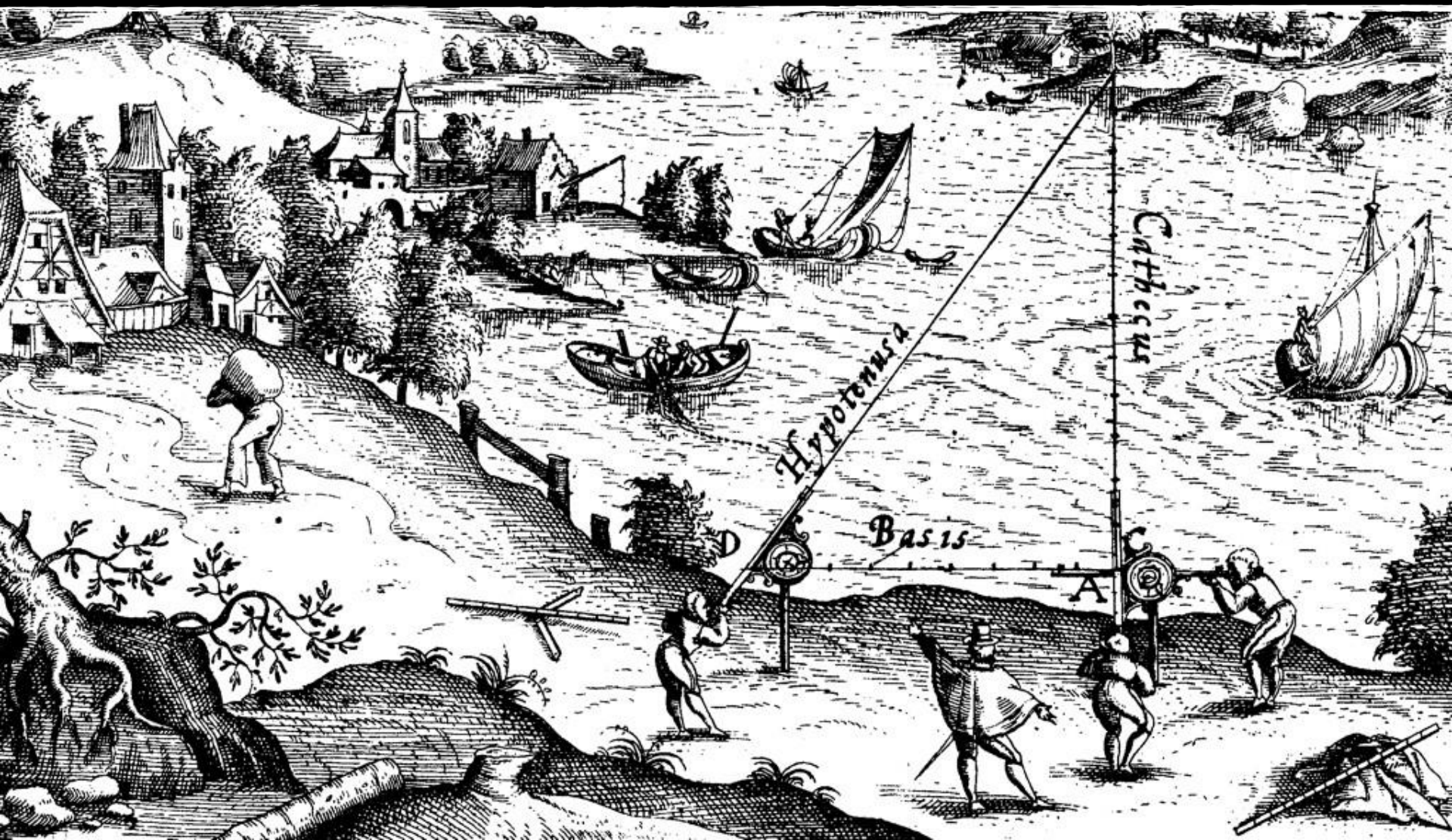
The estimated pose is thus the rotation (`rvec`) and the translation (`tvec`) vectors that allow transforming a 3D point expressed in the world frame into the camera frame:

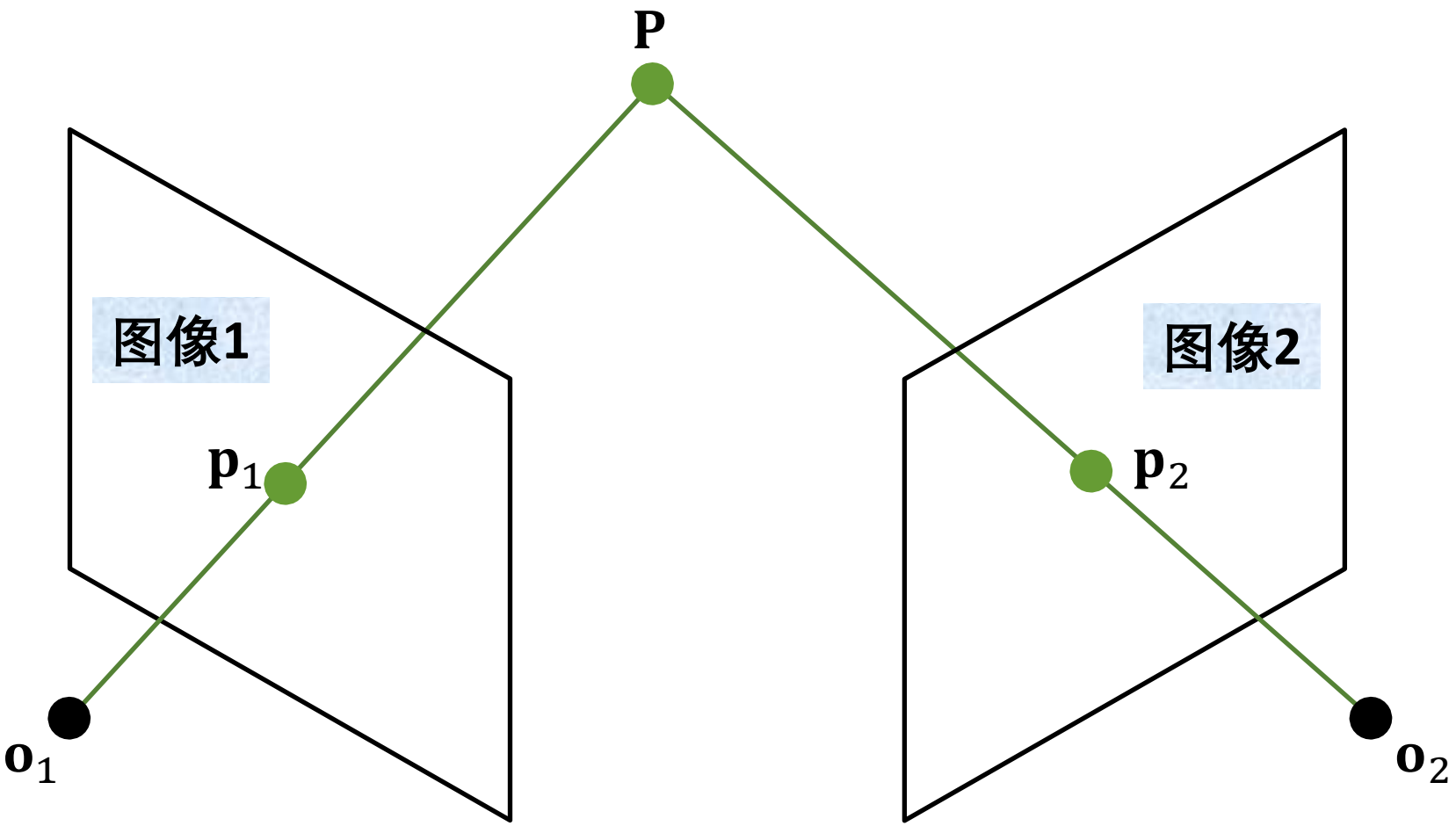
如果没有3D空间对应点呢？

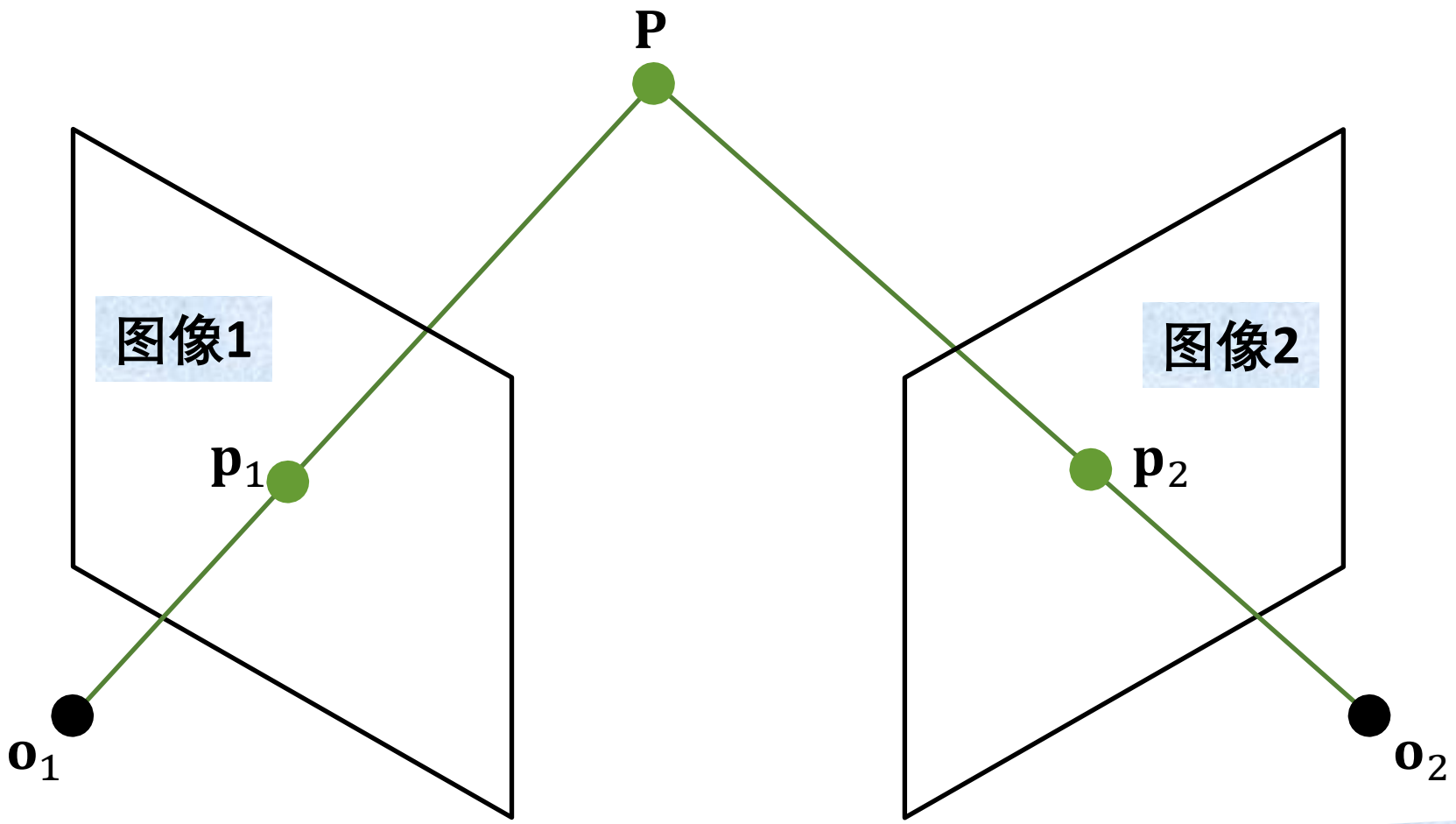
如果没有3D空间对应点呢？

找到多幅图像点对应关系，求解相机模型参数

三角測量



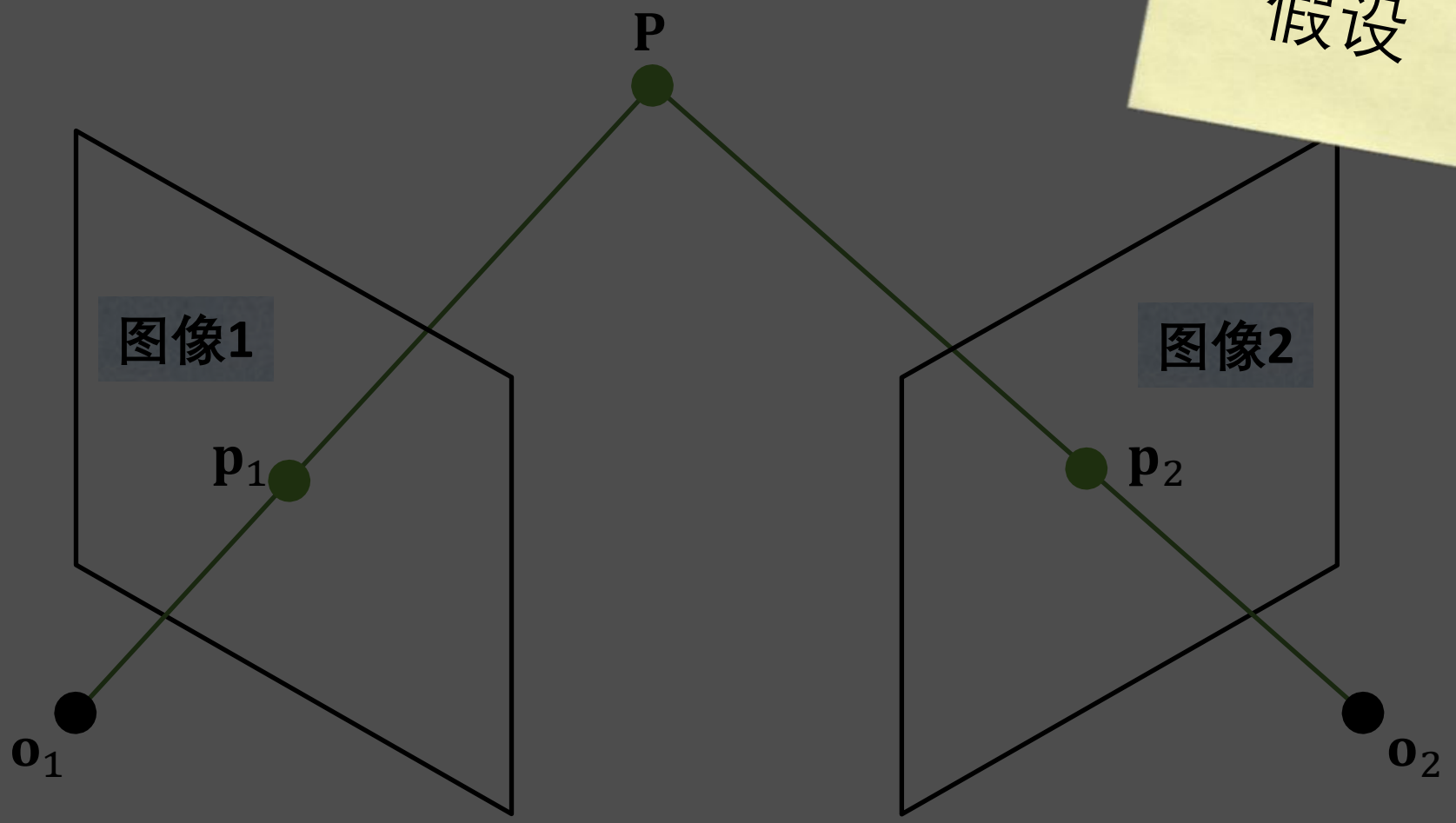

P
图像1
图像2
p_1
p_2
o_1
o_2

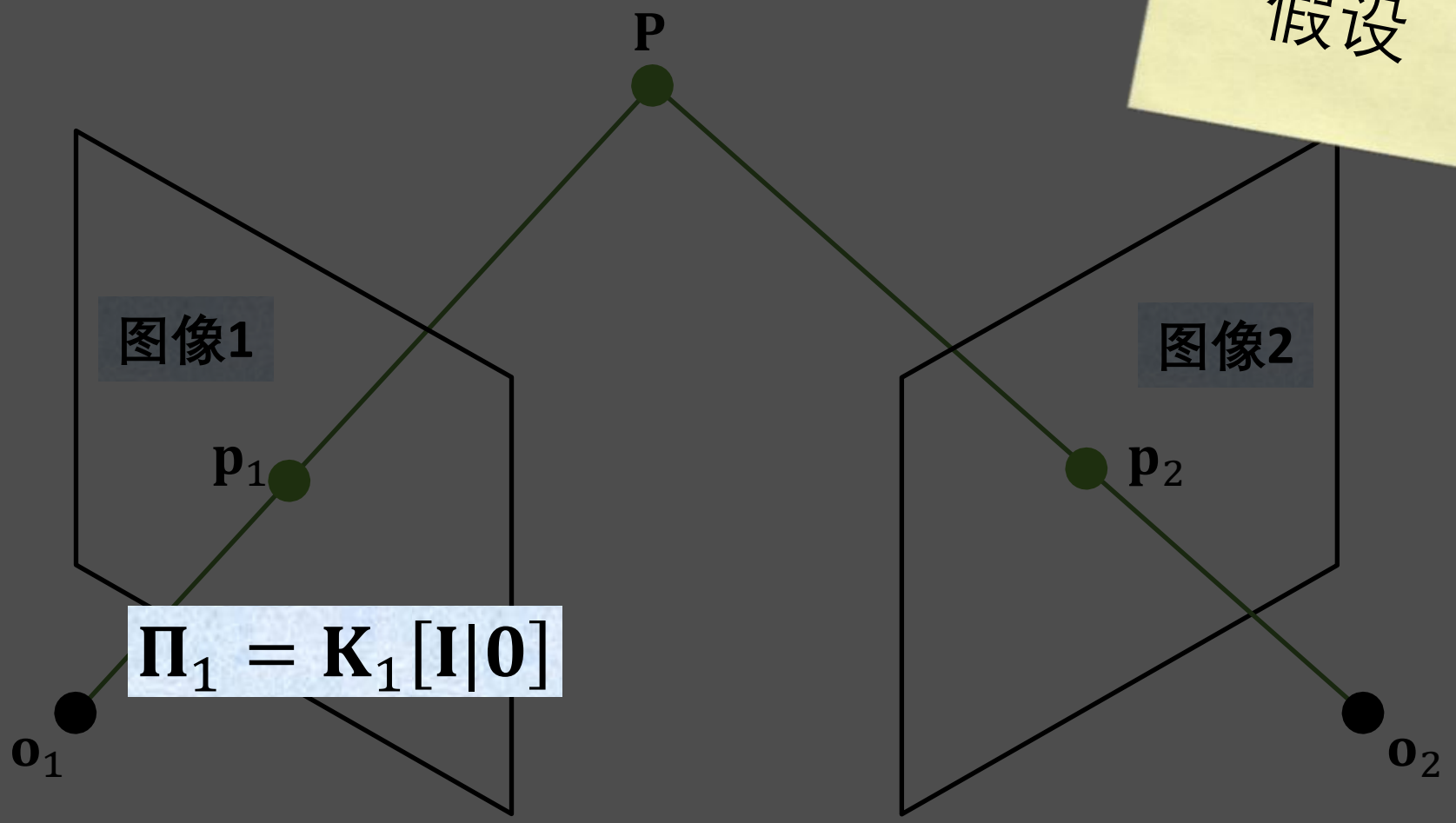


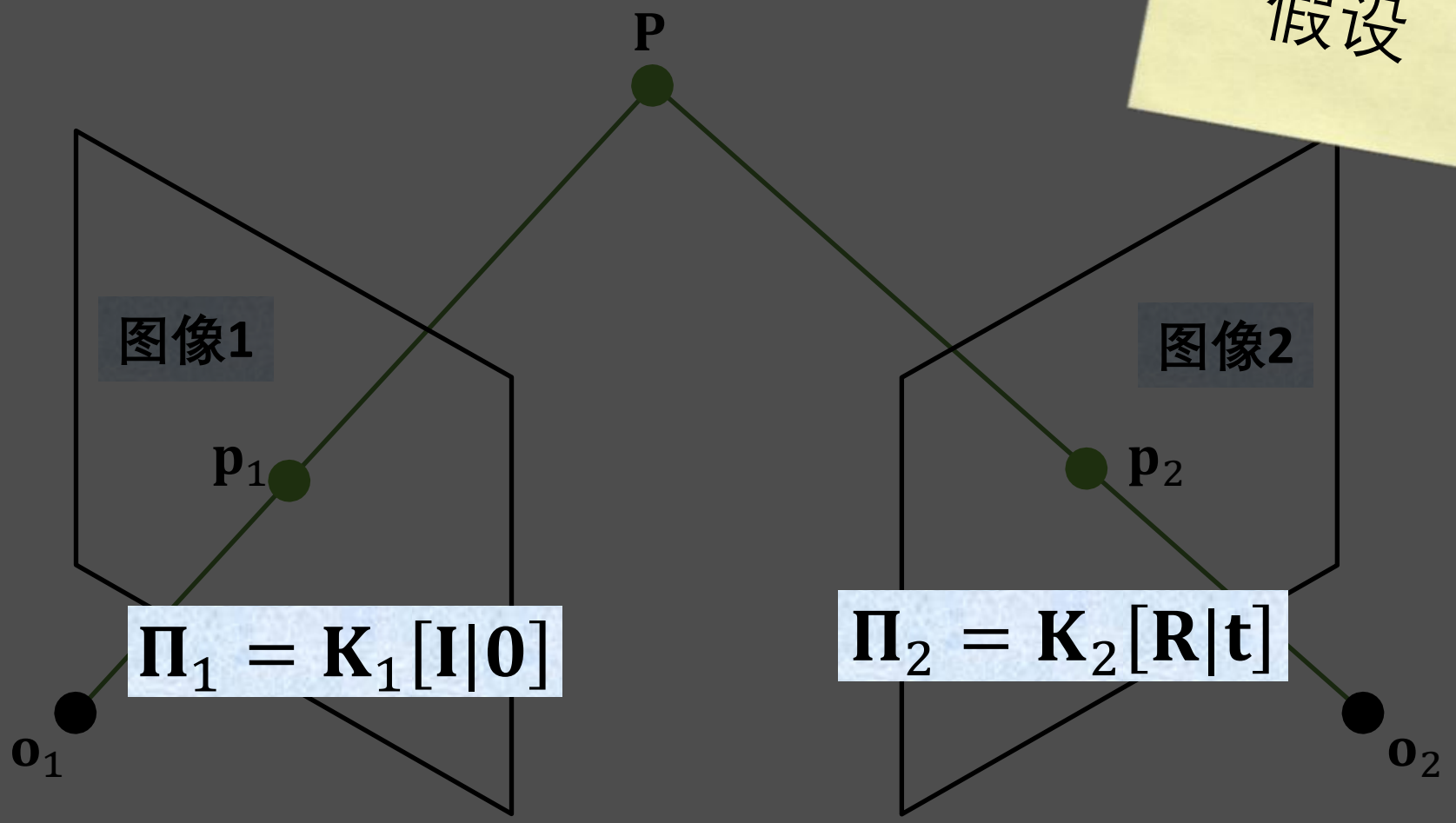
如何关联2D-2D对应点？

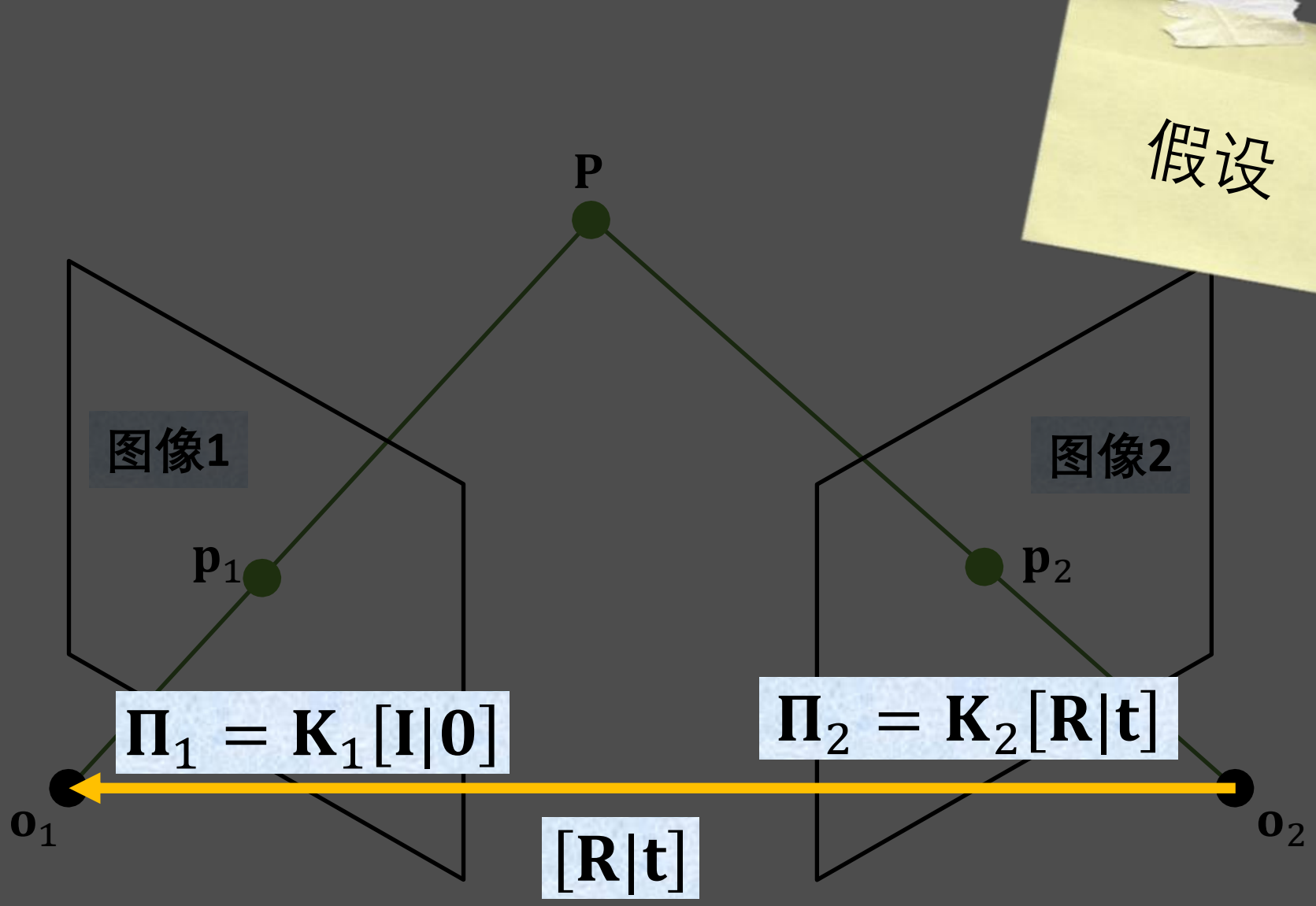
$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \mathbf{F} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = 0$$

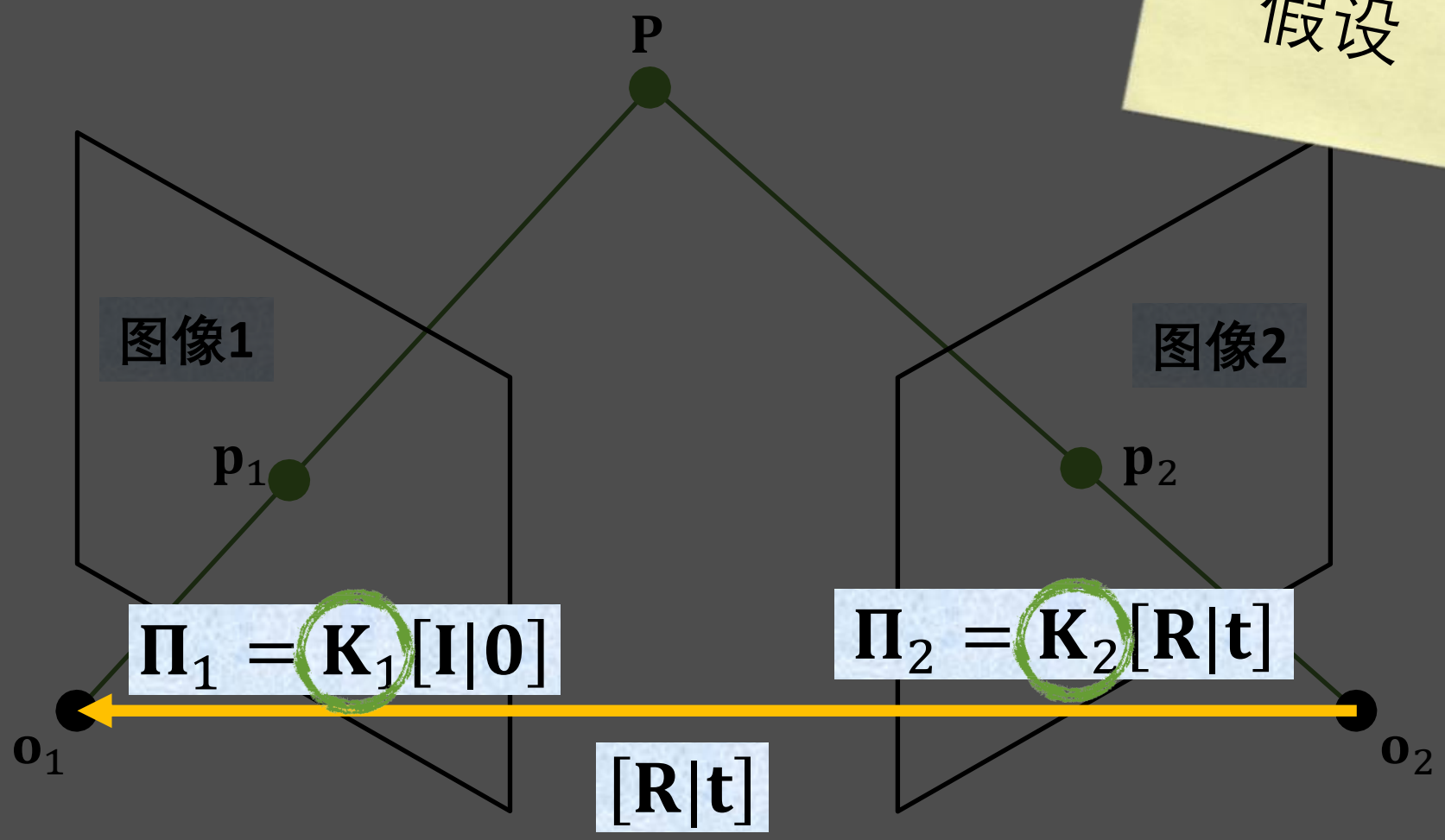
基础矩阵

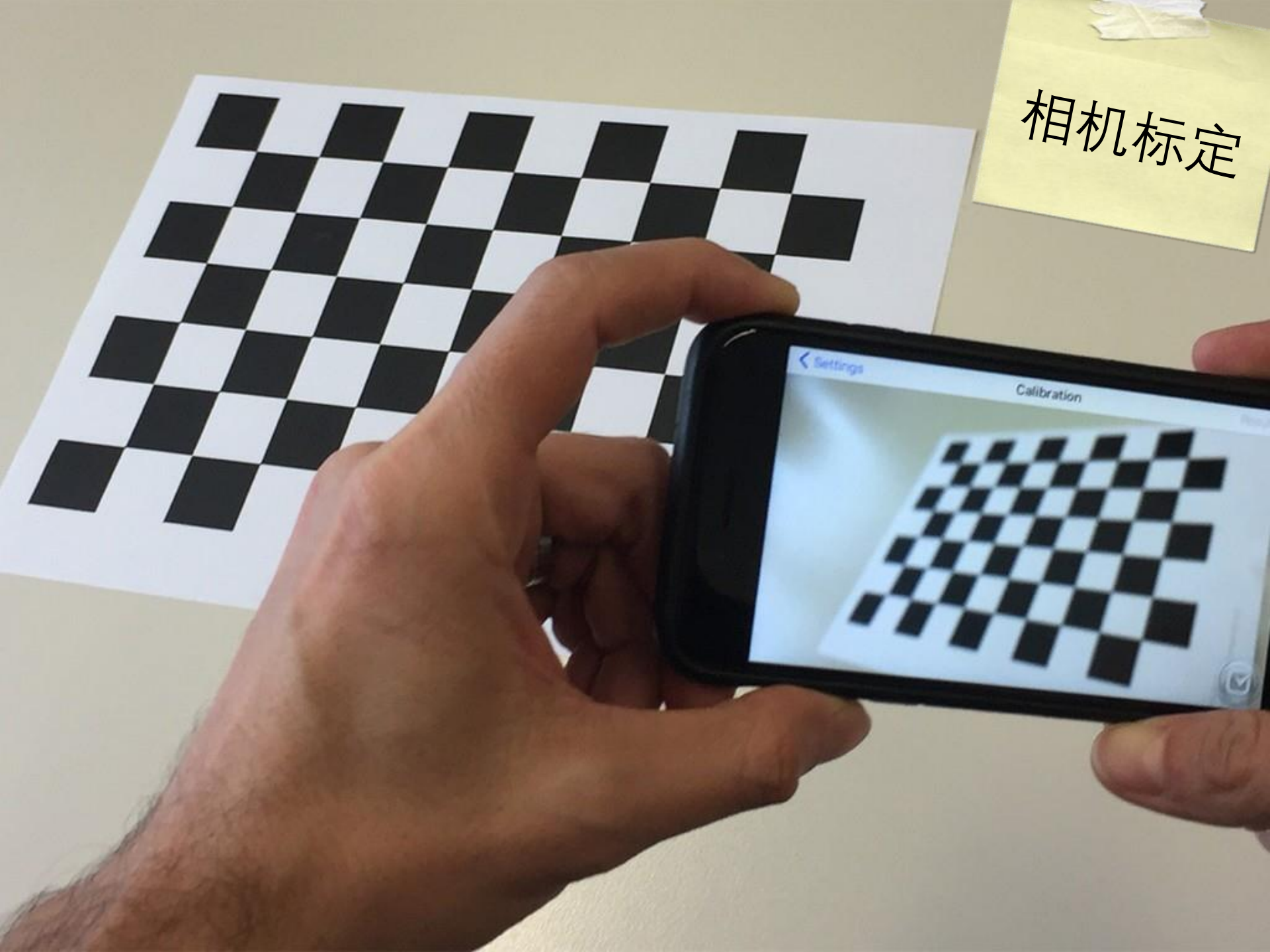

假设
P
图像1
图像2
p_1
p_2
o_1
o_2


假设
P
图像1
图像2
p_1
p_2
$\Pi_1 = K_1[I|0]$
o_1
o_2


假设
P
图像1
图像2
p_1
p_2
$\Pi_1 = K_1[I|0]$
$\Pi_2 = K_2[R|t]$
o_1
o_2


假设
P
图像1
图像2
p_1
p_2
$\Pi_1 = K_1[I|0]$
$\Pi_2 = K_2[R|t]$
o_1
o_2
$[R|t]$


假设
P
图像1
图像2
p_1
p_2
$\Pi_1 = K_1[I|0]$
$\Pi_2 = K_2[R|t]$
o_1
o_2
$[R|t]$


相机标定
Settings
Calibration

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \mathbf{K}_2^{-\mathrm{T}} \mathbf{E} \mathbf{K}_1^{-1} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = 0$$

本质矩阵

$$\mathbf{E} = [\mathbf{t}_{\times}]\mathbf{R}$$

本质矩阵

$$\mathbf{E} = [\mathbf{t}_\times]\mathbf{R}$$

$$[\mathbf{t}_\times] = \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{pmatrix}$$

本质矩阵

$$\mathbf{E} = [\mathbf{t}_\times]\mathbf{R}$$

$$[\mathbf{t}_\times] = \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{pmatrix}$$

本质矩阵

反对称矩阵

回顾：线性代数

谱分解

定理：令$\mathbf{M}$是一个$d \times d$实反对称矩阵，它具有非零虚特征值$i\lambda_1, \cdots, i\lambda_d$，则存在正交矩阵$\mathbf{U}$，使得：

谱分解

定理： 令$\mathbf{M}$是一个$d \times d$实反对称矩阵，它具有非零虚特征值$i\lambda_1, \cdots, i\lambda_d$，则存在正交矩阵$\mathbf{U}$，使得：

$$\mathbf{M} = \mathbf{U} \begin{pmatrix} \begin{matrix} 0 & \lambda_1 \\ -\lambda_1 & 0 \end{matrix} & & & \\ & \begin{matrix} 0 & \lambda_2 \\ -\lambda_2 & 0 \end{matrix} & & \\ & & \ddots & \\ & & & \begin{matrix} 0 & \lambda_{\lfloor d/2 \rfloor} \\ -\lambda_{\lfloor d/2 \rfloor} & 0 \end{matrix} \\ & & & & \begin{matrix} 0 & & \\ & \ddots & \\ & & 0 \end{matrix} \end{pmatrix} \mathbf{U}^T$$

初等行变换

定义：矩阵的初等行变换指

初等行变换

定义：矩阵的初等行变换指

1. 互换矩阵中两行的位置

初等行变换

定义：矩阵的初等行变换指

1. 互换矩阵中两行的位置
2. 将矩阵的一行乘以非零常数

初等行变换

定义：矩阵的初等行变换指

1. 互换矩阵中两行的位置
2. 将矩阵的一行乘以非零常数
3. 将矩阵的某一行乘以任意一个数，加到另一行

初等行变换

定义：矩阵的初等行变换指

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$$\mathbf{Ax} = \mathbf{b}$$

初等行变换

定义：矩阵的初等行变换指

1. 互换矩阵中两行的位置
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3. 将矩阵的某一行乘以任意一个数，加到另一行

$$\mathbf{Ax} = \mathbf{b}$$

$$[\mathbf{A}|\mathbf{b}]$$

初等
行变换

定义：矩阵的初等行变换指

1. 互换矩阵中两行的位置
2. 将矩阵的一行乘以非零常数
3. 将矩阵的某一行乘以任意一个数，加到另一行

$$\mathbf{Ax} = \mathbf{b}$$

$$[\mathbf{A}|\mathbf{b}]$$

增广矩阵

初等行变换

定义： 矩阵的初等行变换指

1. 互换矩阵中两行的位置
2. 将矩阵的一行乘以非零常数
3. 将矩阵的某一行乘以任意一个数，加到另一行

$$\mathbf{Ax} = \mathbf{b}$$

$$[\mathbf{A}|\mathbf{b}]$$

增广矩阵

对增广矩阵进行初等变换不影响方程组的解

回顾：线性代数

$$\mathbf{E} = [\mathbf{t}_\times]\mathbf{R}$$

$$[\mathbf{t}_\times] = \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{pmatrix}$$

本质矩阵

反对称矩阵

$$\mathbf{E} = [\mathbf{t}_{\times}]\mathbf{R}$$

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谱分解

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谱分解

$$[\mathbf{t}_\times] = \lambda \mathbf{U} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{U}^{\mathrm{T}}$$

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代入

$$\mathbf{E} = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}\mathbf{R}$$

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代入

$$\mathbf{E} = \mathbf{U}\mathbf{Z}\mathbf{U}^\mathrm{T}\mathbf{R}$$

λ去哪了？

$$\mathbf{P}_r^{\mathrm{T}}\mathbf{E}\mathbf{P}_l = 0$$

在齐次坐标系中是确定的，且具有尺度不变性

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谱分解

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代入

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我们有：

$$\mathbf{E} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$$

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正交矩阵

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比较两个式子，为了把E写成SVD的形式，需要把Z构造成对角矩阵和正交矩阵相乘的形式

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$$[\mathbf{Z}|\mathbf{I}] = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

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初等矩阵变换

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初等矩阵变换

$$\underbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]}_{\mathbf{W}}$$

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$\mathbf{W}$

正交矩阵，且$\mathbf{W}^{-1} = \mathbf{W}^{\mathrm{T}}$

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$\mathbf{W}$

正交矩阵，且$\mathbf{W}^{-1} = \mathbf{W}^{\mathrm{T}}$

$$\mathbf{W} = \mathbf{R}_Z\left(\frac{\pi}{2}\right) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[\mathbf{Z}|\mathbf{I}] = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

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正交矩阵，且$\mathbf{W}^{-1} = \mathbf{W}^{\mathrm{T}}$

$$\mathbf{ZW} = \mathrm{diag}(1,1,0)$$

$$\mathbf{ZW}^{\mathrm{T}} = -\mathrm{diag}(1,1,0)$$

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代入$\mathbf{E} = \mathbf{UZU}^{\mathrm{T}}\mathbf{R}$

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代入$\mathbf{E} = \mathbf{UZU}^{\mathrm{T}}\mathbf{R}$

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\left(\mathbf{W}^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\mathbf{R}\right)$$

或

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正交矩阵

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$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

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这意味着什么？

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改写

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

这意味着什么？

本质矩阵秩为2，且奇异值相等

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

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代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

$$\mathbf{E} = \mathbf{UZWV}^{\mathrm{T}}$$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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我们有：

$$\mathbf{E} = [\mathbf{t}_{\times}]\mathbf{R} = \left(\mathbf{UZU}^{\mathrm{T}}\right)\mathbf{R}$$

旋转矩阵

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对比

旋转矩阵

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代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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我们有：

$$\mathbf{E} = [\mathbf{t}_{\times}]\mathbf{R} = \left(\mathbf{UZU}^{\mathrm{T}}\right)\mathbf{R}$$

对比

$$\mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

$$\mathbf{E} = \mathbf{UZWV}^{\mathrm{T}}$$

我们有：

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对比

$$\mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$$

我们又有：

$$\mathbf{ZW}^{\mathrm{T}} = -\,\mathrm{diag}(1,1,0)$$

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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对比

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我们又有：

$$\mathbf{ZW}^{\mathrm{T}} = -\,\mathrm{diag}(1,1,0)$$

并服从如下约束：

$$\det \mathbf{R} = 1$$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

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我们有：

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同理

旋转矩阵

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代入$\mathbf{ZW} = \mathrm{diag}(1,1,0)$

$$\mathbf{E} = \mathbf{UZWV}^{\mathrm{T}}$$

我们有：

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$$\mathbf{ZW}^{\mathrm{T}} = -\,\mathrm{diag}(1,1,0)$$

并服从如下约束：

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同理

$$\mathbf{R_2} = \mathbf{UW}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$$

旋转矩阵

$$\mathbf{E} = \mathbf{U}\,\mathrm{diag}(1,1,0)\,\mathbf{V}^{\mathrm{T}}$$

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并服从如下约束：

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旋转矩阵

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}$$

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我们有：

$$[\mathbf{t}_\times]\mathbf{t} = \mathbf{0}$$

平移向量

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}$$

我们有：

$$[\mathbf{t}_\times]\mathbf{t} = \mathbf{0}$$

$\mathbf{t}$是$[\mathbf{t}_\times]$的零空间

平移向量

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}$$

我们有：

$$[\mathbf{t}_\times]\mathbf{t} = \mathbf{0}$$

t是$[\mathbf{t}_\times]$的零空间

t是U的最后一列$\mathbf{u}_3$

平移向量

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}$$

我们有：

$$[\mathbf{t}_\times]\mathbf{t} = \mathbf{0}$$

$\mathbf{t}$是$[\mathbf{t}_\times]$的零空间

$\mathbf{t}$是$\mathbf{U}$的最后一列$\mathbf{u}_3$

考虑一个尺度因子$\lambda = \pm 1$：

平移向量

$$[\mathbf{t}_\times] = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}$$

我们有：

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$\mathbf{t}$是$[\mathbf{t}_\times]$的零空间

$\mathbf{t}$是$\mathbf{U}$的最后一列$\mathbf{u}_3$

考虑一个尺度因子$\lambda = \pm 1$：

$$\mathbf{t}_1 = \mathbf{u}_3$$

$$\mathbf{t}_2 = -\mathbf{u}_3$$

平移向量

$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R_1} = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}$$

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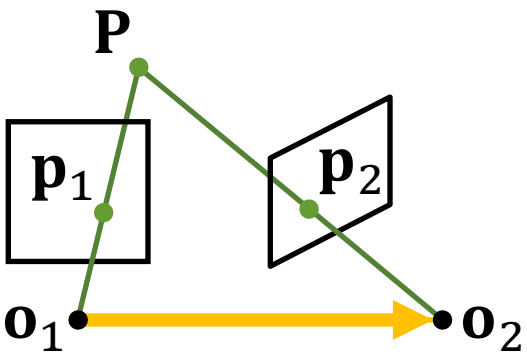
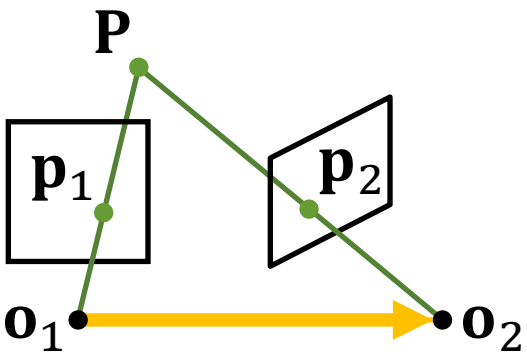


$$[\mathbf{R}_1|\mathbf{t}_1]$$

$\mathbf{t}_1 = \mathbf{u}_3$ $\quad\quad$ $\mathbf{R_1} = \mathbf{UWV}^{\mathrm{T}}$

$\mathbf{t}_2 = -\mathbf{u}_3$ $\quad\quad$ $\mathbf{R}_2 = \mathbf{UW}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}$



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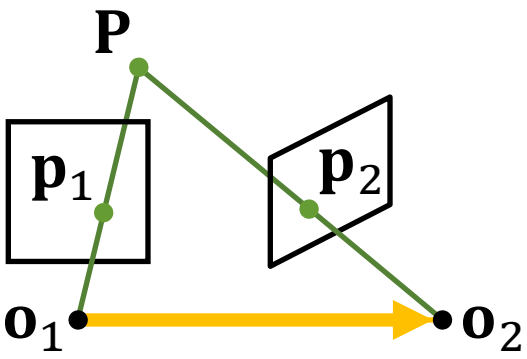


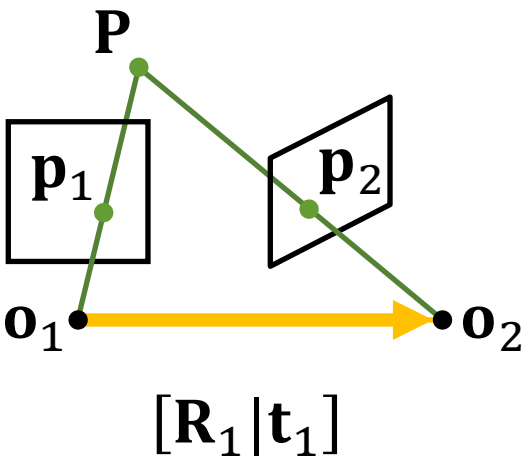
$[\mathbf{R}_1|\mathbf{t}_1]$

$\mathbf{t}_1 = \mathbf{u}_3$

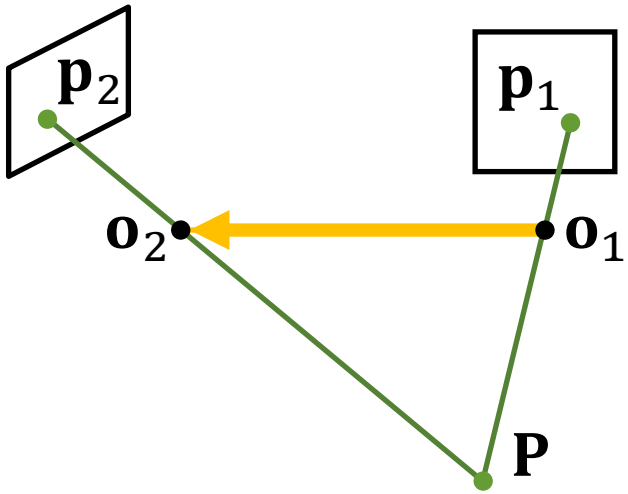
$\mathbf{t}_2 = -\mathbf{u}_3$

$\mathbf{R_1} = \mathbf{UWV}^T$

$\mathbf{R}_2 = \mathbf{UW}^T\mathbf{V}^T$



$[\mathbf{R}_1|\mathbf{t}_1]$



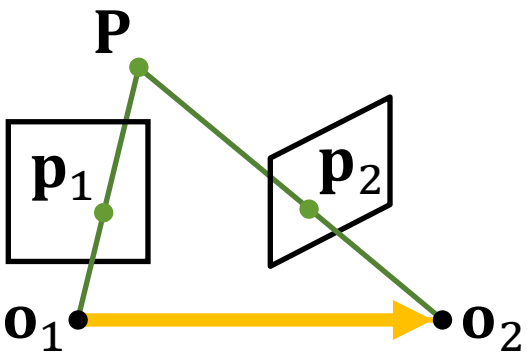
$[\mathbf{R}_1|\mathbf{t}_2]$

$$\mathbf{t}_1 = \mathbf{u}_3$$

$$\mathbf{t}_2 = -\mathbf{u}_3$$

$$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

$$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$

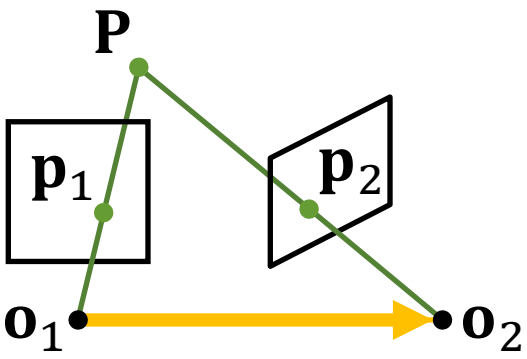


$[\mathbf{R}_1|\mathbf{t}_1]$

$[\mathbf{R}_1|\mathbf{t}_2]$

$$\mathbf{t}_1 = \mathbf{u}_3$$

$$\mathbf{t}_2 = -\mathbf{u}_3$$

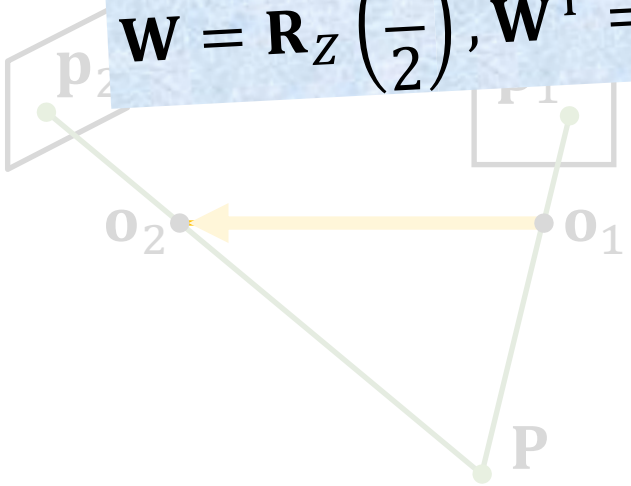


$$[\mathbf{R}_1|\mathbf{t}_1]$$

$$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

$$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$

$$\mathbf{W} = \mathbf{R}_Z\left(\frac{\pi}{2}\right), \mathbf{W}^T = \mathbf{R}_Z\left(-\frac{\pi}{2}\right)$$



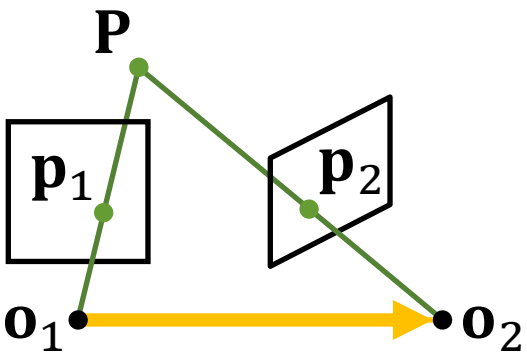
$$[\mathbf{R}_1|\mathbf{t}_2]$$

$\mathbf{t}_1 = \mathbf{u}_3$

$\mathbf{t}_2 = -\mathbf{u}_3$

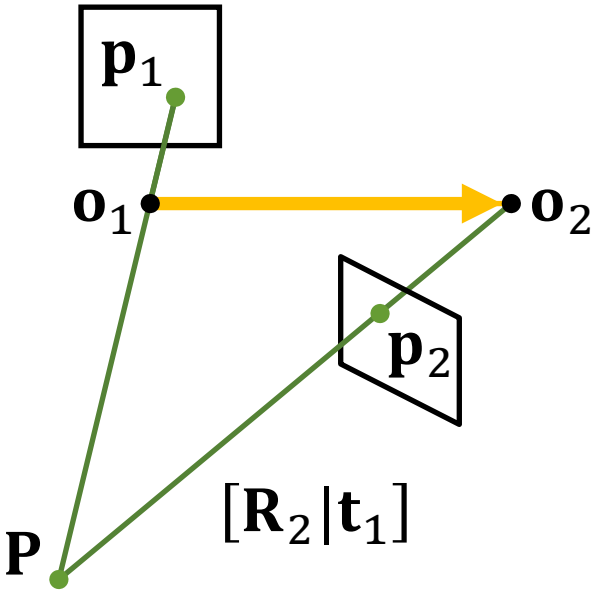
$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$

$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$



$[\mathbf{R}_1|\mathbf{t}_1]$

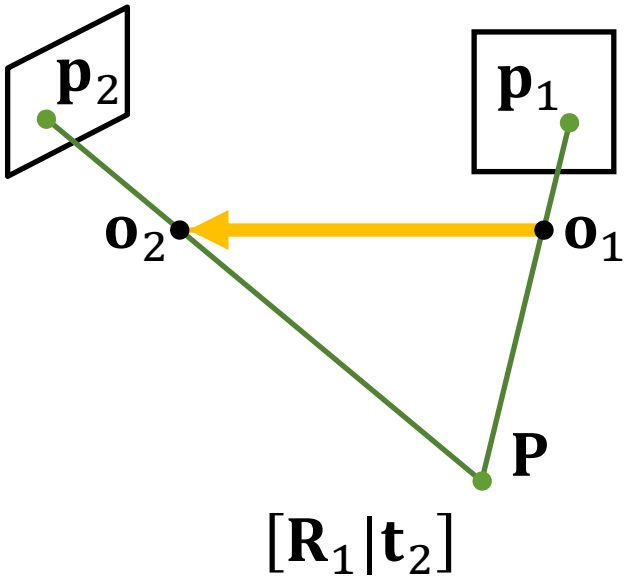
$[\mathbf{R}_1|\mathbf{t}_2]$



$[\mathbf{R}_2|\mathbf{t}_1]$

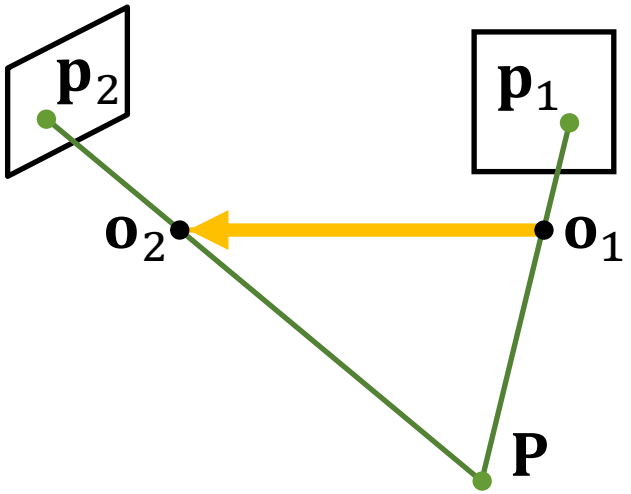
$\mathbf{t}_1 = \mathbf{u}_3$ $\mathbf{R}_1 = \mathbf{UWV}^T$

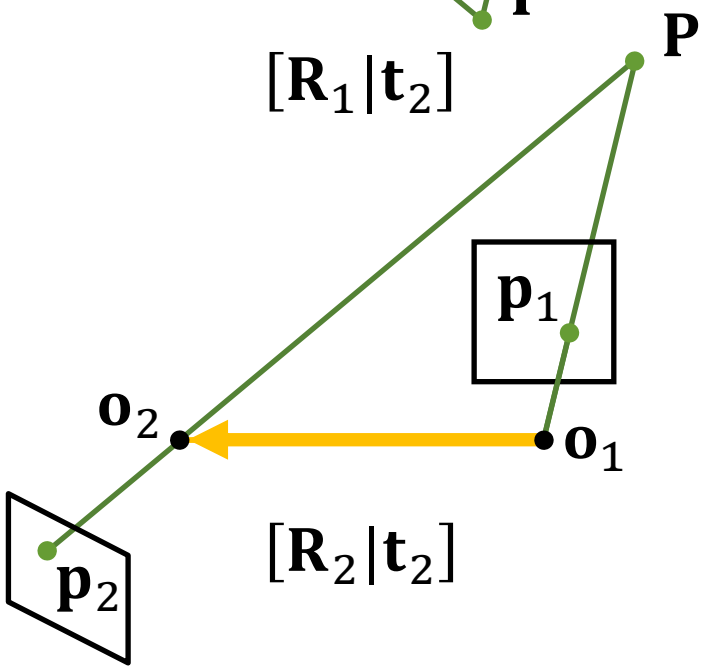
$\mathbf{t}_2 = -\mathbf{u}_3$ $\mathbf{R}_2 = \mathbf{UW}^T\mathbf{V}^T$



$\mathbf{t}_1 = \mathbf{u}_3$ $\mathbf{R}_1 = \mathbf{UWV}^T$

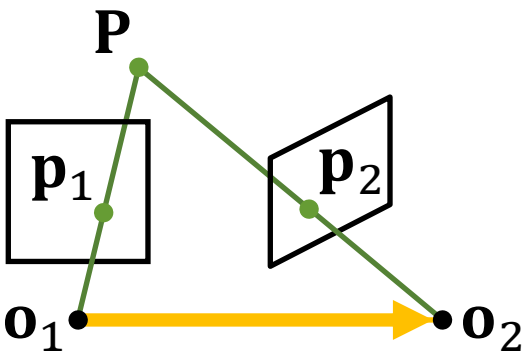
$\mathbf{t}_2 = -\mathbf{u}_3$ $\mathbf{R}_2 = \mathbf{UW}^T\mathbf{V}^T$



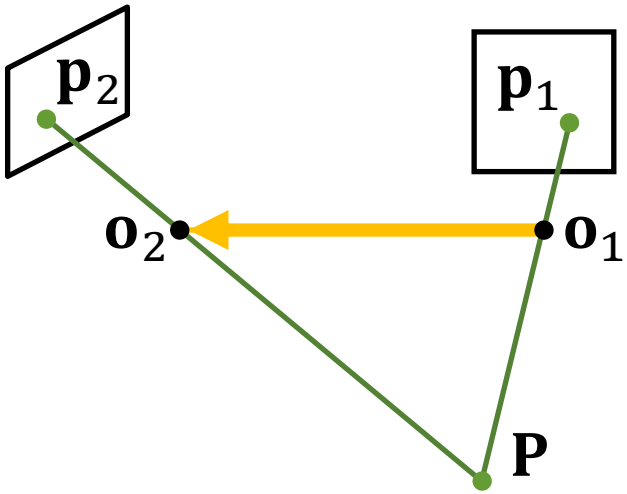


$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

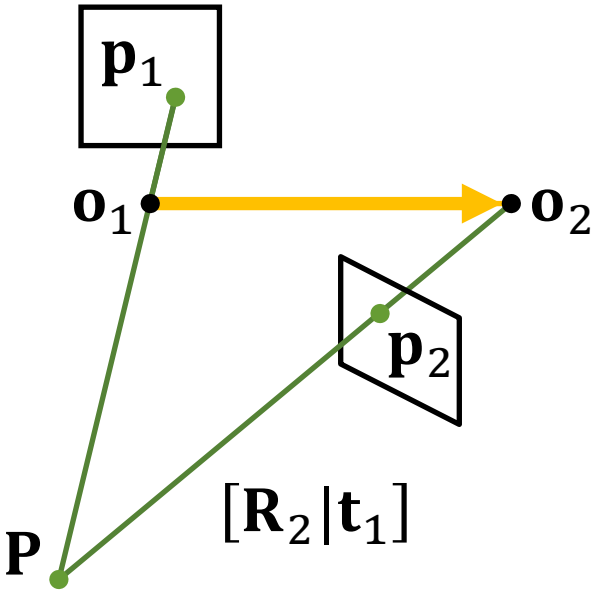
$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$



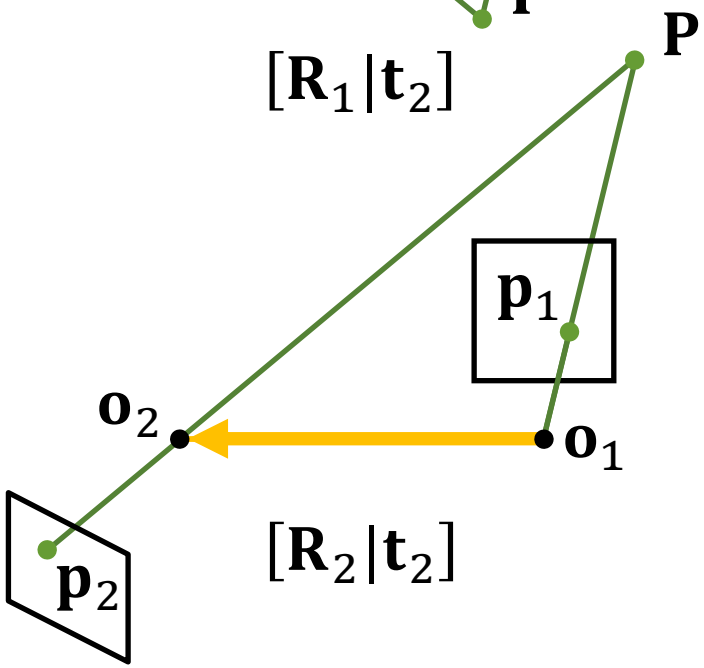
$[\mathbf{R}_1|\mathbf{t}_1]$



$[\mathbf{R}_1|\mathbf{t}_2]$



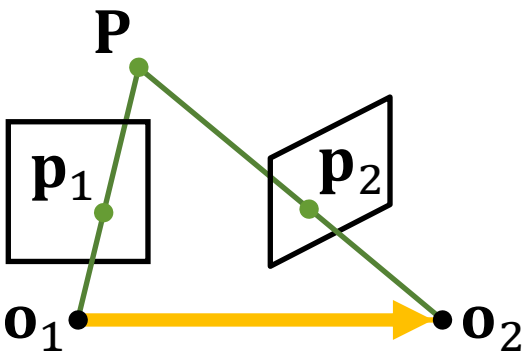
$[\mathbf{R}_2|\mathbf{t}_1]$



$[\mathbf{R}_2|\mathbf{t}_2]$

$$\mathbf{t}_1 = \mathbf{u}_3 \qquad \mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^T$$

$$\mathbf{t}_2 = -\mathbf{u}_3 \qquad \mathbf{R}_2 = \mathbf{U}\mathbf{W}^T\mathbf{V}^T$$



$[\mathbf{R}_1|\mathbf{t}_1]$

$[\mathbf{R}_1|\mathbf{t}_2]$

$[\mathbf{R}_2|\mathbf{t}_1]$

$[\mathbf{R}_2|\mathbf{t}_2]$

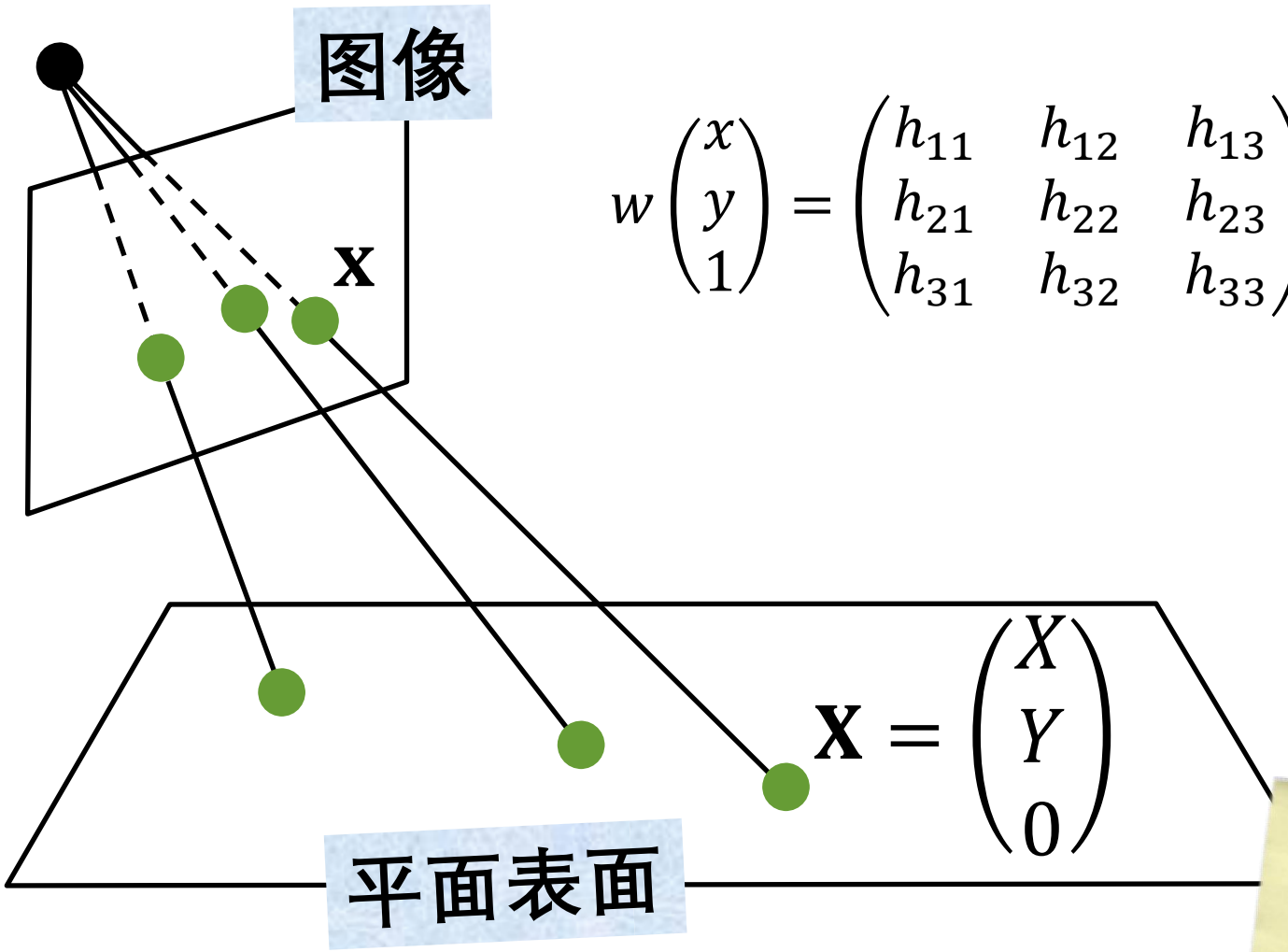
如果世界表面为平面，或平移分量太小，
本质/基础矩阵退化

如果世界表面为平面，或平移分量太小，
本质/基础矩阵退化

约束是线性相关的

$$\omega \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

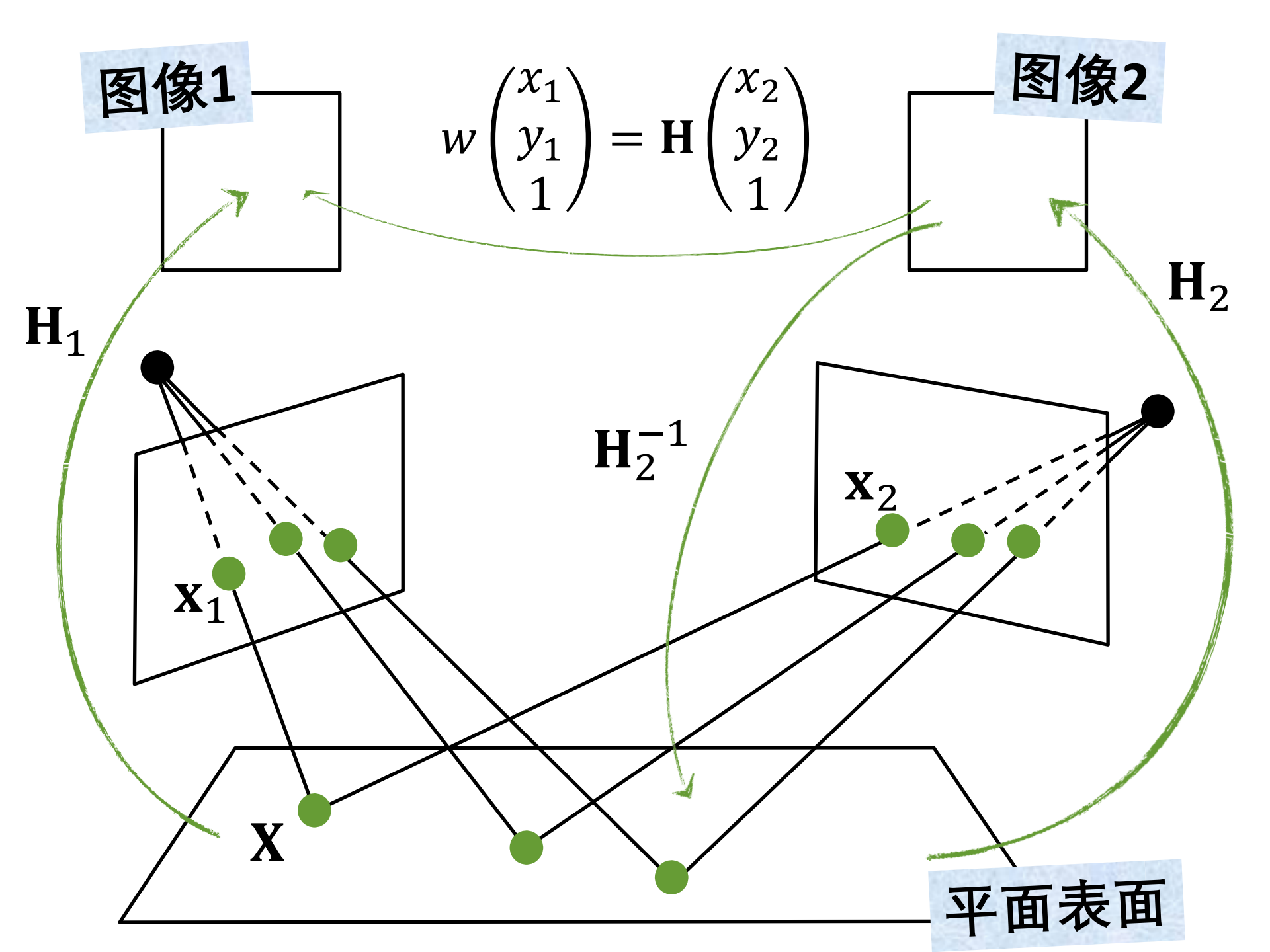
单应矩阵

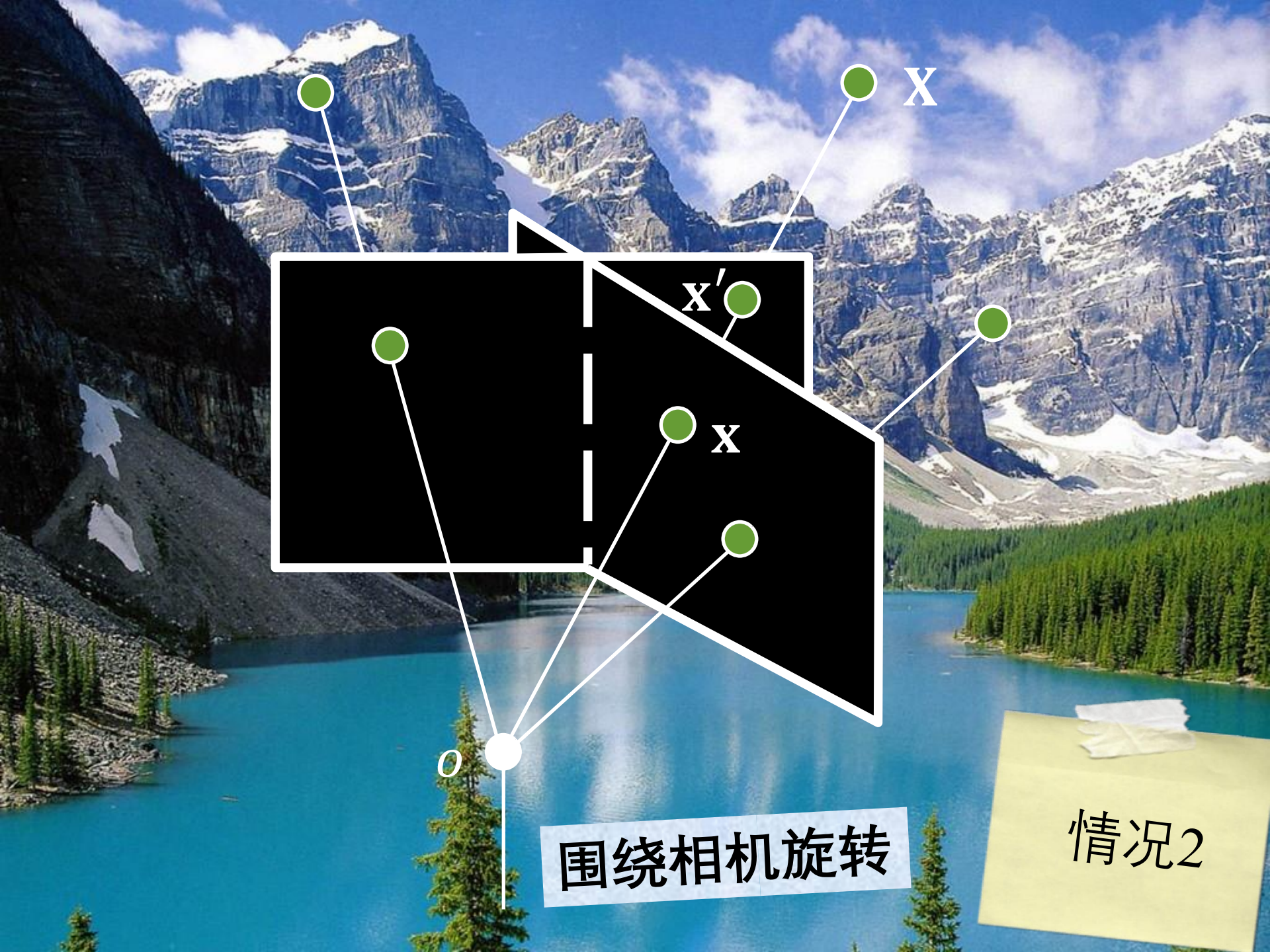


$$w\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

情况1

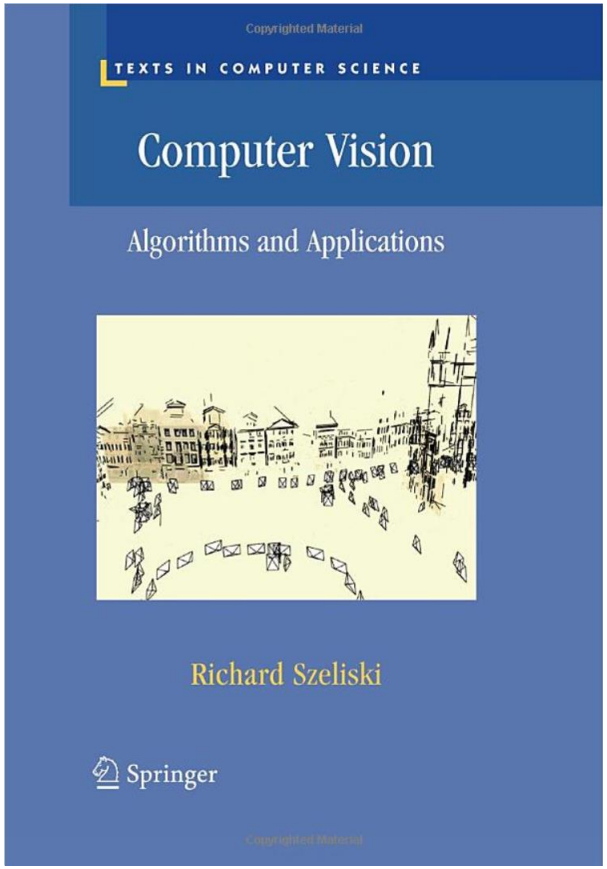

图像1
$w\begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{H}\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$
图像2
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_2^{-1}$
$\mathbf{x}_1$
$\mathbf{x}_2$
$\mathbf{X}$
平面表面


X
x′
x
o
围绕相机旋转
情况2

$$\omega \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

单应矩阵

不能通过伪逆求解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Computer Vision
Algorithms and Applications
Richard Szeliski
Springer
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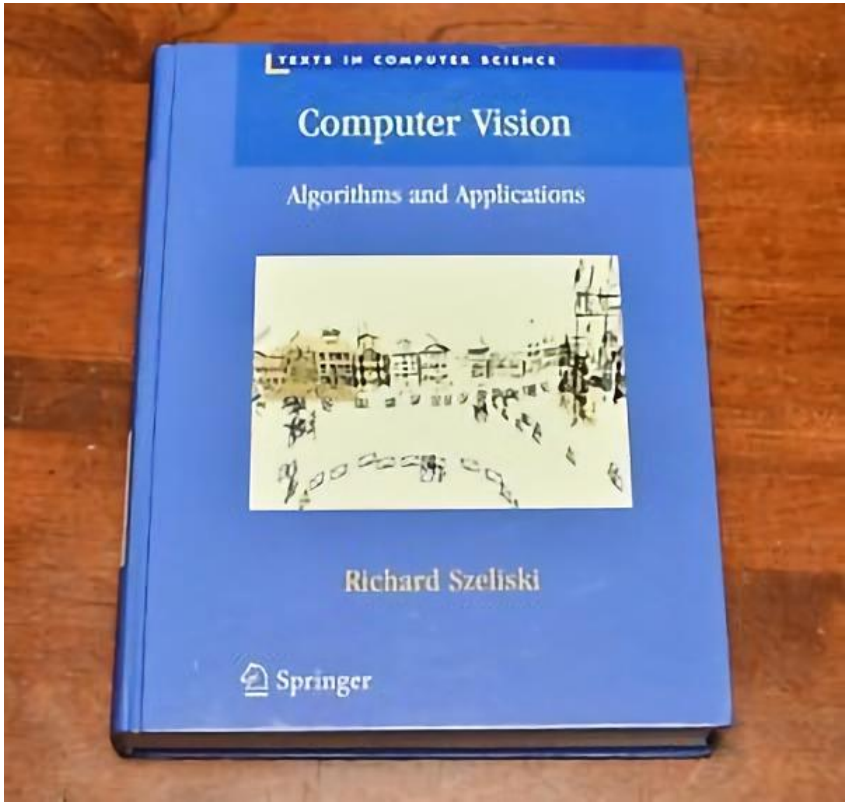
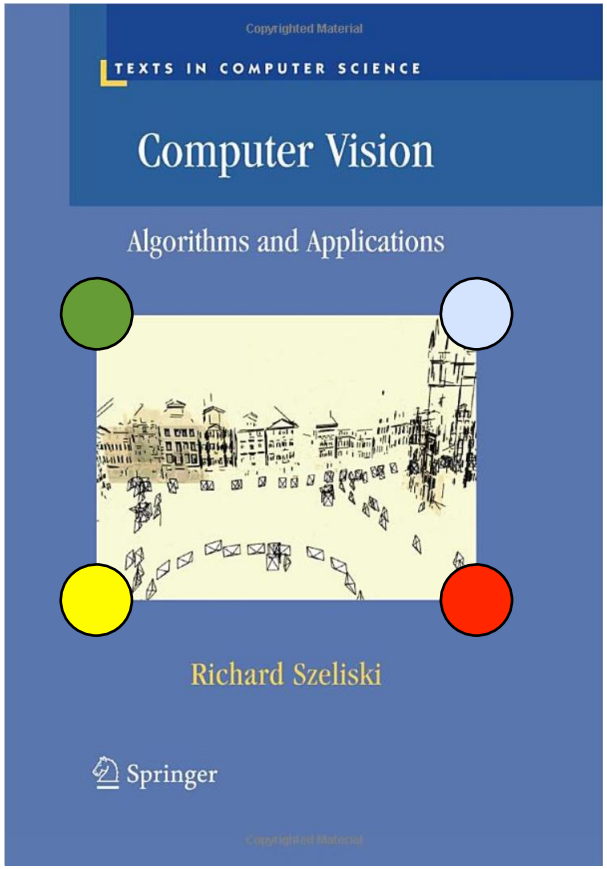
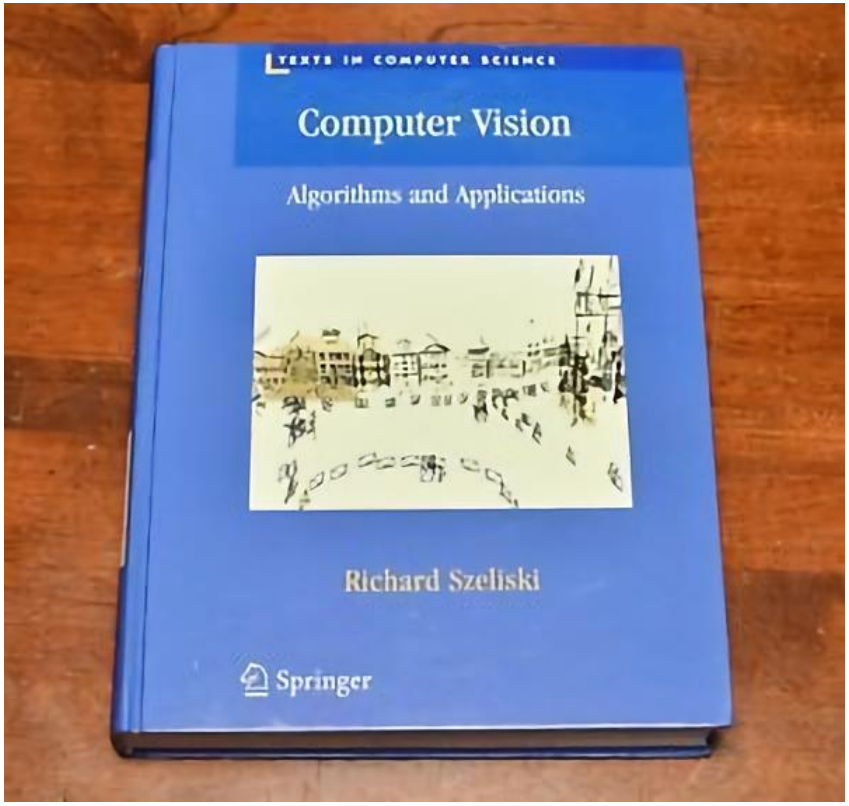

TEXTS IN COMPUTER SCIENCE
Computer Vision
Algorithms and Applications
Richard Szeliski
Springer

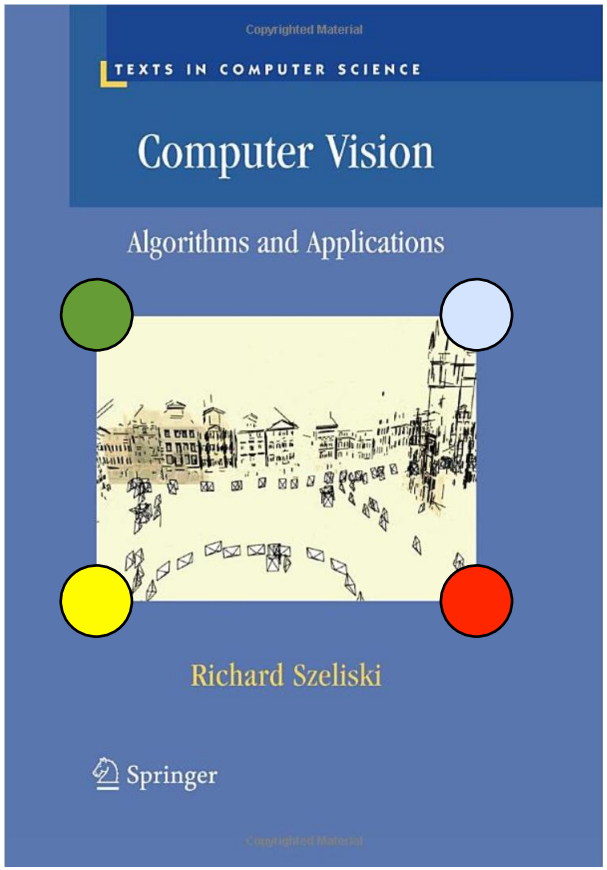
$$\mathbf{x}_i = (x_i, y_i)$$

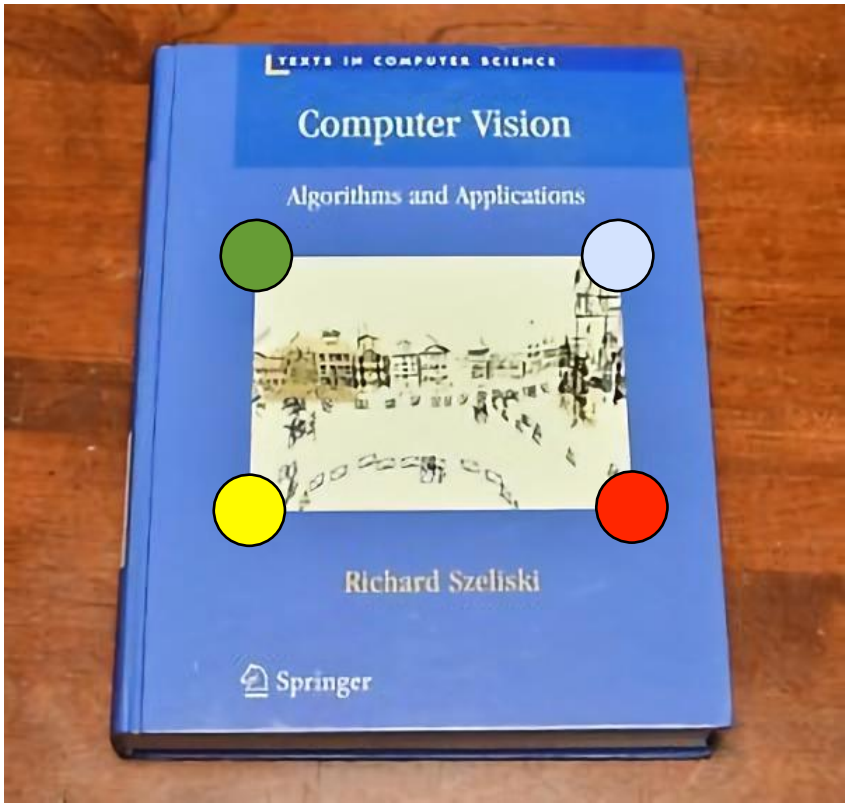




$$\mathbf{x}_i = (x_i, y_i)$$

$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$





$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

展开

$$\omega \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

展开

$$\mathbf{x}_i' = \begin{pmatrix} \dfrac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}} \\ \dfrac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}} \end{pmatrix}$$

$$\mathbf{x}_i' = \begin{pmatrix} \dfrac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}} \\ \dfrac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}} \end{pmatrix}$$

改写成齐次方程组

$$\mathbf{x}_i' = \begin{pmatrix} \dfrac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}} \\ \dfrac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}} \end{pmatrix}$$

改写成齐次方程组

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

至少需要4点来估计h

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2$$

不能用伪逆估计

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2 \text{ subject to } \|\mathbf{h}\| = 1$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ & & & & & \vdots & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -x_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Nx_N' & -x_Nx_N' & -y_N' \end{pmatrix} \mathbf{h} = \mathbf{0}$$

使用最小二乘法估计映射

$$\mathbf{h}^* = \arg\min_{\mathbf{h}} \|\mathbf{A}\mathbf{h}\|^2 \text{ subject to } \|\mathbf{h}\| = 1$$

计算A的SVD，$\mathbf{h}^*$就是V的最后一列

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix}$$

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} = \mathbf{M}_{\text{int}} \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{pmatrix}$$

如何恢复相机模型参数?

decomposeHomographyMat()

```
int cv::decomposeHomographyMat ( InputArray          H,
                                 InputArray          K,
                                 OutputArrayOfArrays rotations,
                                 OutputArrayOfArrays translations,
                                 OutputArrayOfArrays normals
                               )
```

Python:

```
cv.decomposeHomographyMat( H, K[, rotations[, translations[, normals]]] ) -> retval, rotations, translations, normals
```

```
#include <opencv2/calib3d.hpp>
```

Decompose a homography matrix to rotation(s), translation(s) and plane normal(s).

Parameters

H	The input homography matrix between two images.
K	The input camera intrinsic matrix.
rotations	Array of rotation matrices.
translations	Array of translation matrices.
normals	Array of plane normal matrices.

This function extracts relative camera motion between two views of a planar object and returns up to four mathematical solution tuples of rotation, translation, and plane normal. The decomposition of the homography matrix H is described in detail in **[166]**.

If the homography H, induced by the plane, gives the constraint

$$s_i \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \sim H \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

on the source image points p_i and the destination image points p'_i, then the tuple of rotations[k] and translations[k] is a change of basis from the source camera's coordinate system to the destination camera's coordinate system. However, by decomposing H, one can only get the translation normalized by the (typically unknown) depth of the scene, i.e. its direction but with normalized length.

If point correspondences are available, at least two solutions may further be invalidated, by applying positive depth constraint, i.e. all points must be in front of the camera.

Examples:

samples/cpp/tutorial_code/features2D/Homography/decompose_homography.cpp.

Output

- **motions** Decomposed `H`. A scalar struct with the following fields:
  - **R** Array of rotation matrices. Cell array of 3x3 rotations.
  - **t** Array of translation matrices. Cell array of 3x1 translations.
  - **n** Array of plane normal matrices. Cell array of 3x1 normals.
- **nsols** number of solutions.

This function extracts relative camera motion between two views observing a planar object from the homography `H` induced by the plane. The intrinsic camera matrix `K` must also be provided. The function may return up to four mathematical solution sets. At least two of the solutions may further be invalidated if point correspondences are available by applying positive depth constraint (all points must be in front of the camera). The decomposition method is described in detail in [Malis].

References

[Malis]:

Ezio Malis, Manuel Vargas, and others. "Deeper understanding of the homography decomposition for vision-based control". 2007.

See also

cv.findHomography, cv.decomposeEssentialMat

MATLAB File Help: cv.decompose

https://kyamagu.github.io/mexopencv/matlab/decomposeHomographyMat.html

Output

- **motions** Decomposed `H`. A scalar struct with the following fields:
  - **R** Array of rotation matrices. Cell array of 3x3 rotations.
  - **t** Array of translation matrices. Cell array of 3x1 translations.
  - **n** Array of plane normal matrices. Cell array of 3x1 normals.
- **nsols** number of solutions.

This function extracts relative camera motion between two views observing a planar object from the homography `H` induced by the plane. The intrinsic camera matrix `K` must also be provided. The function may return up to four mathematical solution sets. At least two of the solutions may further be invalidated if point correspondences are available by applying positive

可以通过正深度约束验证

References

[Malis]:

Ezio Malis, Manuel Vargas, and others. "Deeper understanding of the homography decomposition for vision-based control". 2007.

See also

cv.findHomography, cv.decomposeEssentialMat

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Deeper understanding of the homography decomposition for vision-based control

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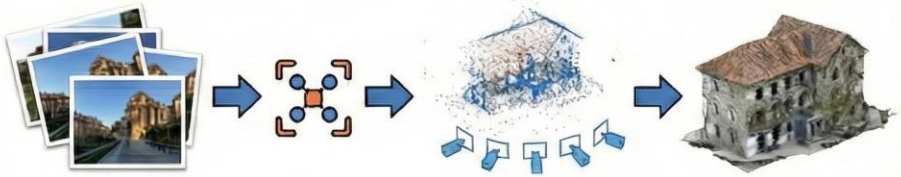
SLAM

Simultaneous Localization and Mapping

同步定位与制图

SfM vs. SLAM：运动恢复结构 与 同时定位与建图

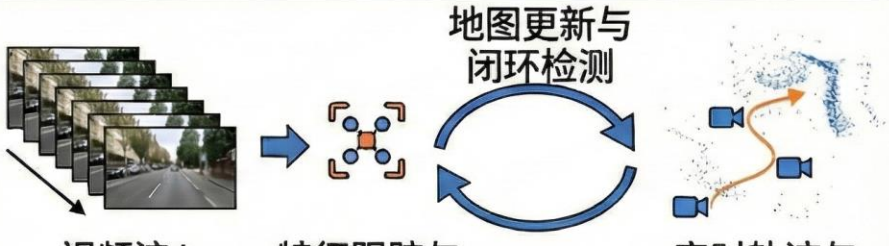
SfM （运动恢复结构）
- 离线重建



批量处理

重点：全局一致性与模型质量

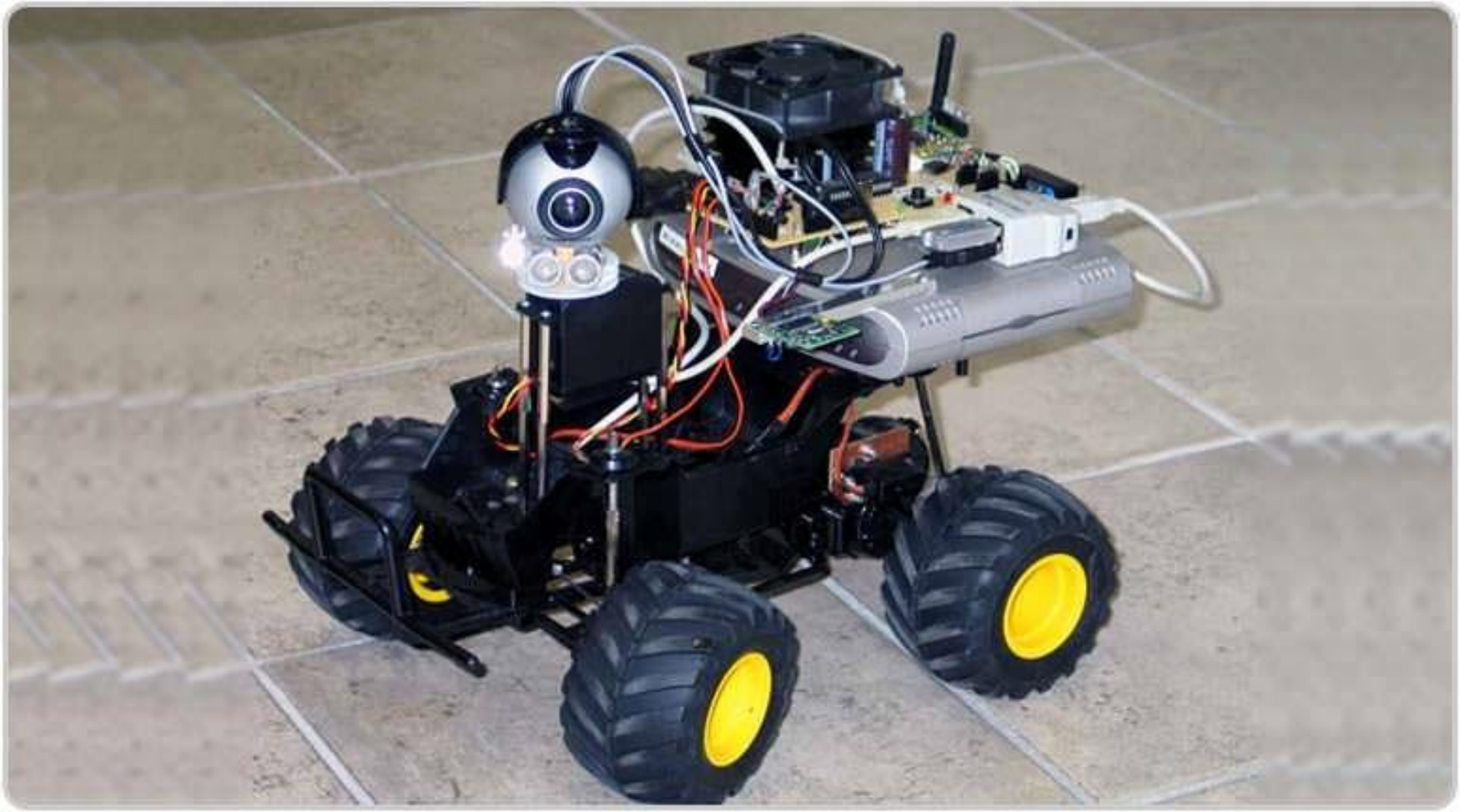
SLAM（同时定位与建图）
- 实时导航



实时/在线处理

重点：定位与导航

[SfM：照片(批量)]	**输入：**	[SLAM：视频(序列)]
[SfM：离线/后处理]	**处理：**	[SLAM：实时]
[SfM：高保真三维模型]	**目标：**	[SLAM：机器人姿态与可导航地图]
[SfM：否]	**实时?**	[SLAM：是]



Real-Time 6-DOF Monocular Visual SLAM in a Large-Scale Environment

Hyon Lim, Jongwoo Lim, H. Jin Kim

ICRA 2014 Video

鸣谢：Hypo Lim et al.