计算机视觉

图像滤波

中国传媒大学

COMMUNICATION UNIVERSITY OF CHINA

本节主题：

生物视觉与色彩

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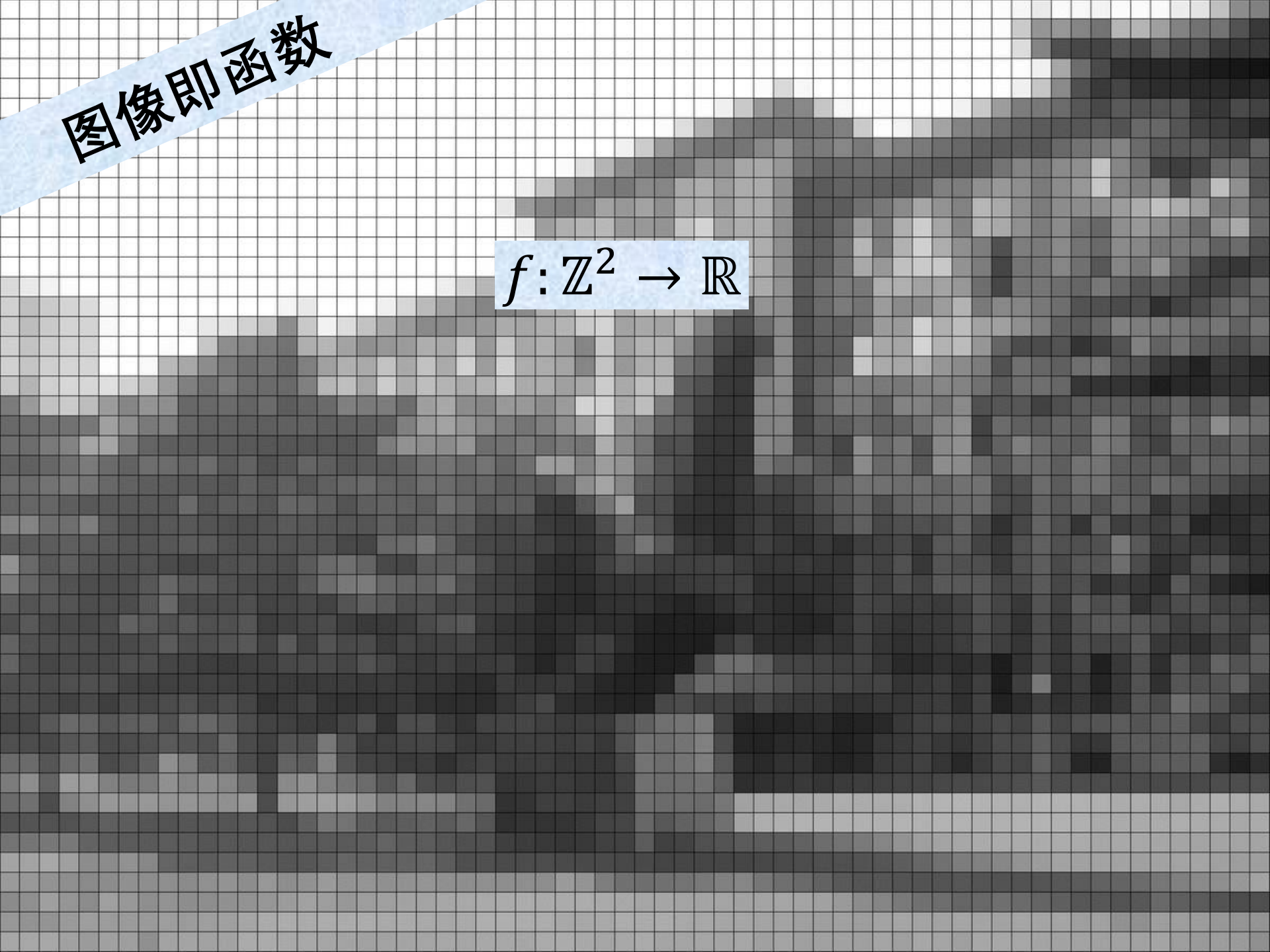
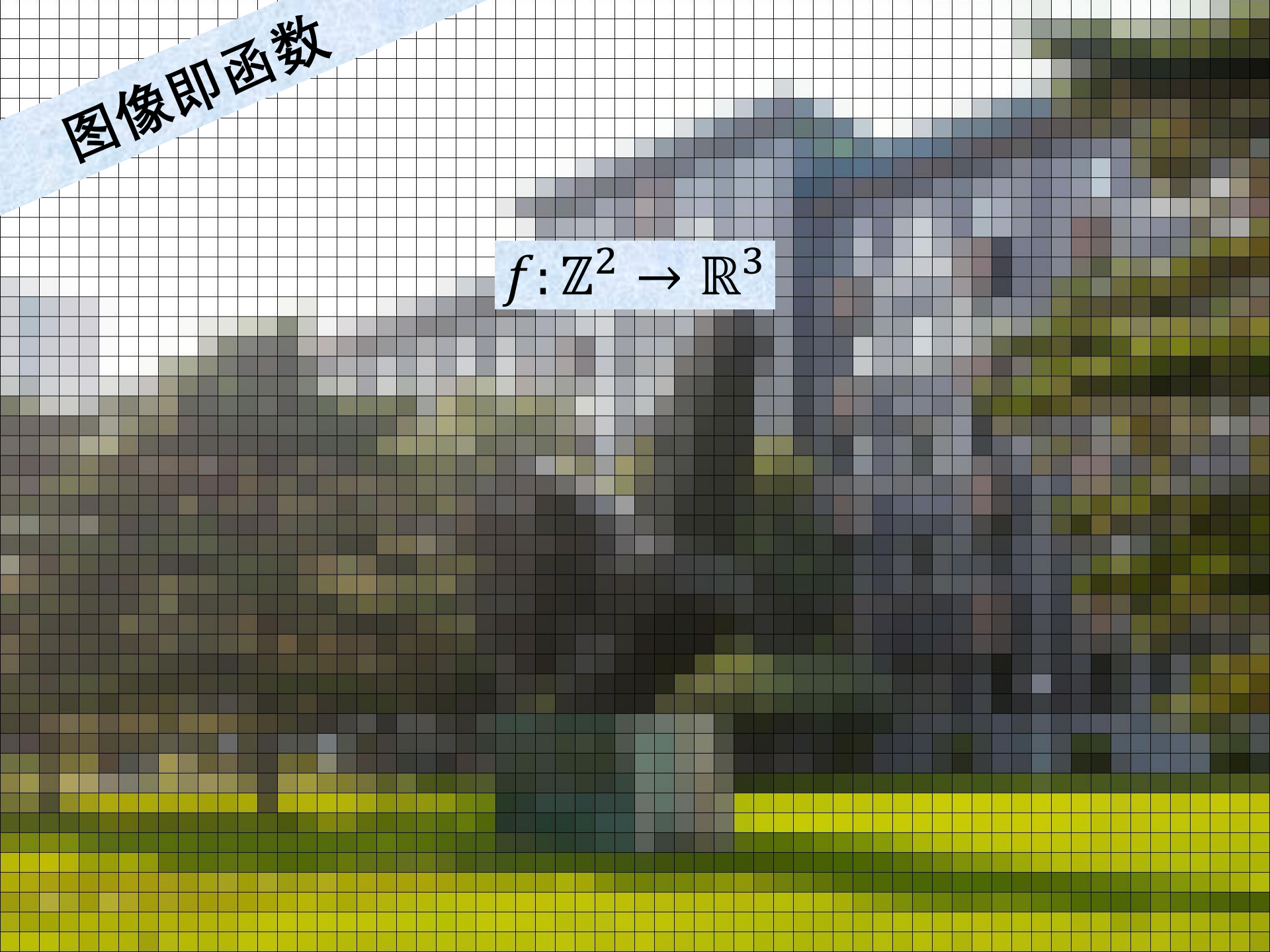
生物视觉与色彩

图像滤波

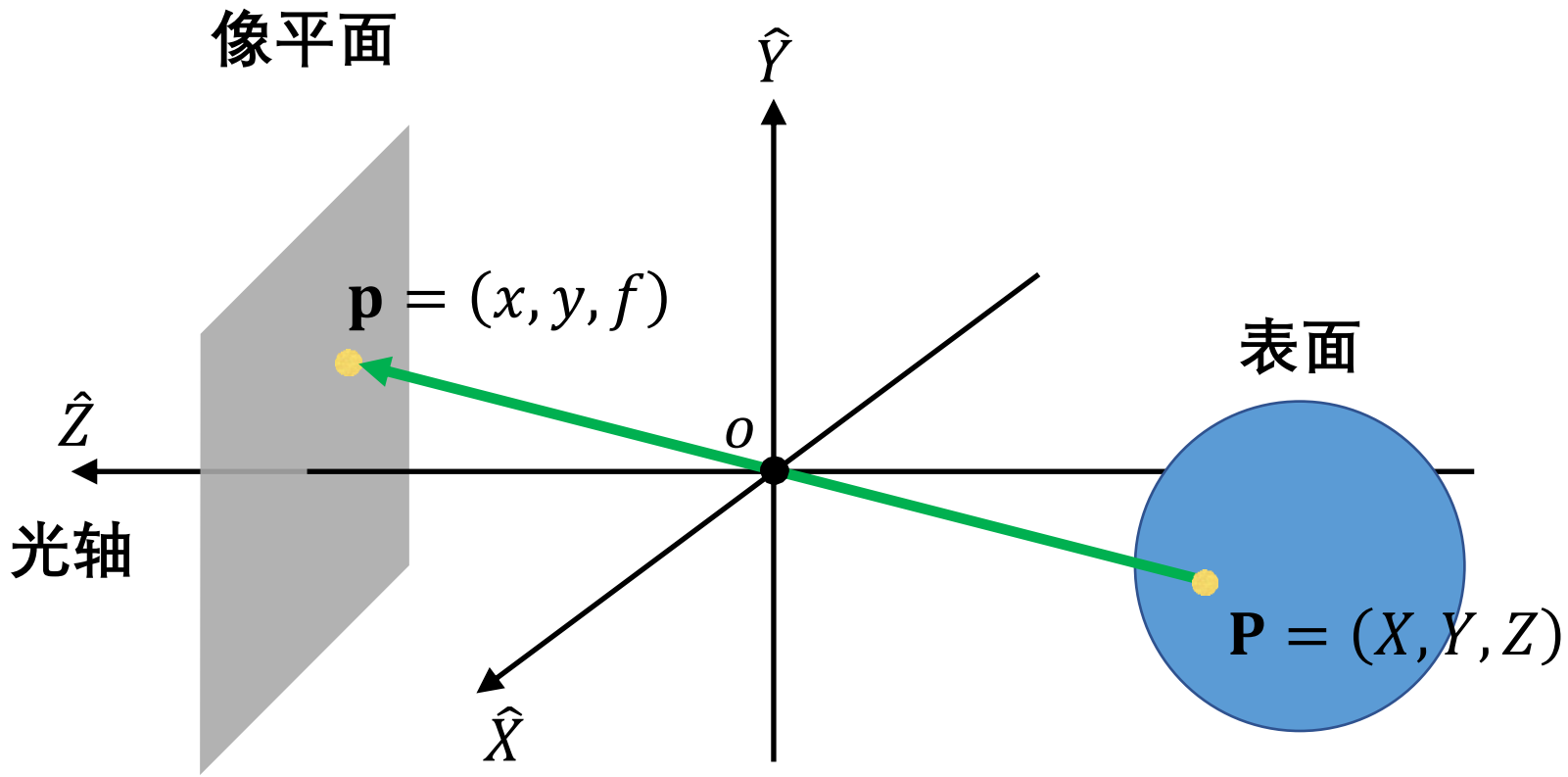
什么是图像?

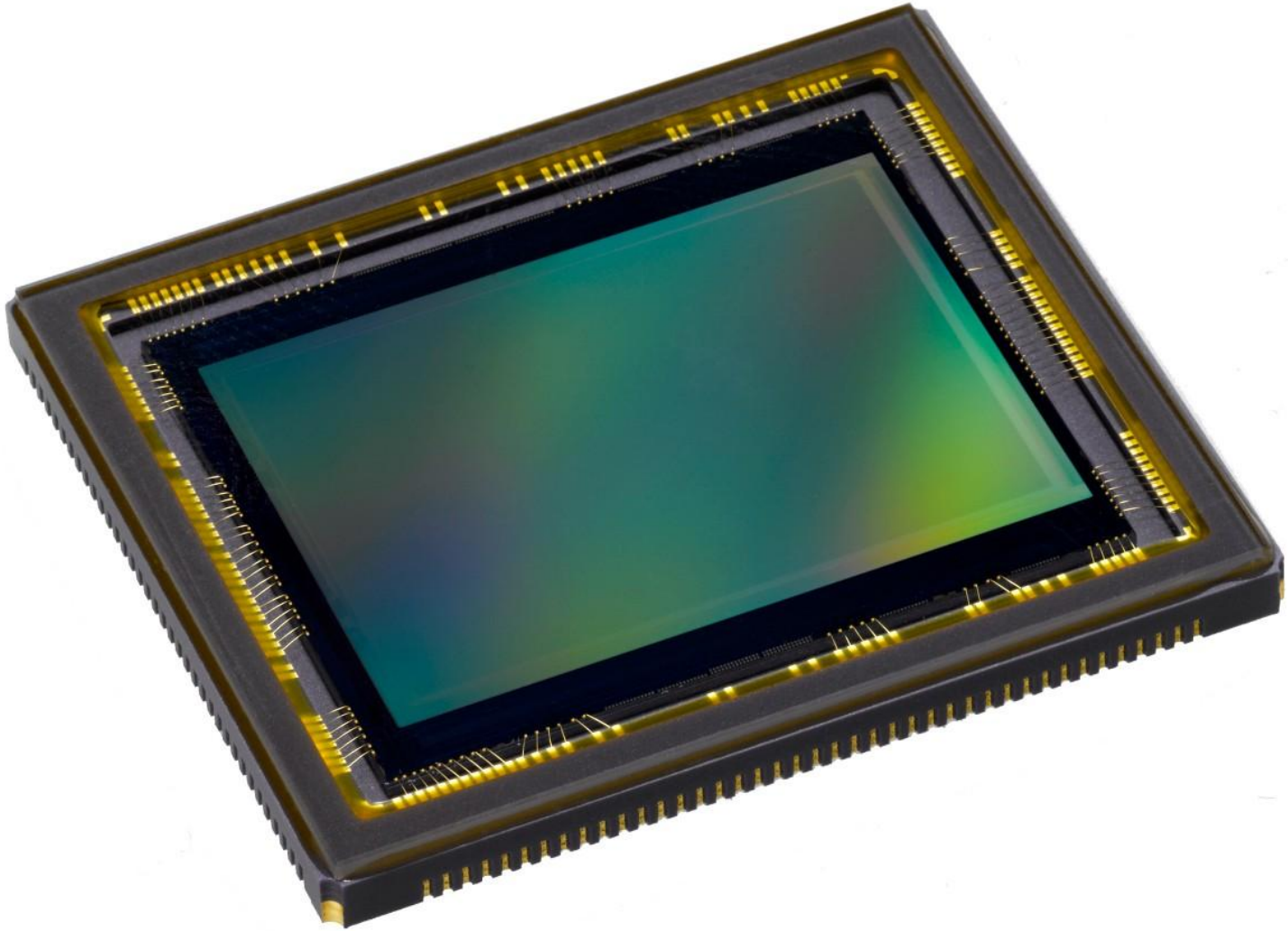

图像即函数


图像即函数
$f: \mathbb{Z}^2 \to \mathbb{R}$

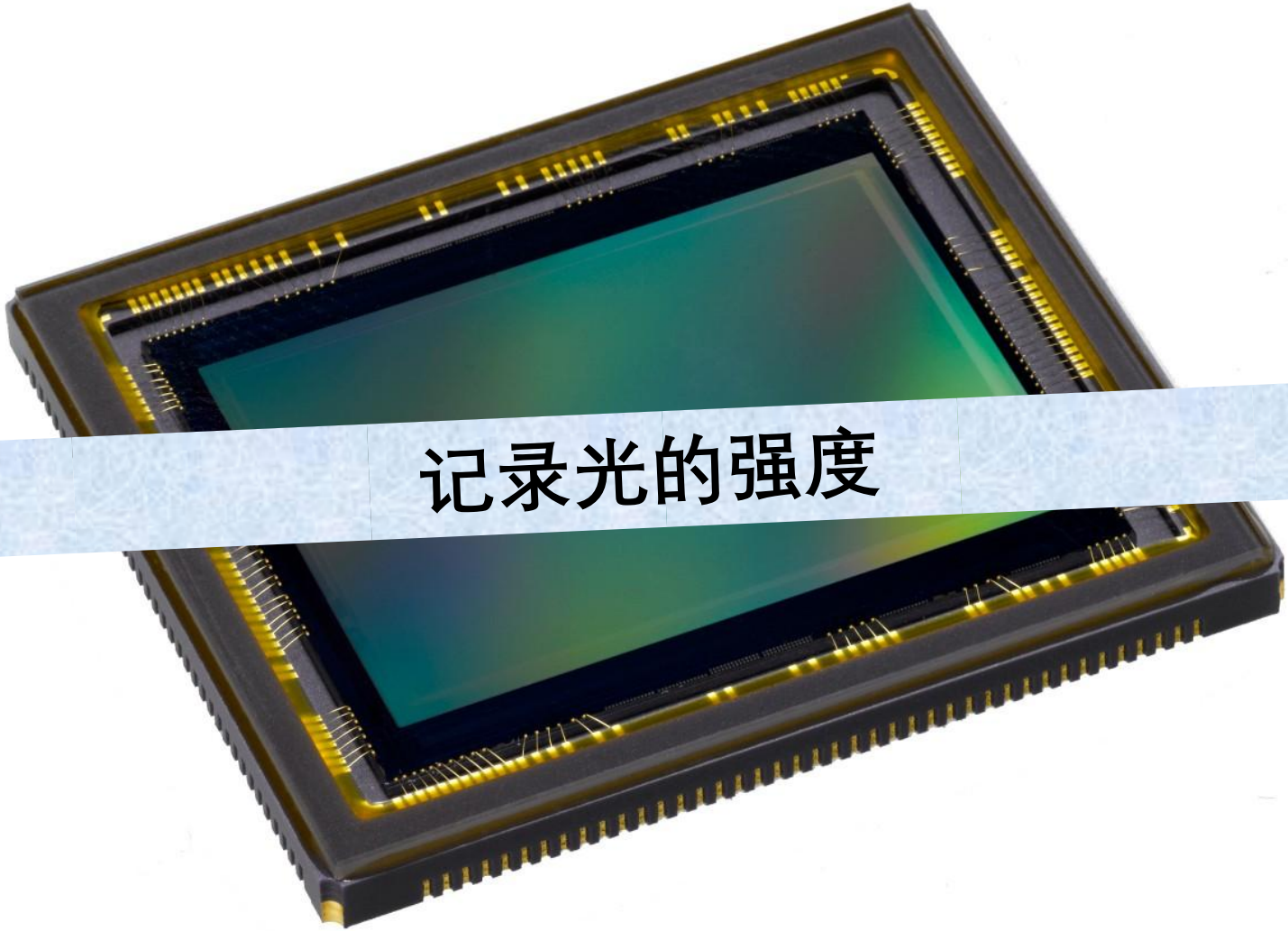

图像即函数
$f: \mathbb{Z}^2 \to \mathbb{R}^3$


图像即函数
$f: \mathbb{R}^2 \to \mathbb{R}^3$


像平面
$\hat{Y}$
$\mathbf{p} = (x, y, f)$
表面
$\hat{Z}$
o
光轴
$\mathbf{P} = (X, Y, Z)$
$\hat{X}$

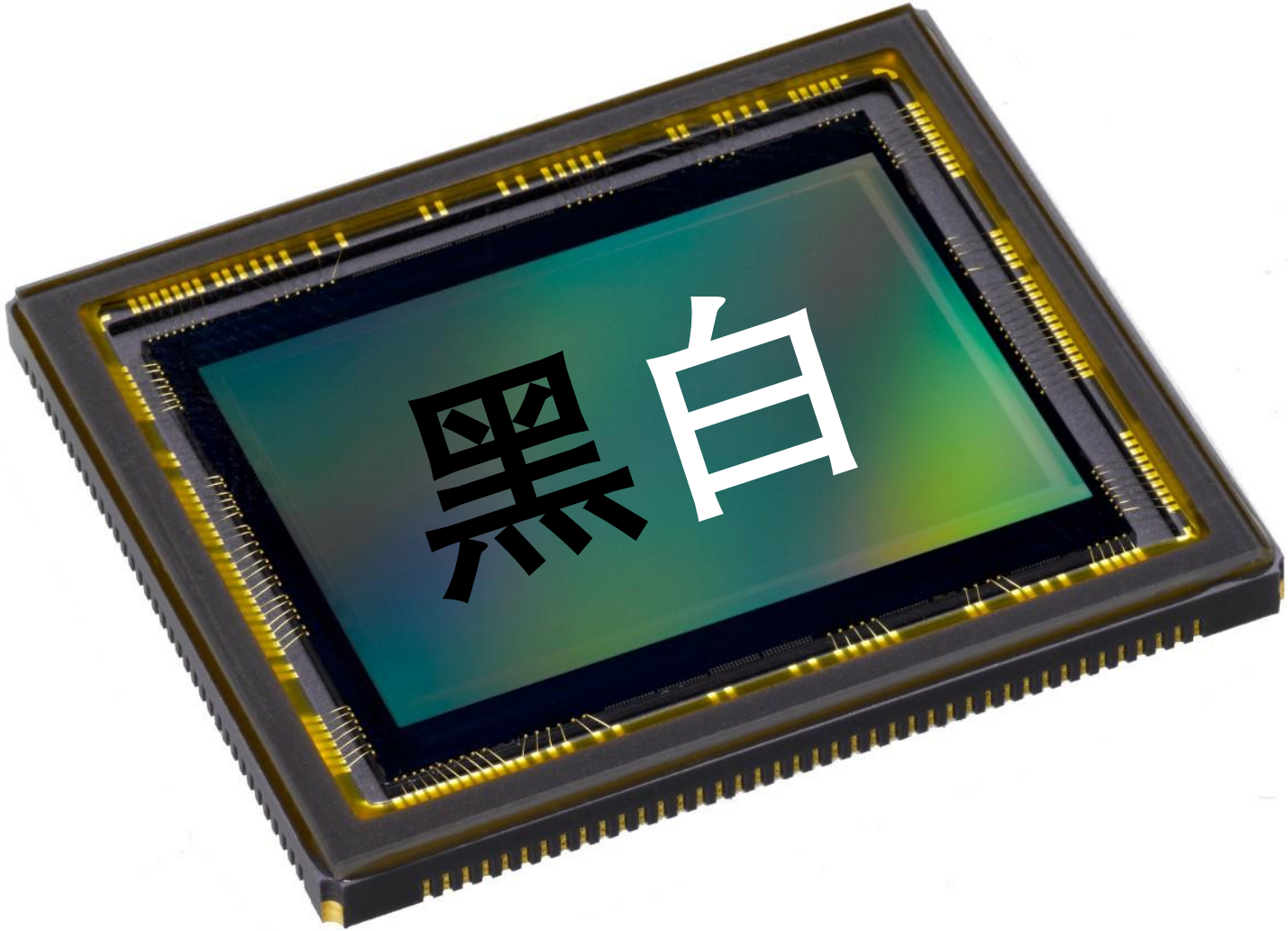


电荷耦合器件（CCD）

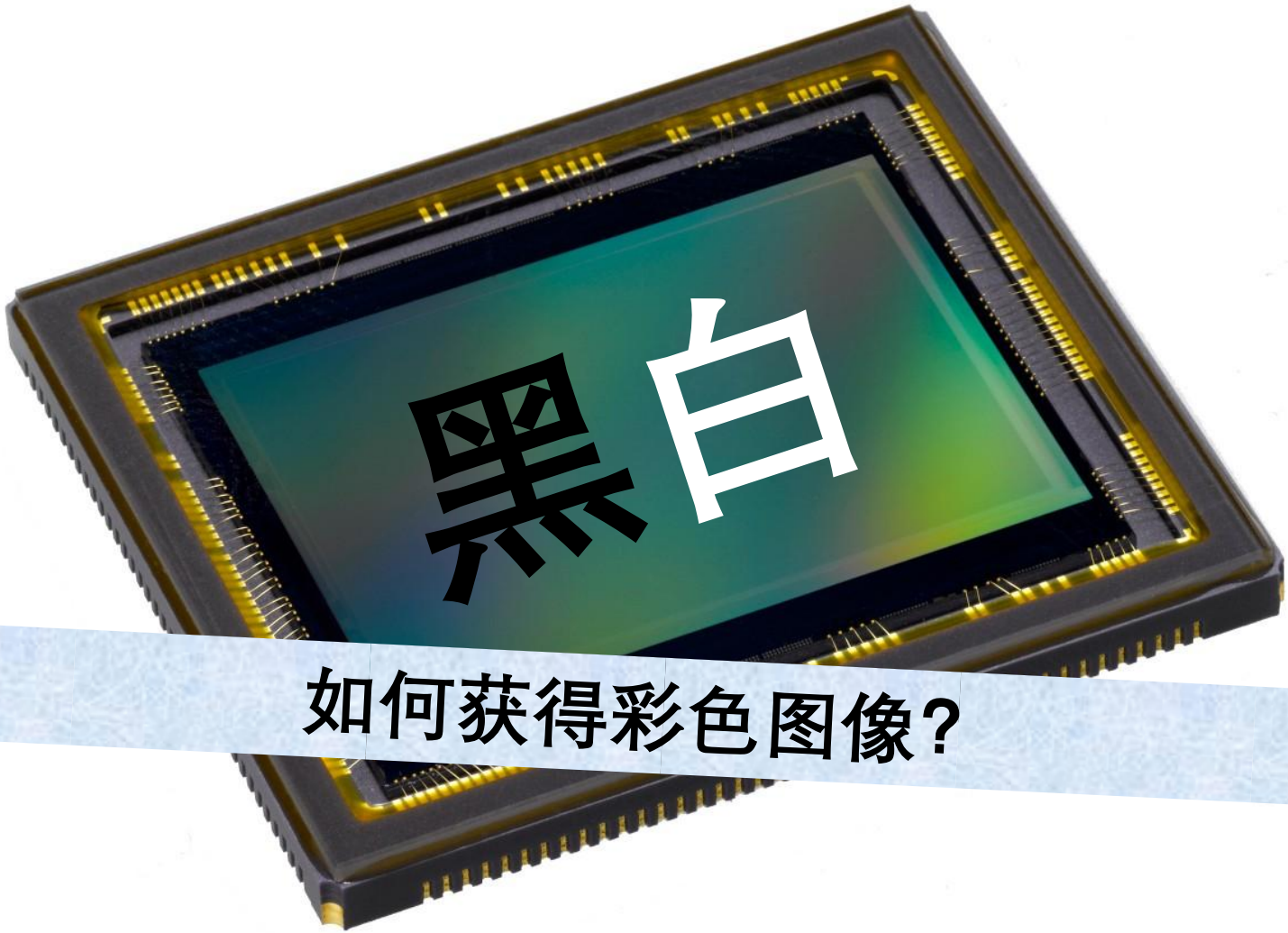


记录光的强度

电荷耦合器件（CCD）

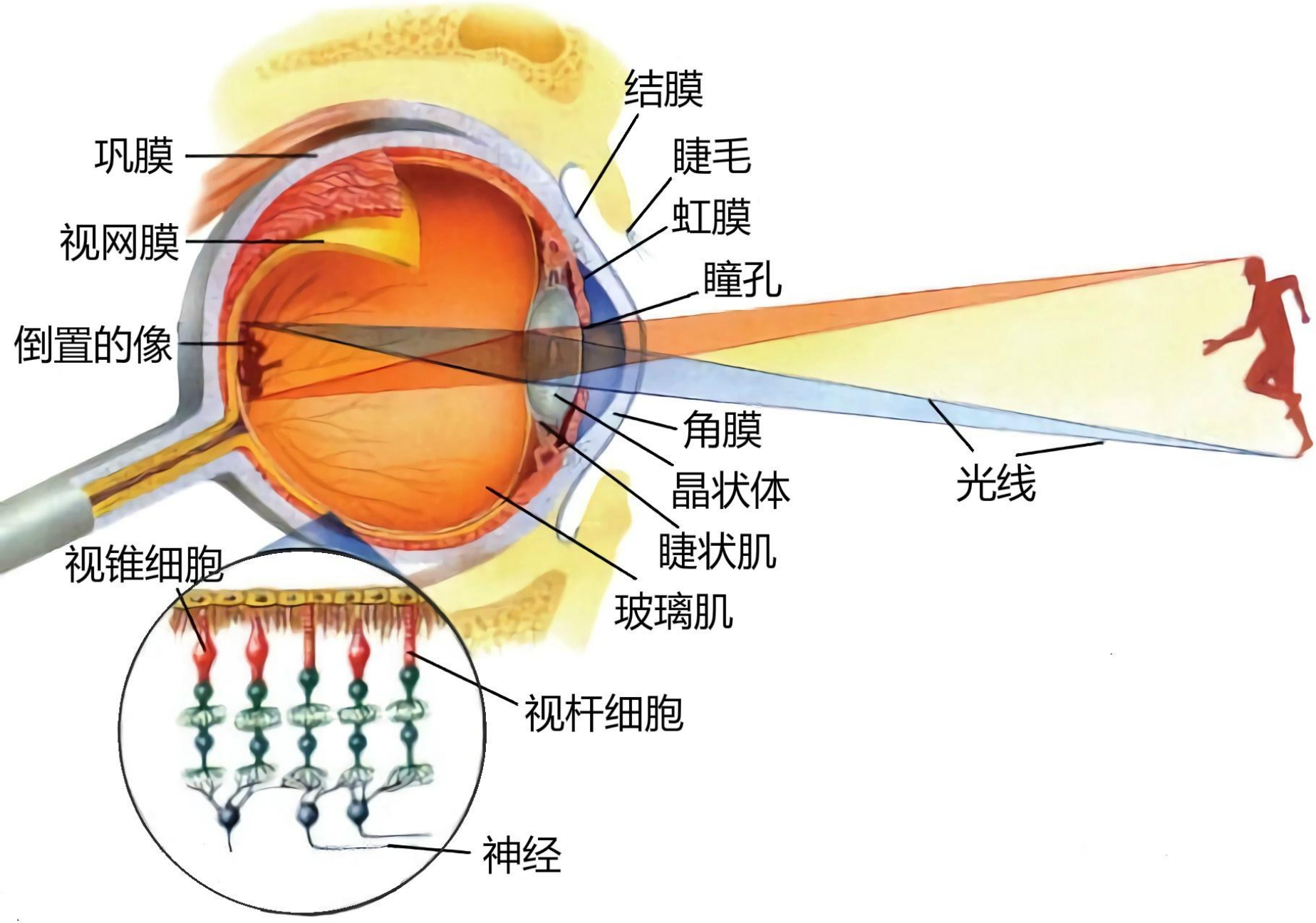


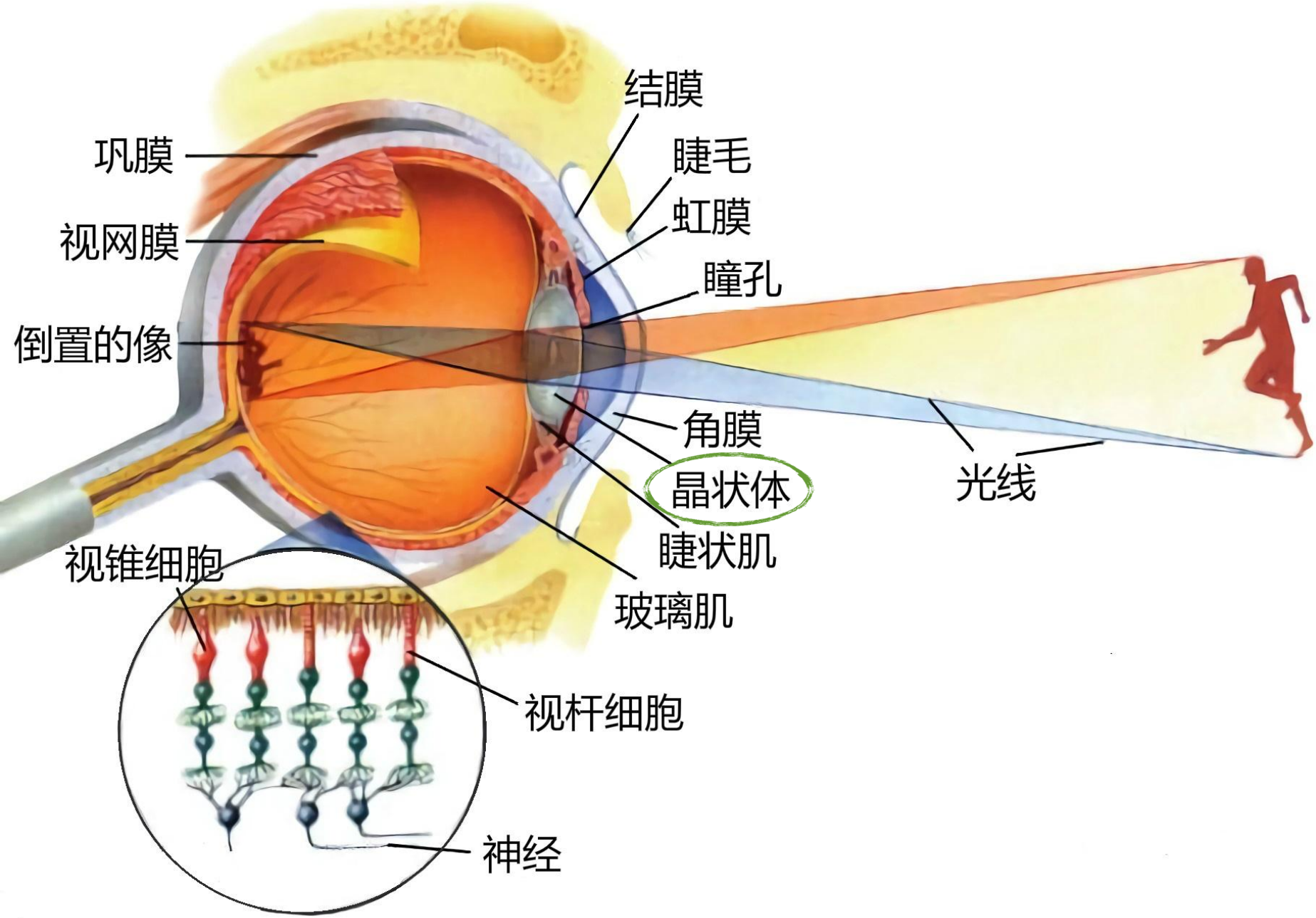
电荷耦合器件（CCD）

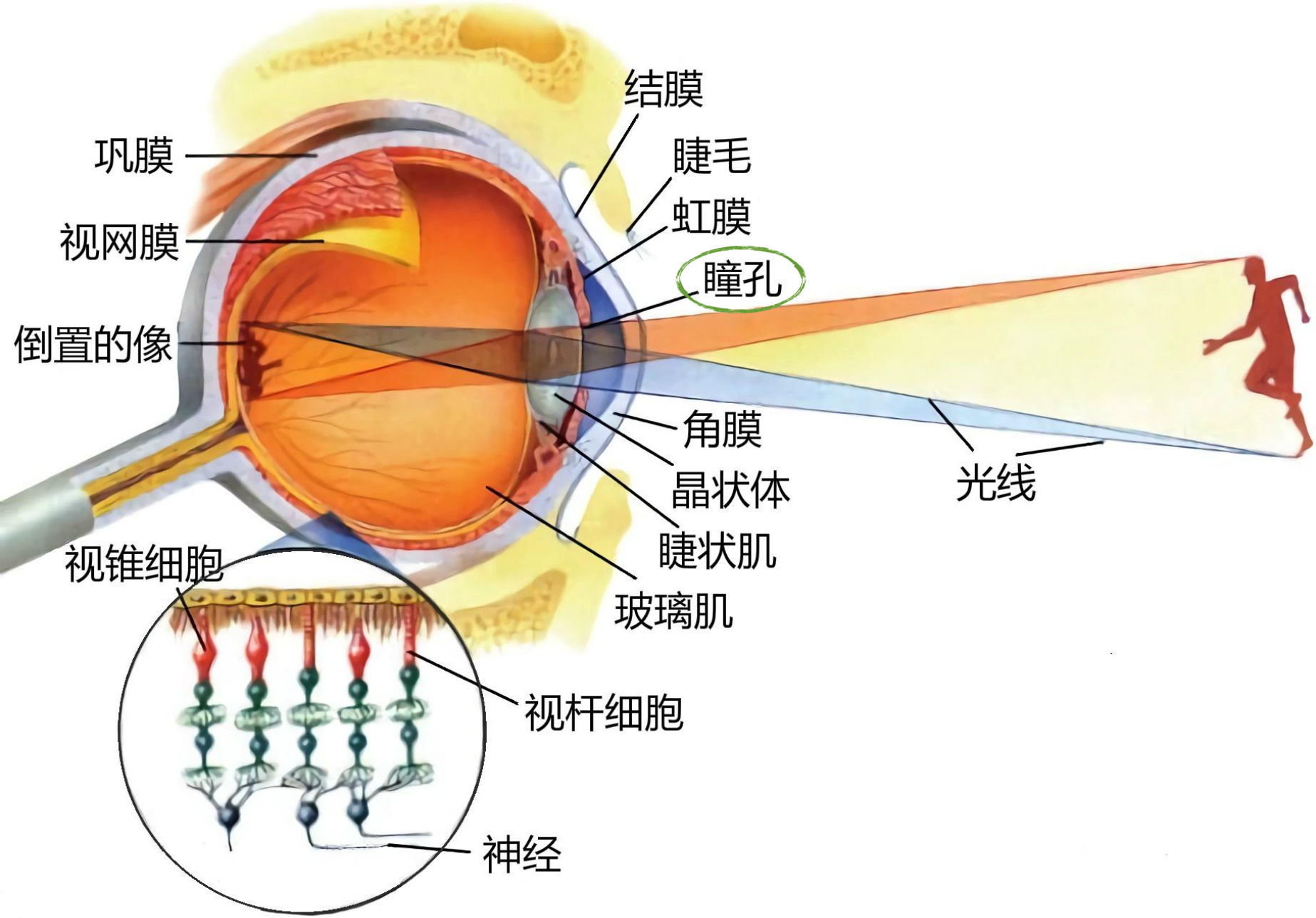


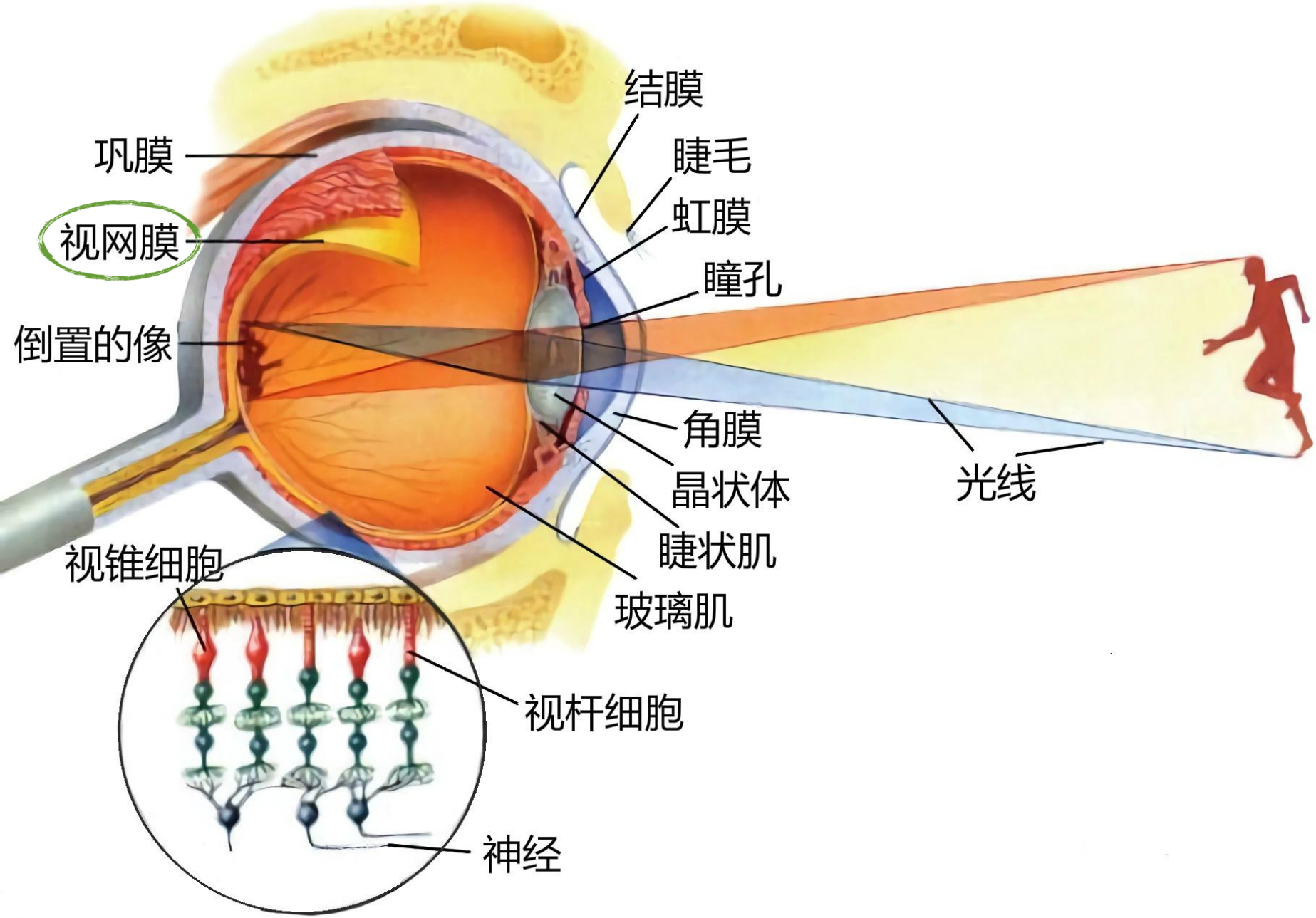
如何获得彩色图像？

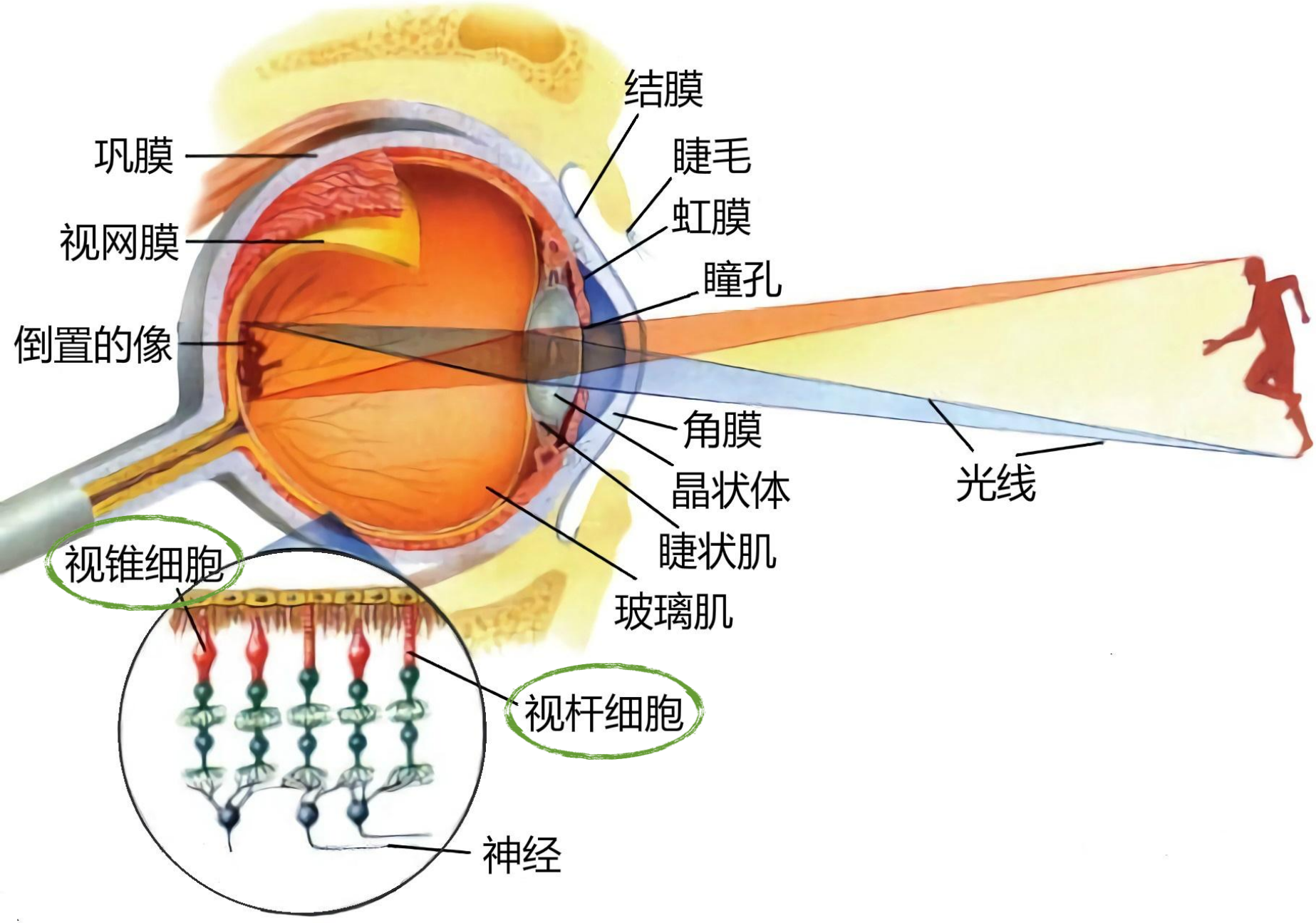
电荷耦合器件（CCD）

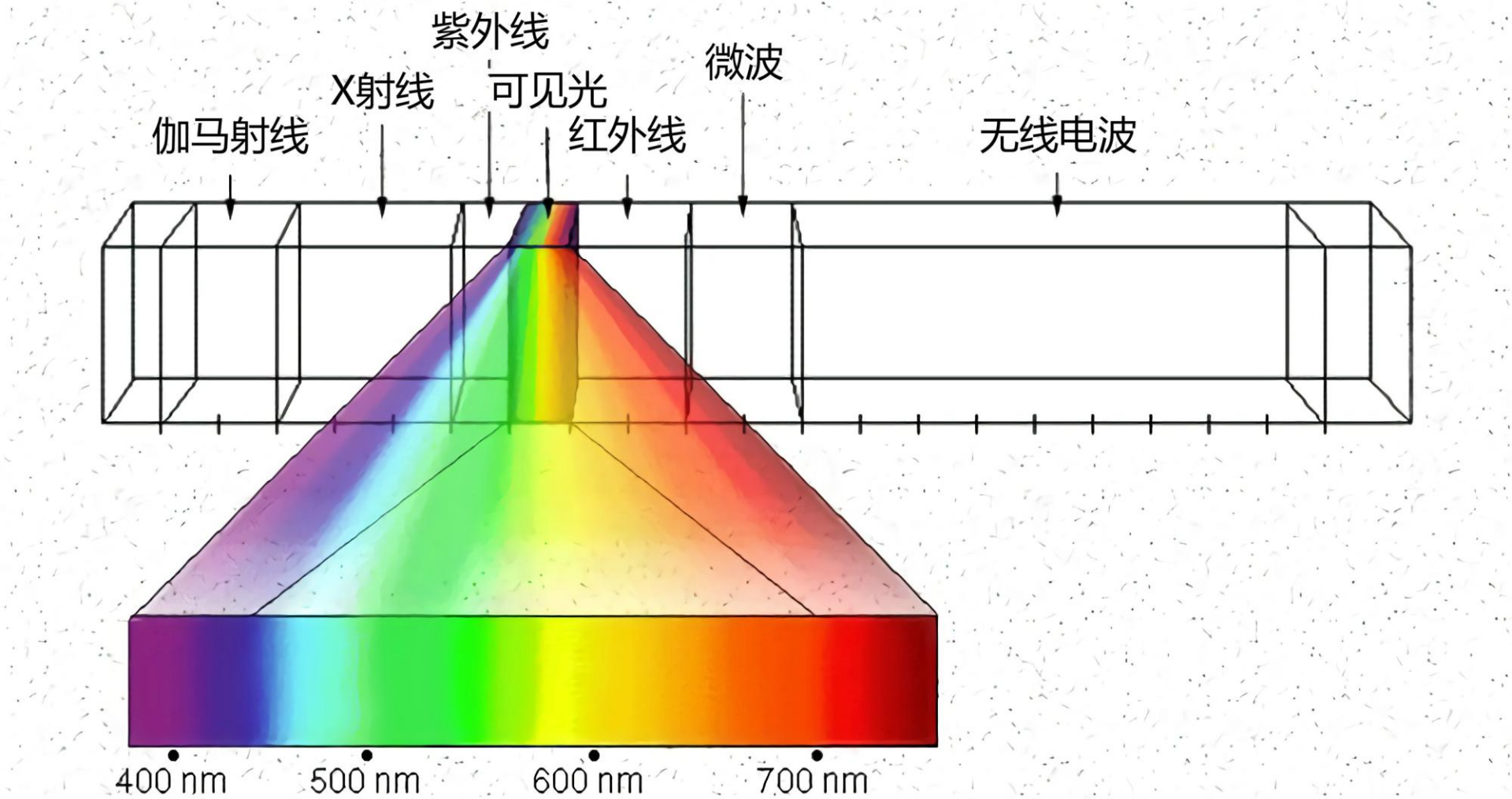

结膜
睫毛
虹膜
瞳孔
巩膜
视网膜
倒置的像
角膜
晶状体
睫状肌
玻璃肌
光线
视锥细胞
视杆细胞
神经

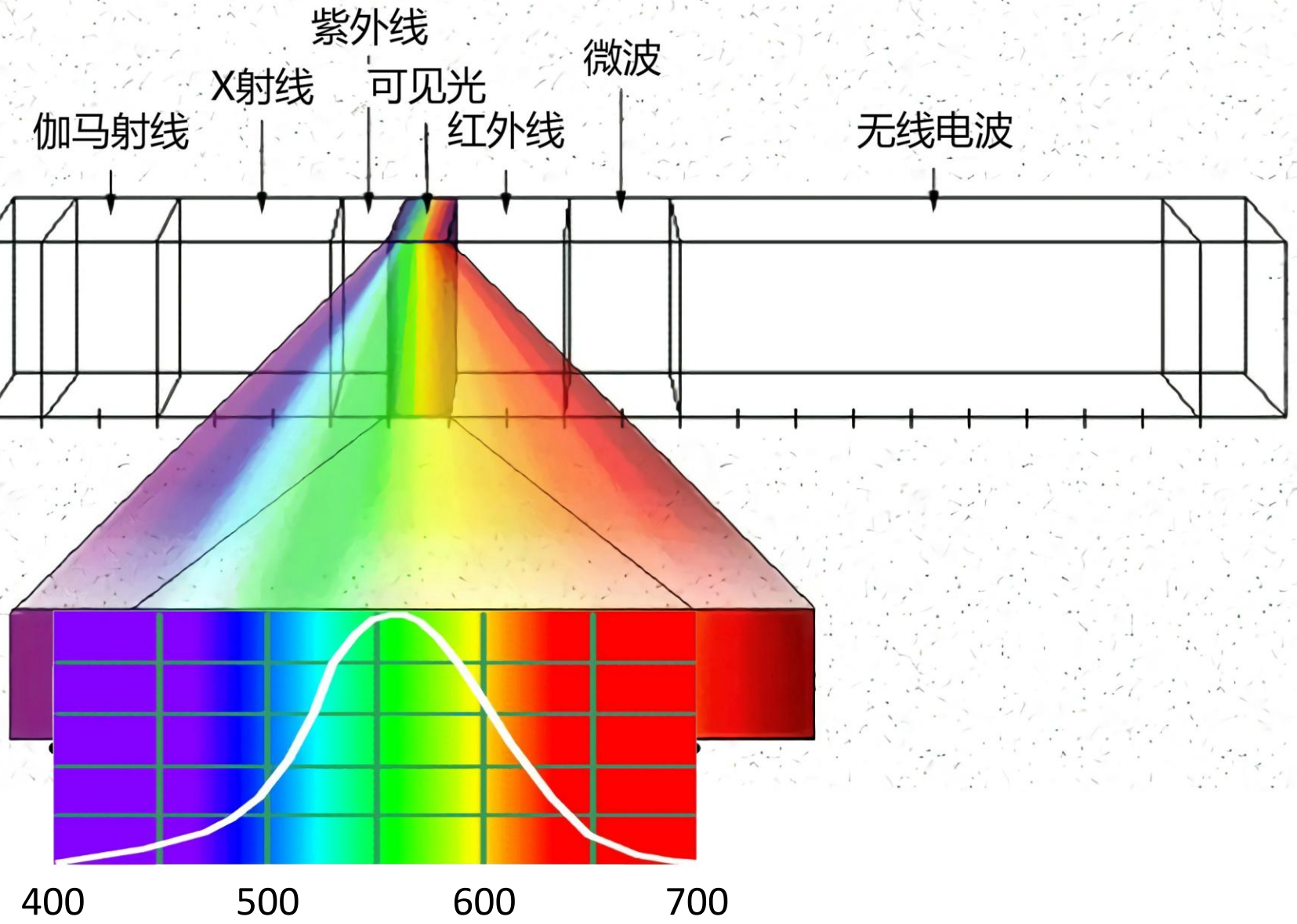

结膜
睫毛
虹膜
瞳孔
巩膜
视网膜
倒置的像
角膜
晶状体
光线
睫状肌
玻璃肌
视锥细胞
视杆细胞
神经

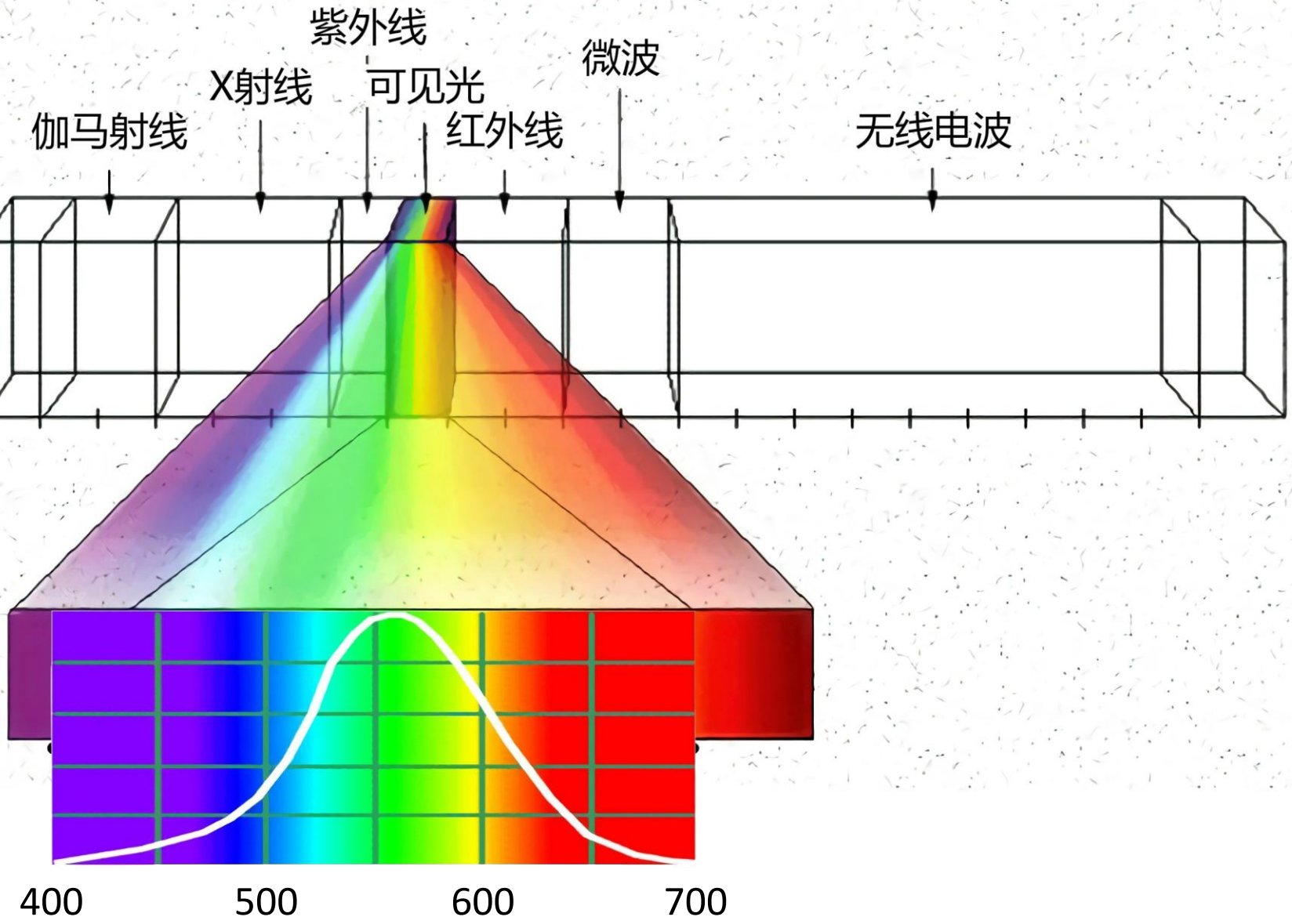

结膜
睫毛
虹膜
瞳孔
巩膜
视网膜
倒置的像
角膜
晶状体
睫状肌
玻璃肌
光线
视锥细胞
视杆细胞
神经


结膜
睫毛
虹膜
瞳孔
巩膜
视网膜
倒置的像
角膜
晶状体
睫状肌
玻璃肌
光线
视锥细胞
视杆细胞
神经

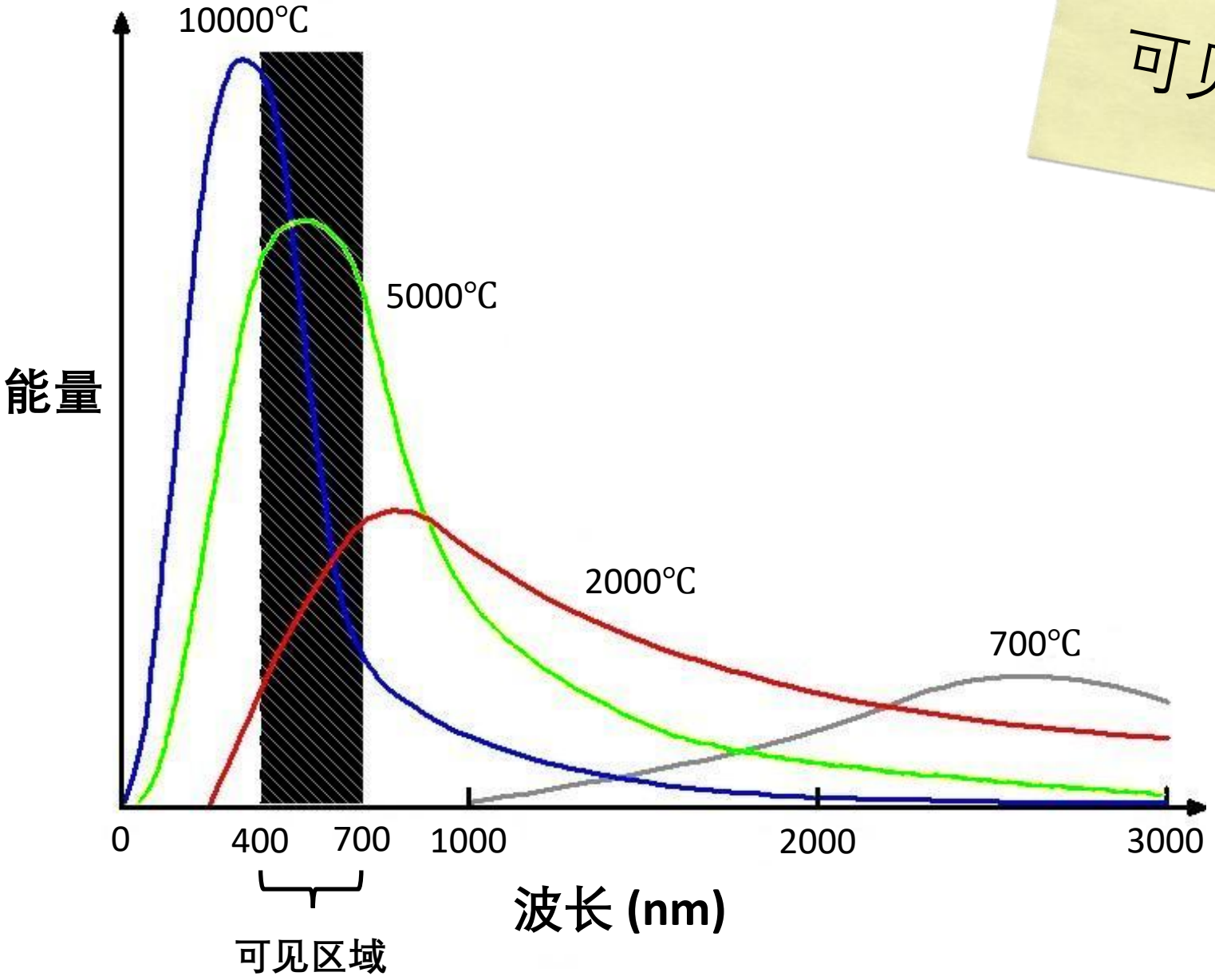

结膜
睫毛
虹膜
瞳孔
巩膜
视网膜
倒置的像
角膜
晶状体
睫状肌
玻璃肌
光线
视锥细胞
视杆细胞
神经


紫外线
X射线
可见光
微波
伽马射线
红外线
无线电波
400 nm
500 nm
600 nm
700 nm

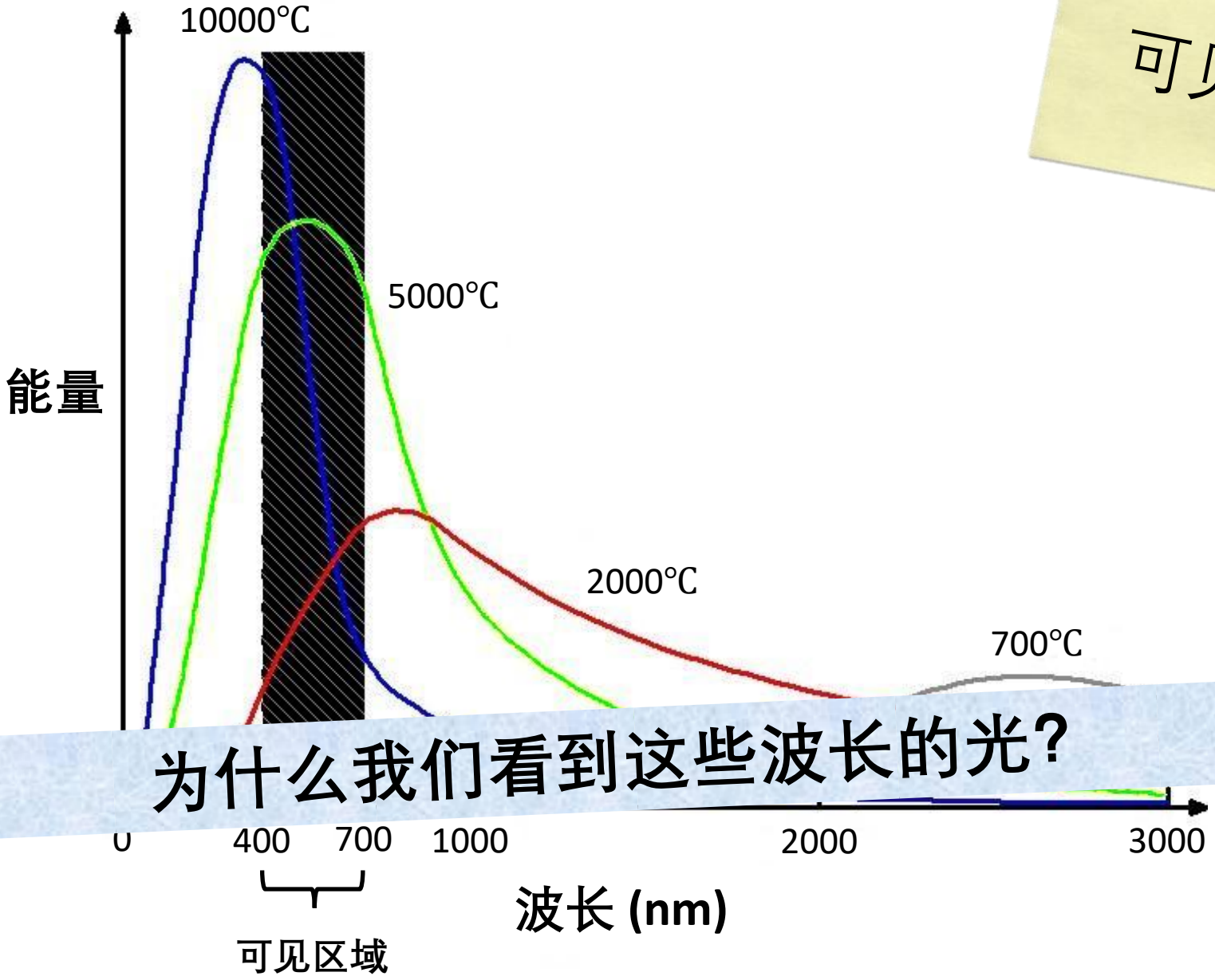

紫外线
X射线
可见光
微波
伽马射线
红外线
无线电波
400
500
600
700



人眼光度敏感函数

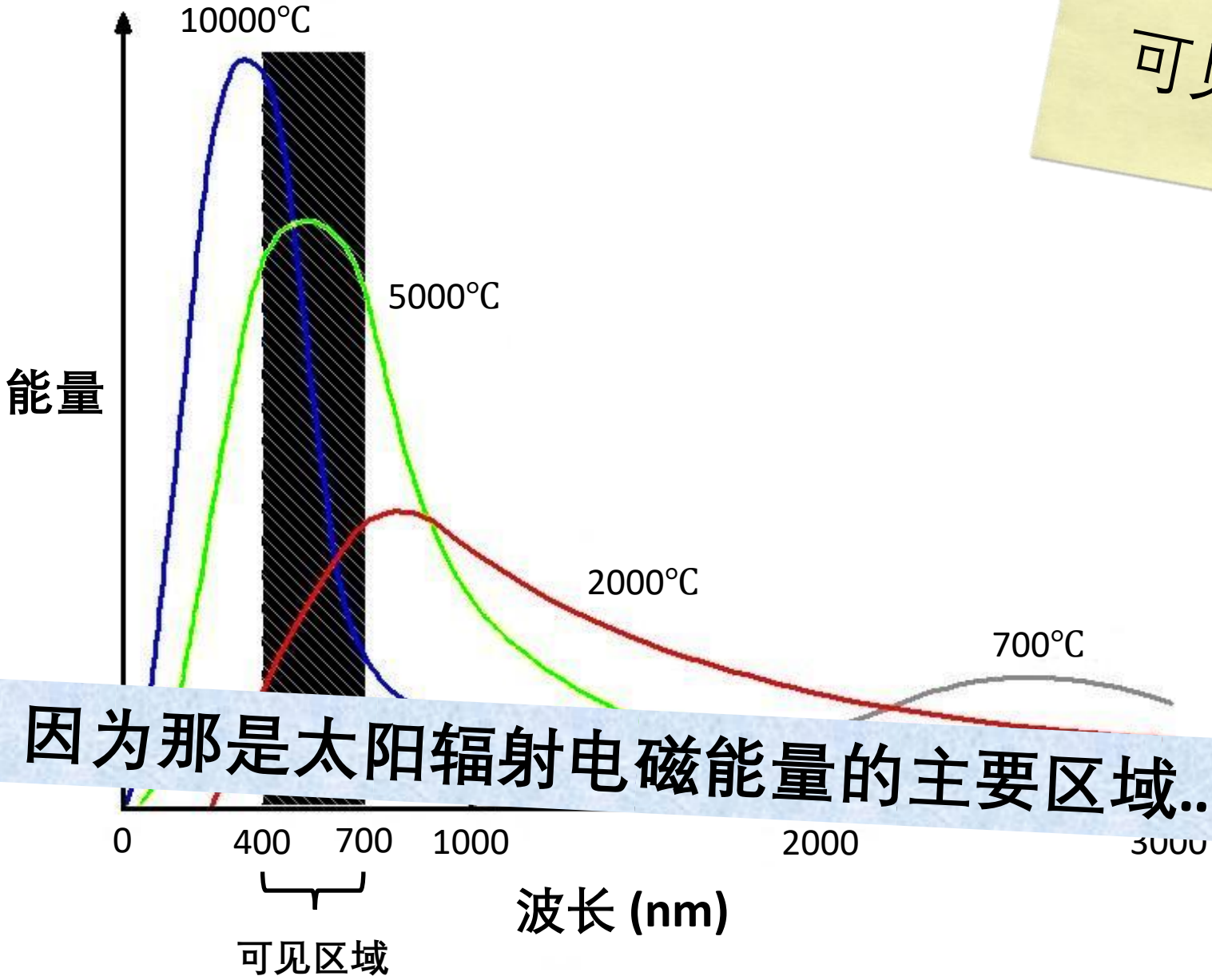

可见光
10000℃
5000℃
2000℃
700℃
能量
0
400
700
1000
2000
3000
波长 (nm)
可见区域

可见光

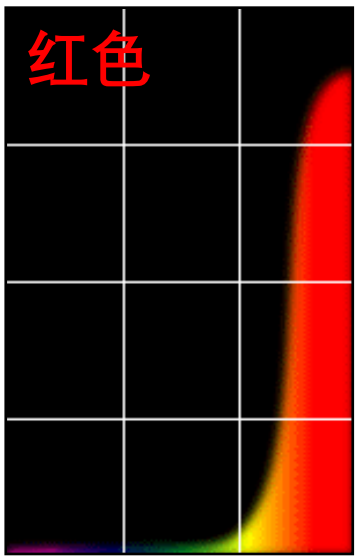


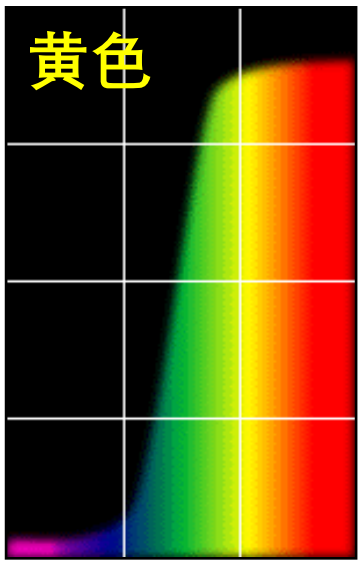
为什么我们看到这些波长的光?

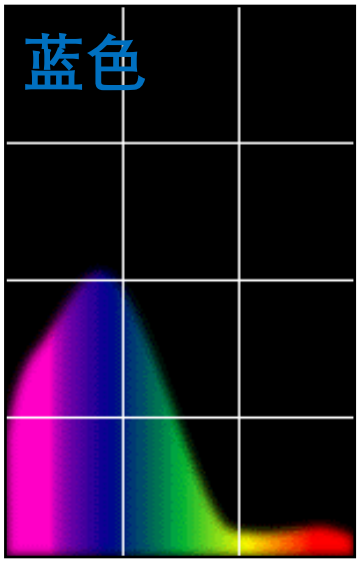
可见光

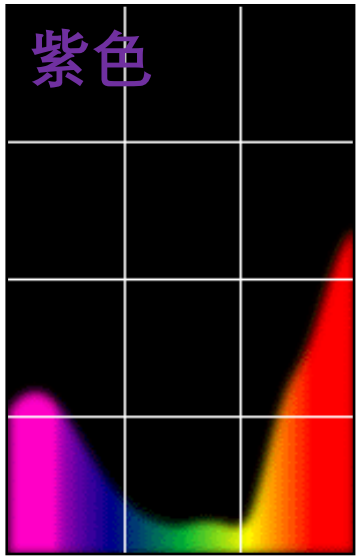


因为那是太阳辐射电磁能量的主要区域...





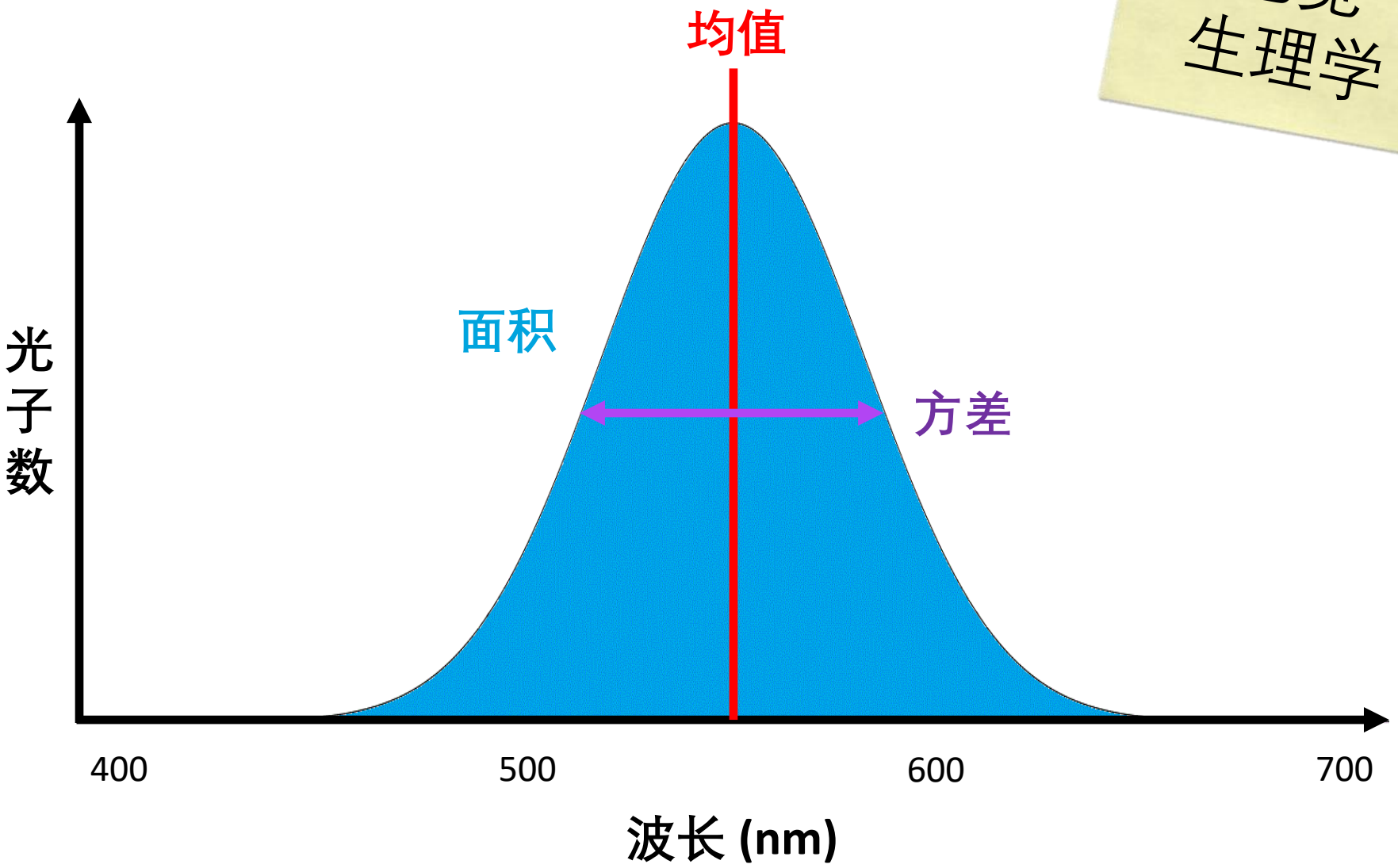




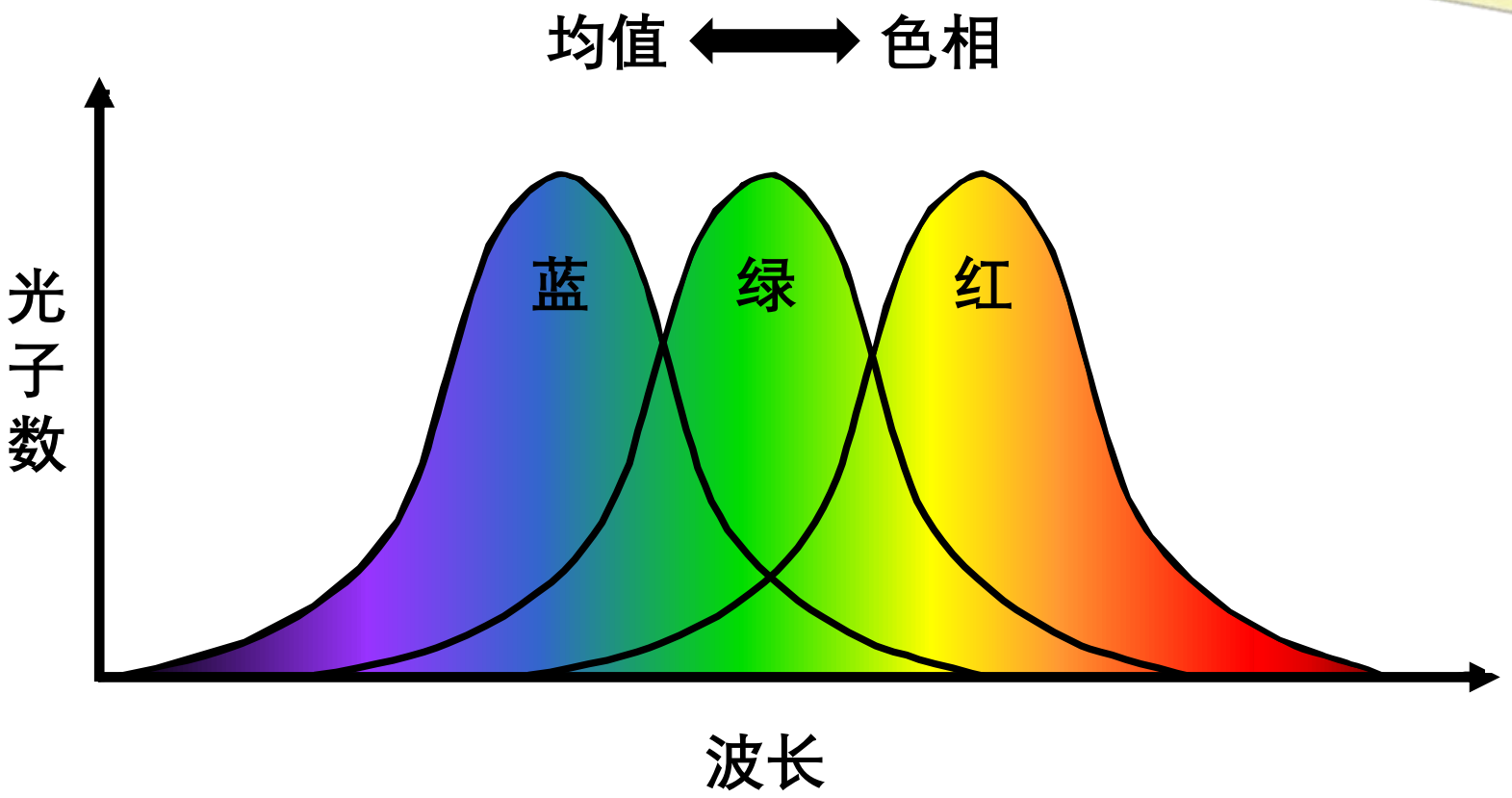
反射光子

波长 (nm)

色觉
生理学

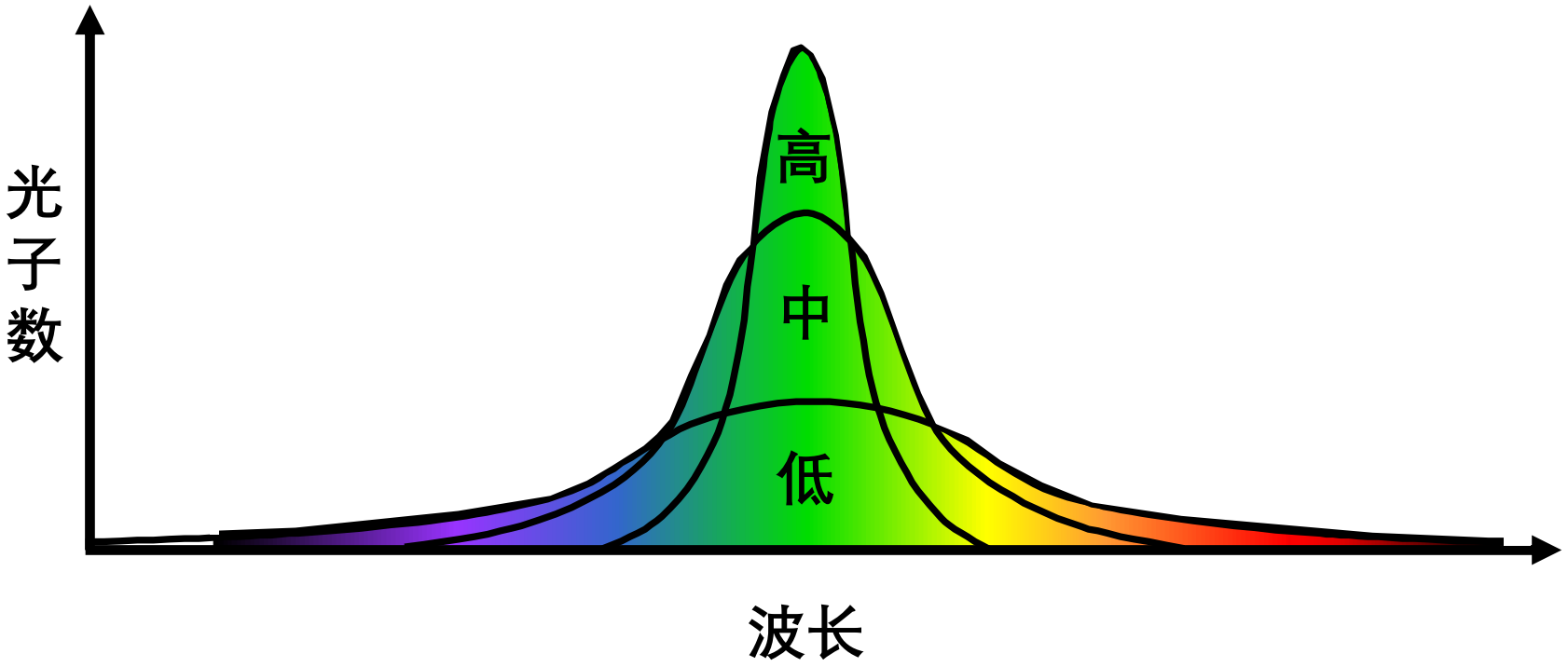


色觉生理学

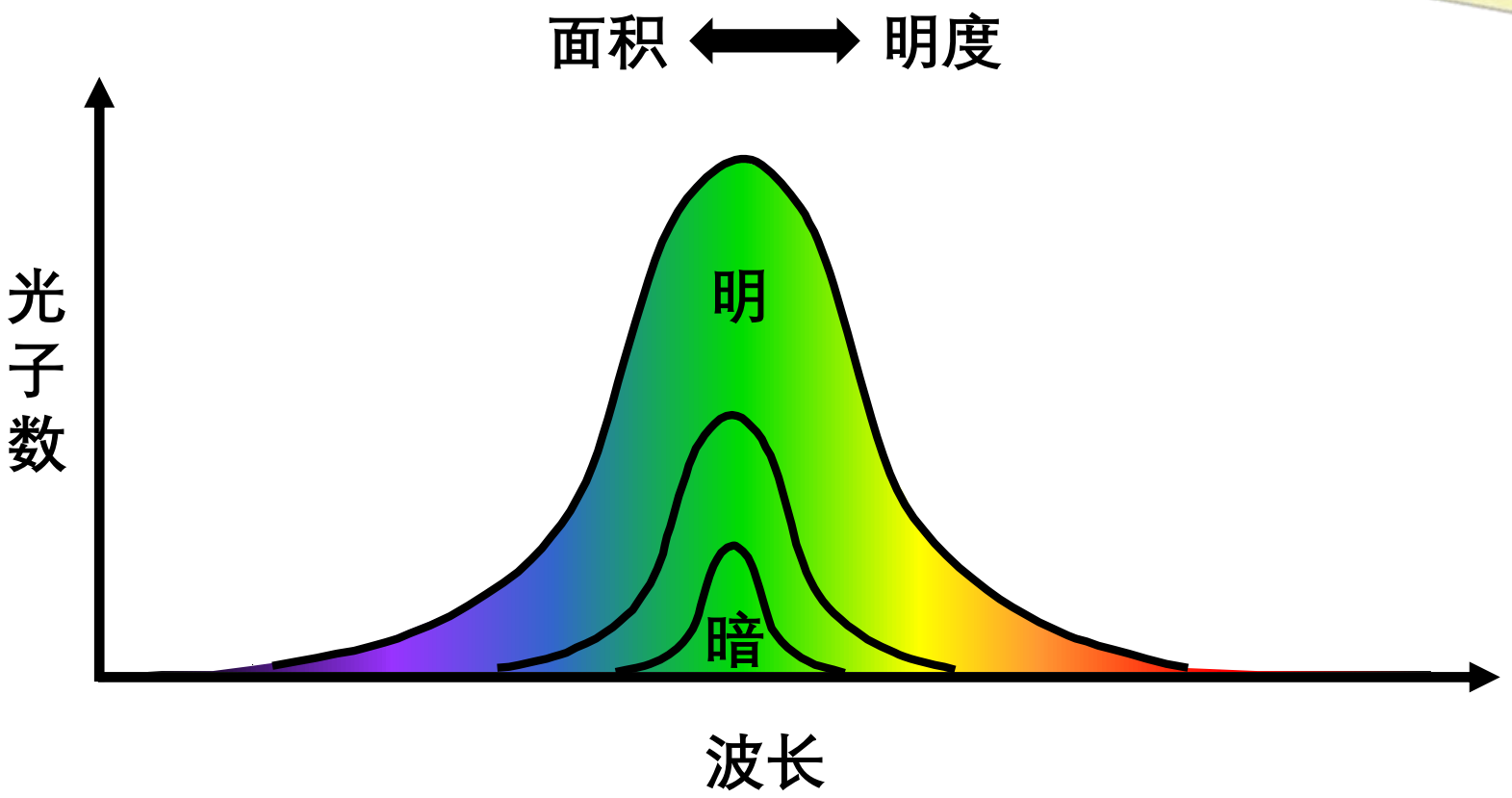


色觉生理学

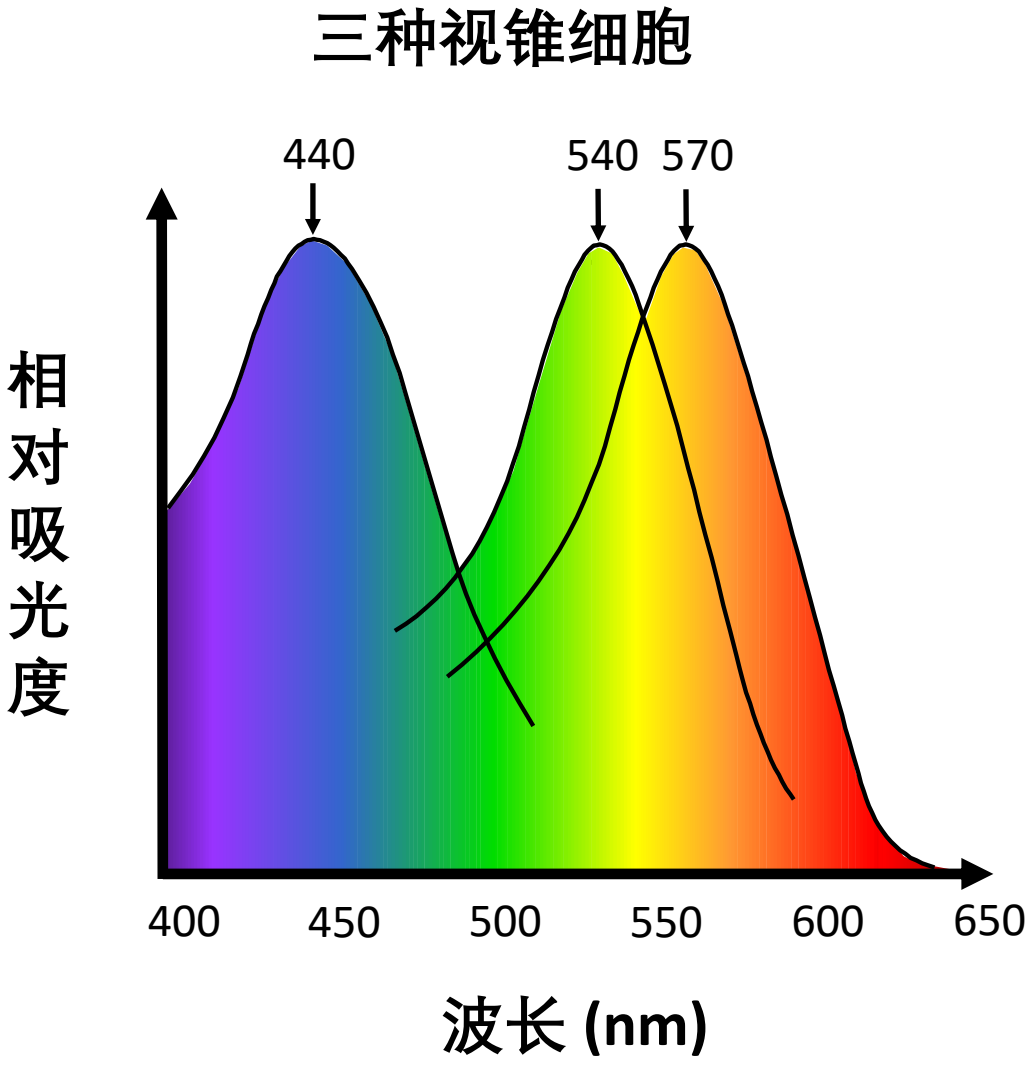
方差 ⟷ 饱和度



色覚生理学

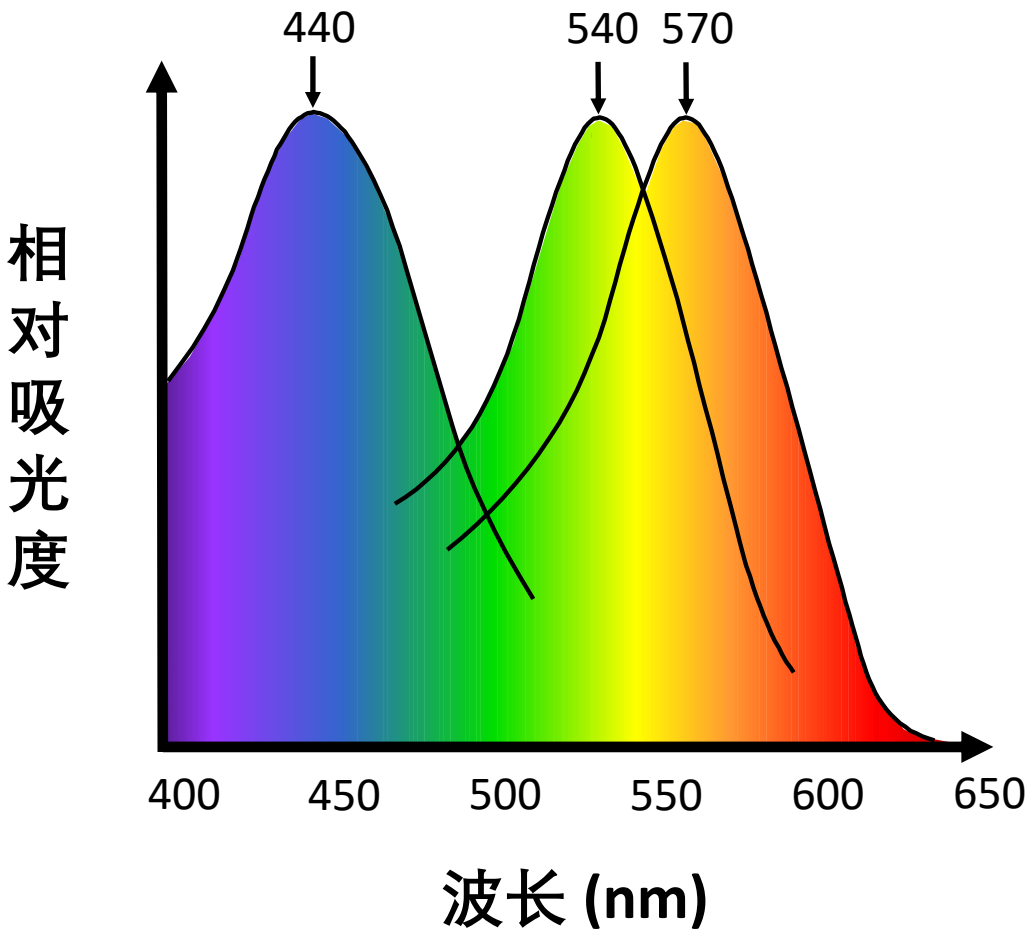


色觉
生理学

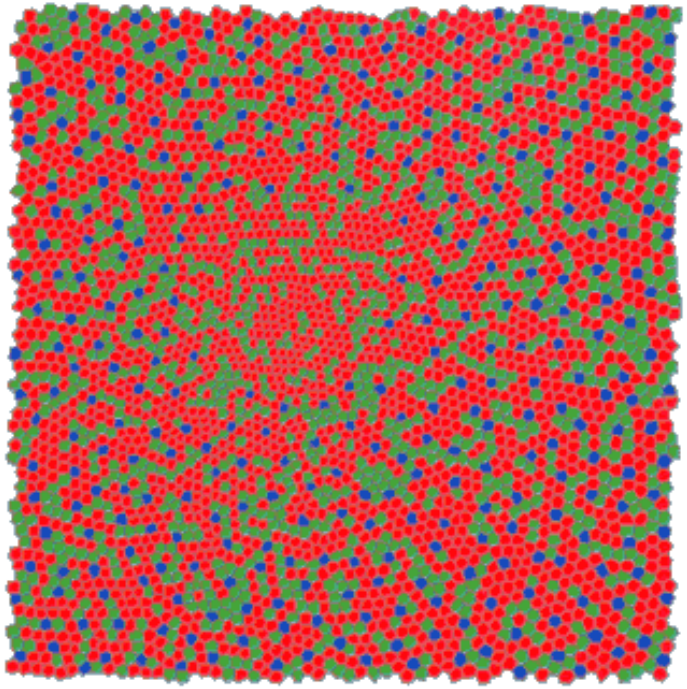


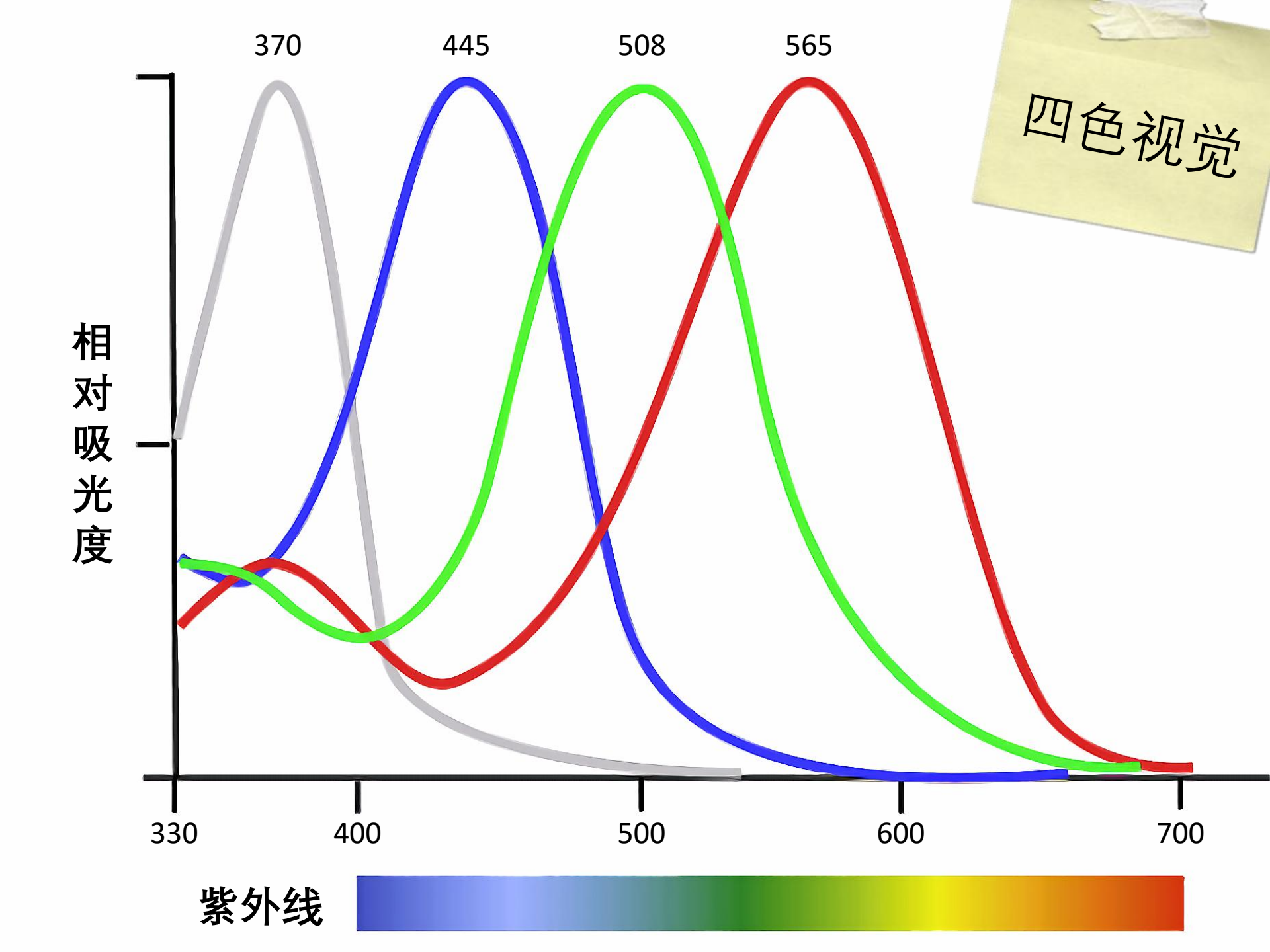
色觉生理学

三种视锥细胞



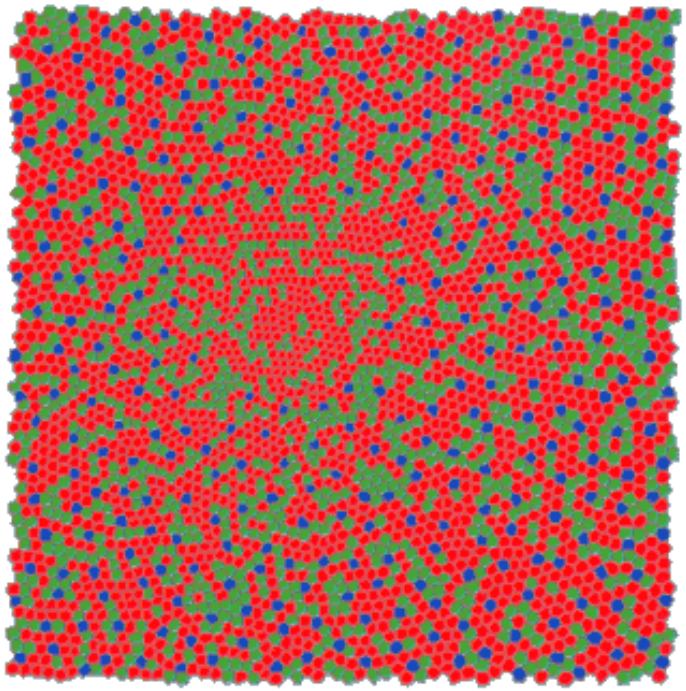
视锥细胞镶嵌




四色视觉
370
445
508
565
相对吸光度
330
400
500
600
700
紫外线

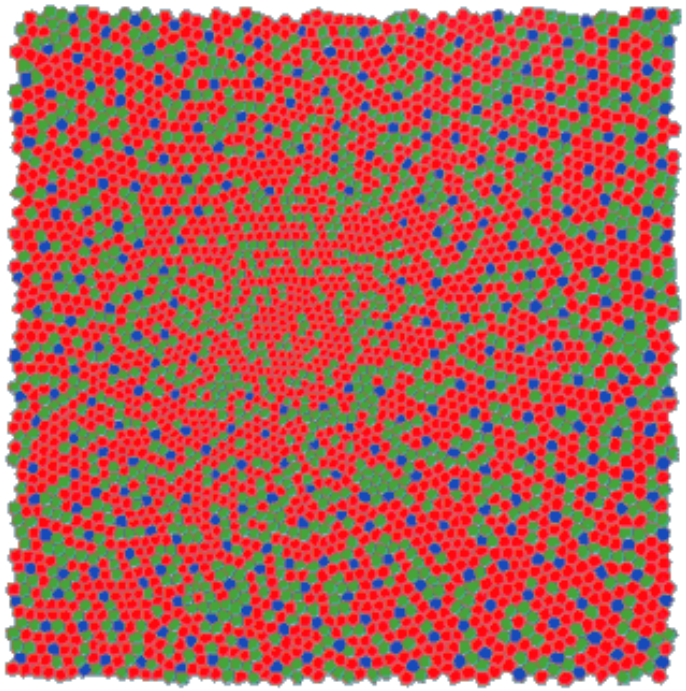
相机色彩感知

视锥细胞镶嵌

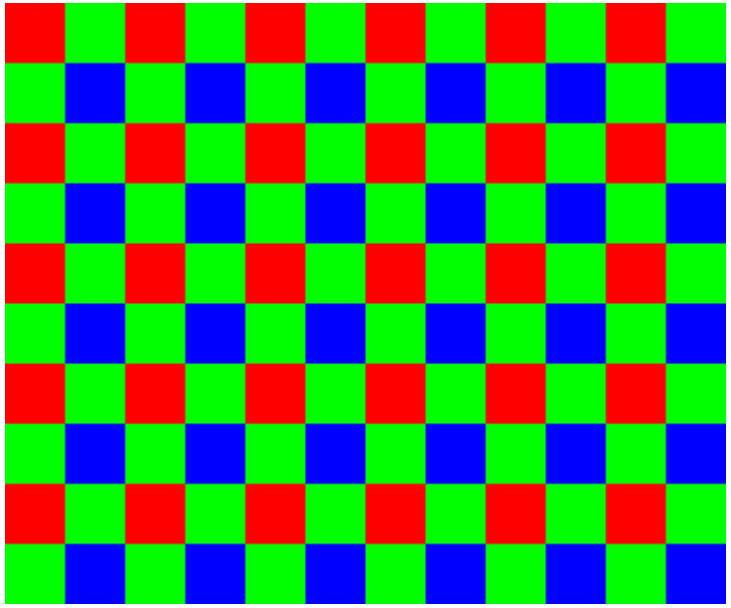


相机色彩感知

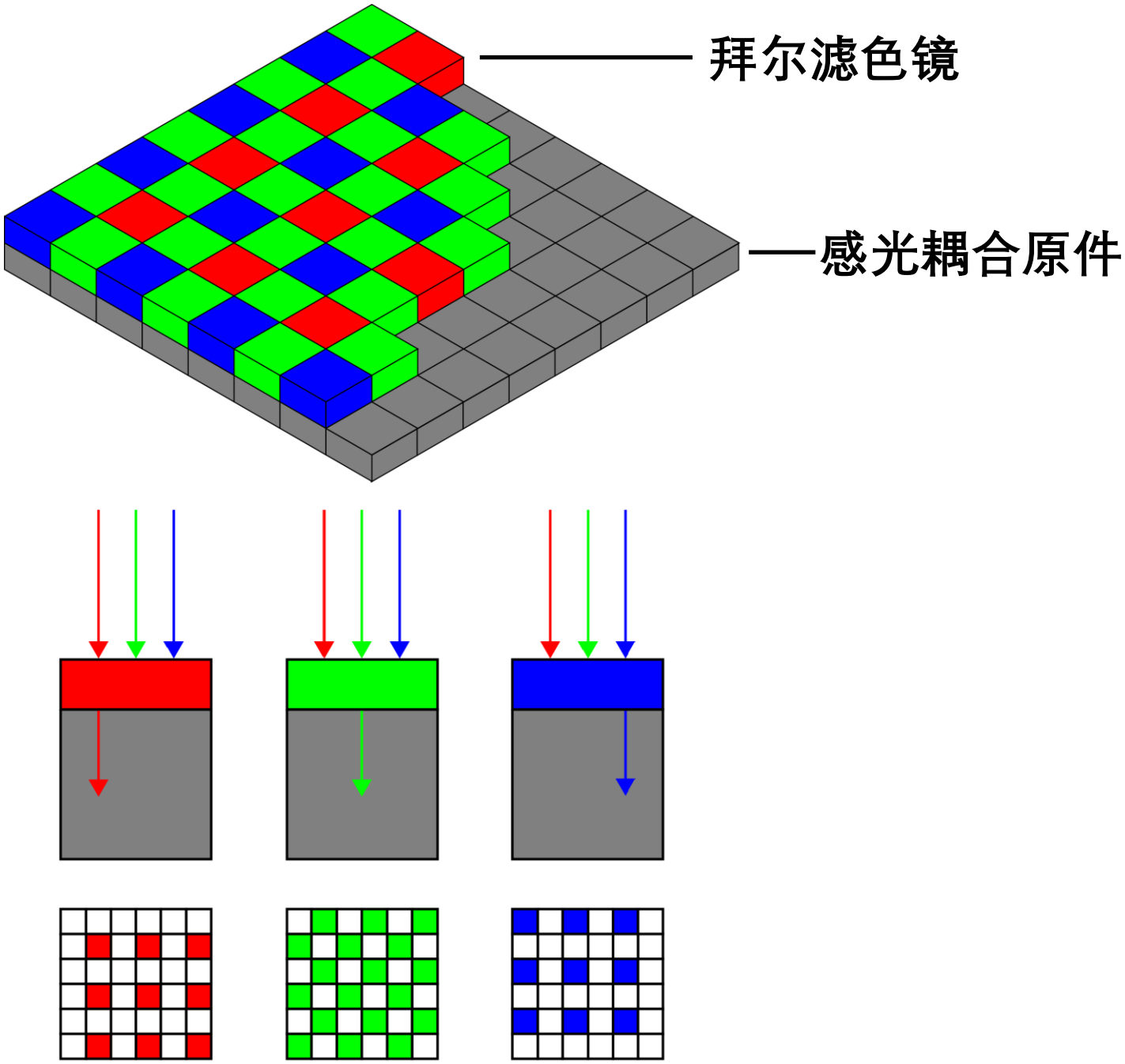
视锥细胞镶嵌

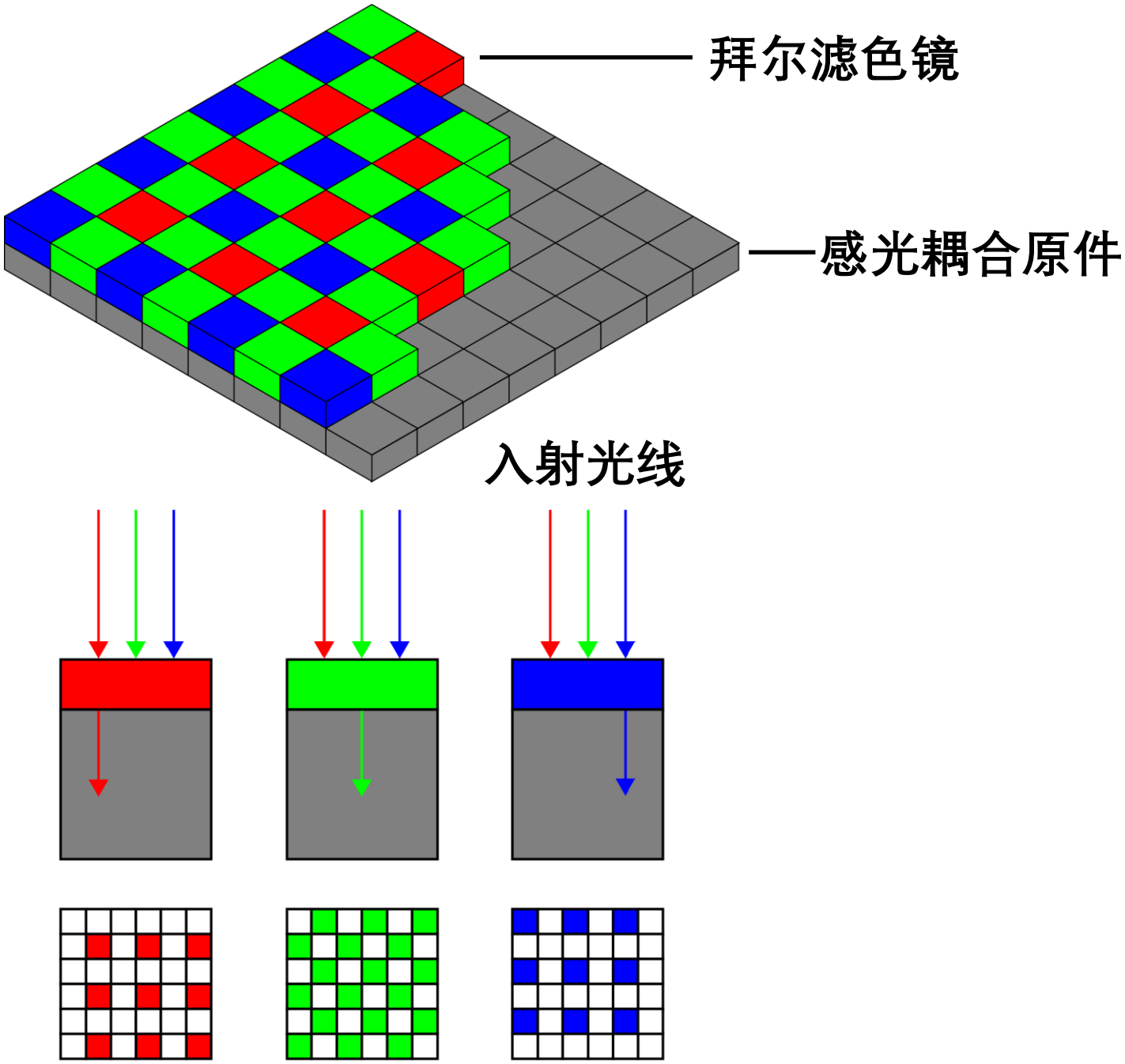


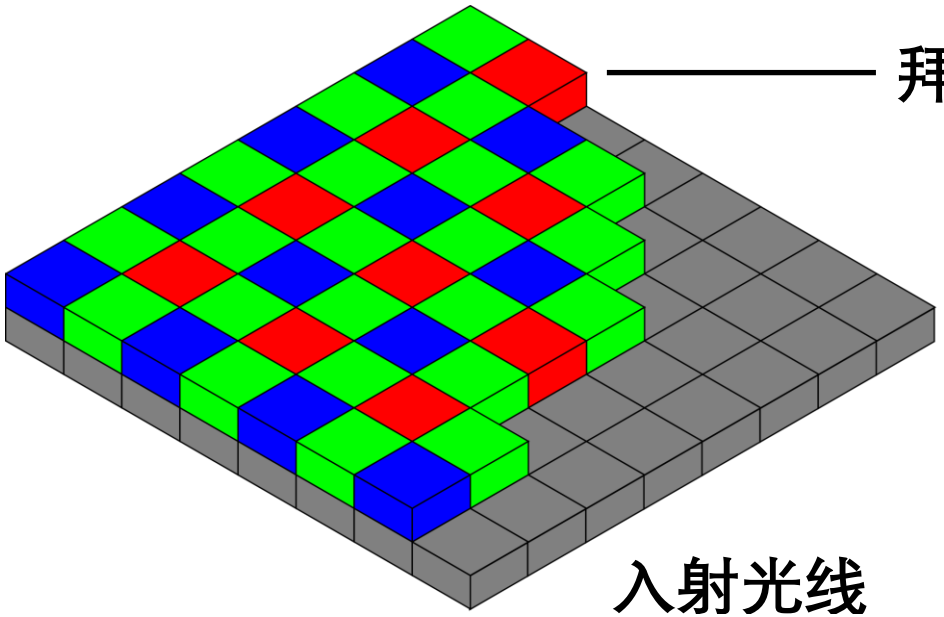
拜尔滤色镜



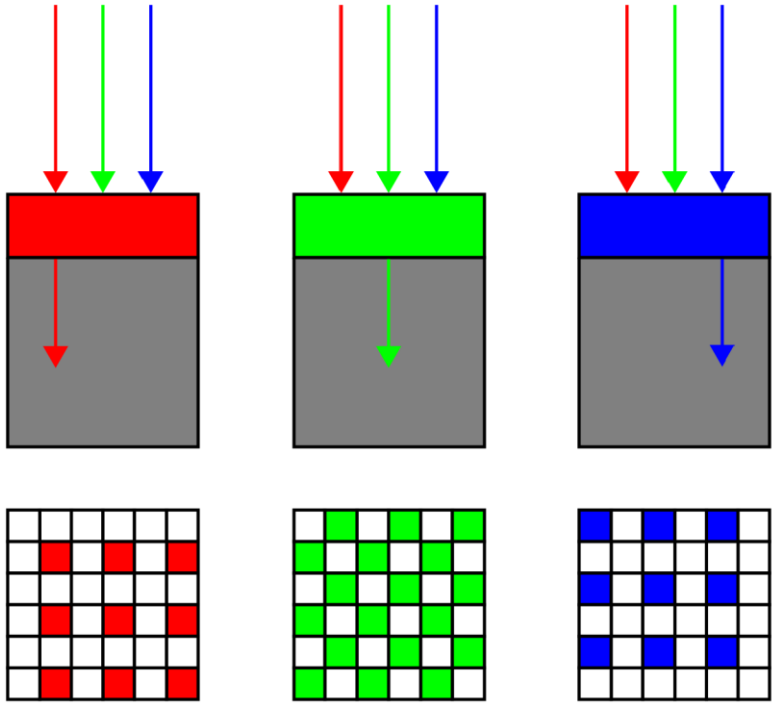
绿色：50%
红色：25%
蓝色：25%

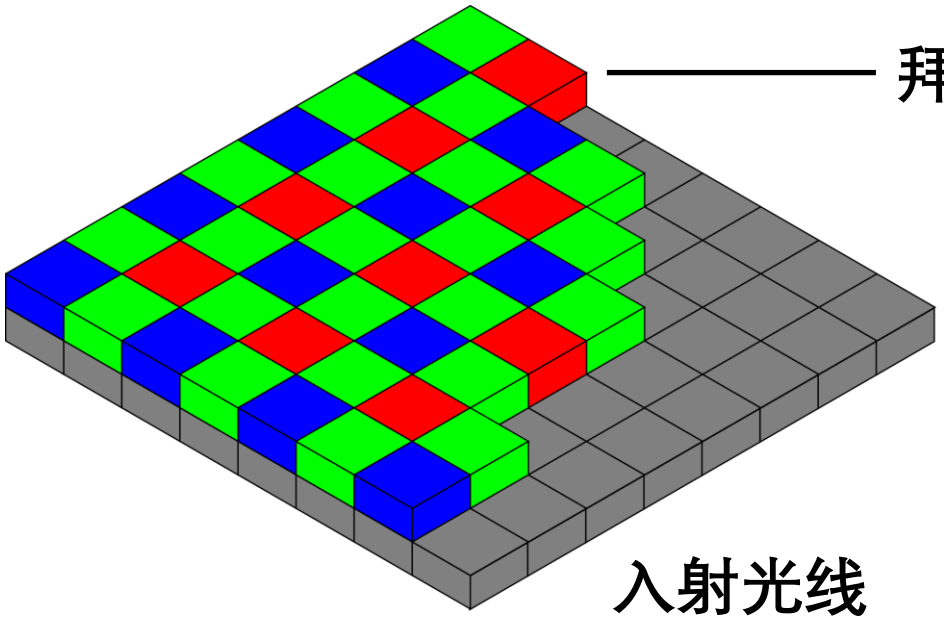

拜尔滤色镜
感光耦合原件


拜尔滤色镜
感光耦合原件
入射光线

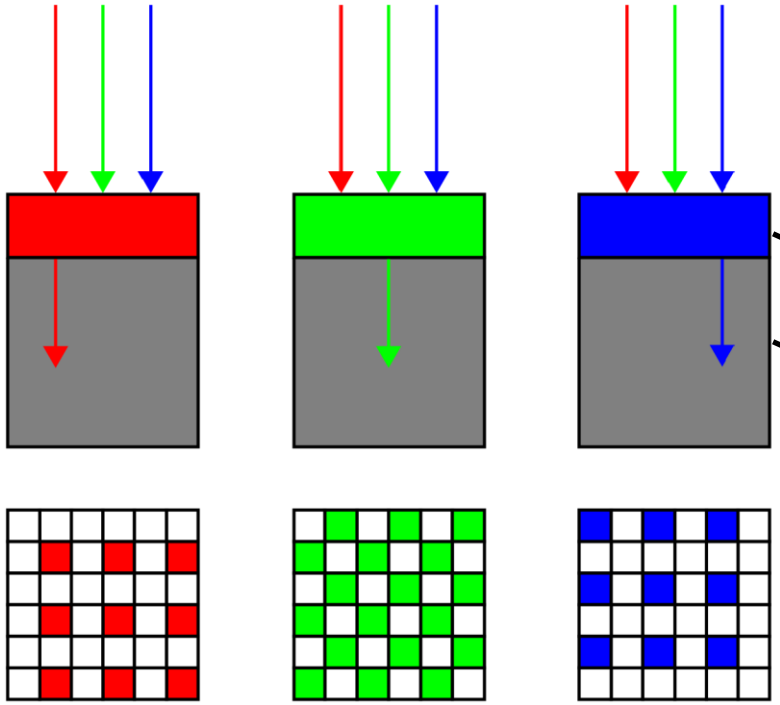


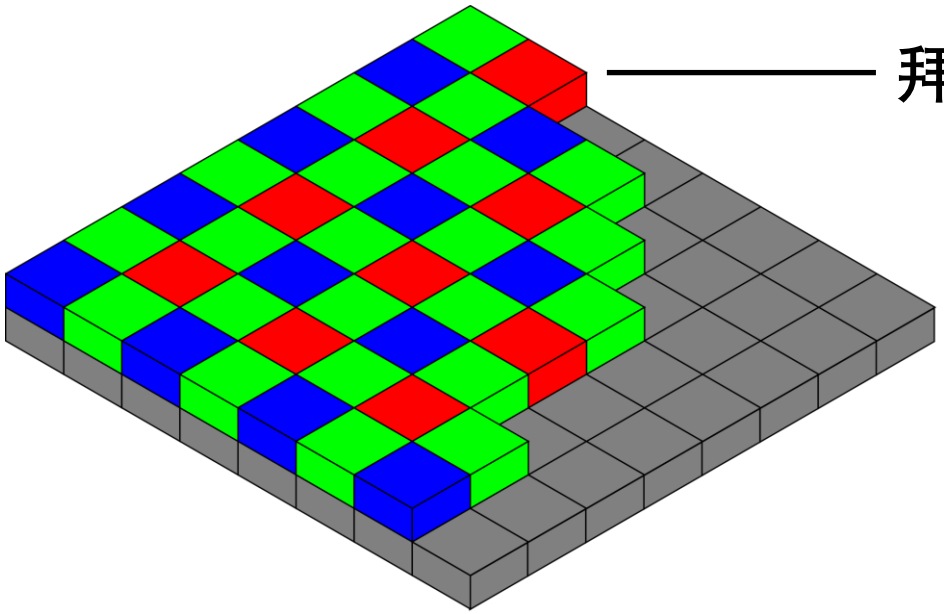
入射光线




拜尔滤色镜
感光耦合原件

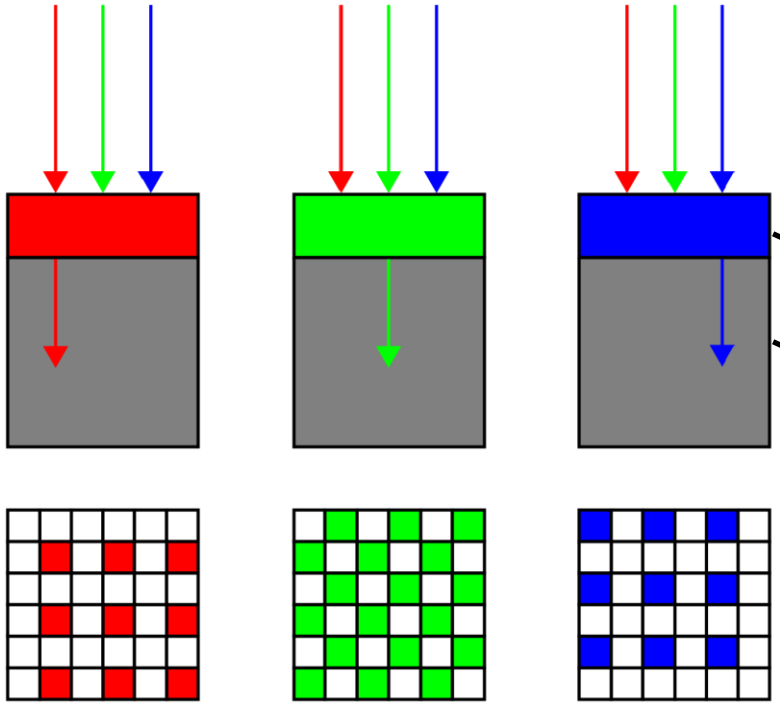

1


入射光线
滤色镜
传感器阵列



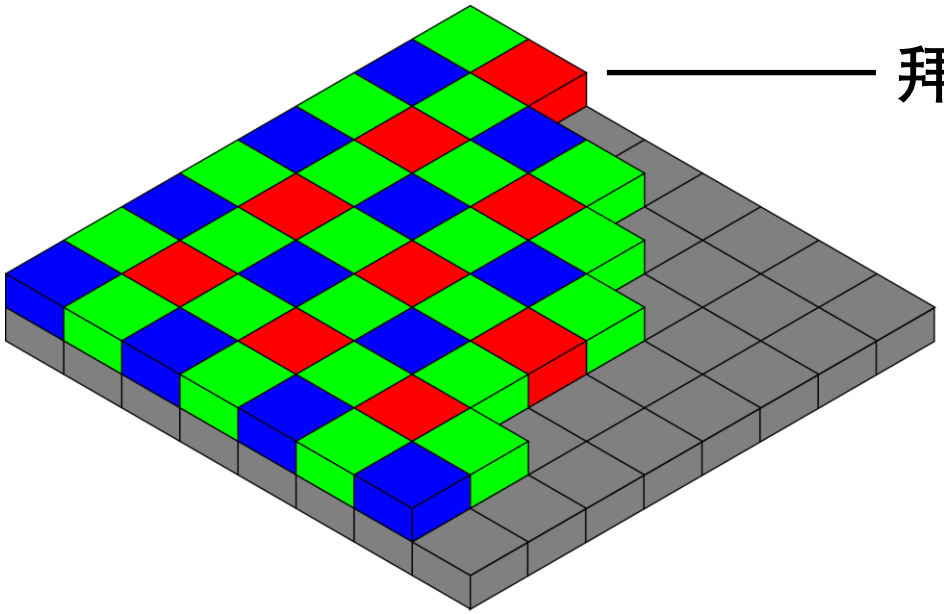
拜尔滤色镜

感光耦合原件



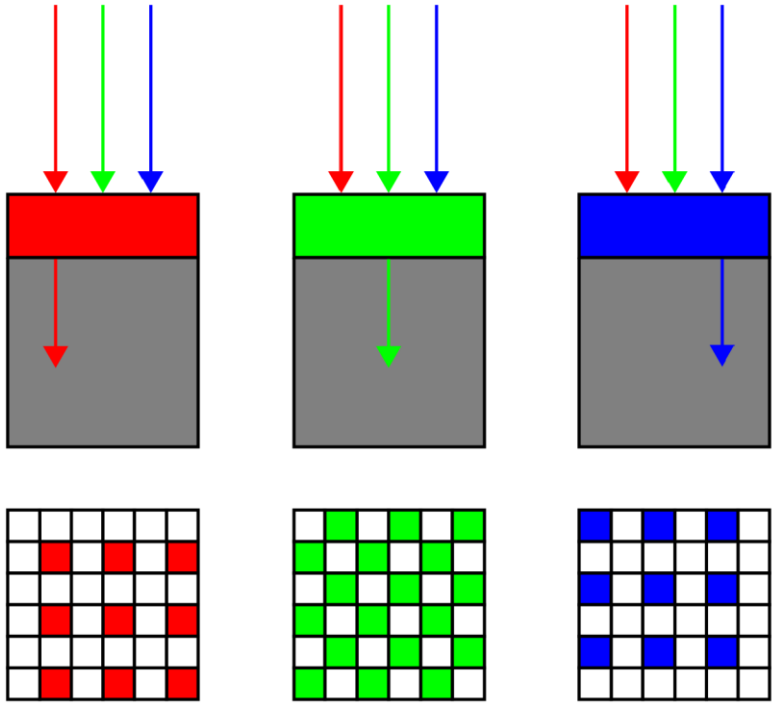
滤色镜

传感器阵列

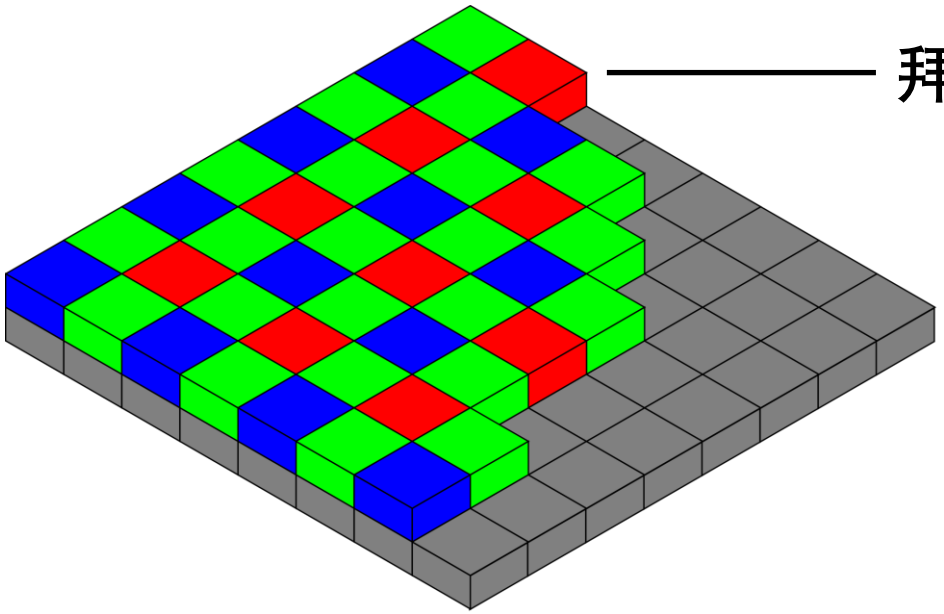


拜尔滤色镜

感光耦合原件

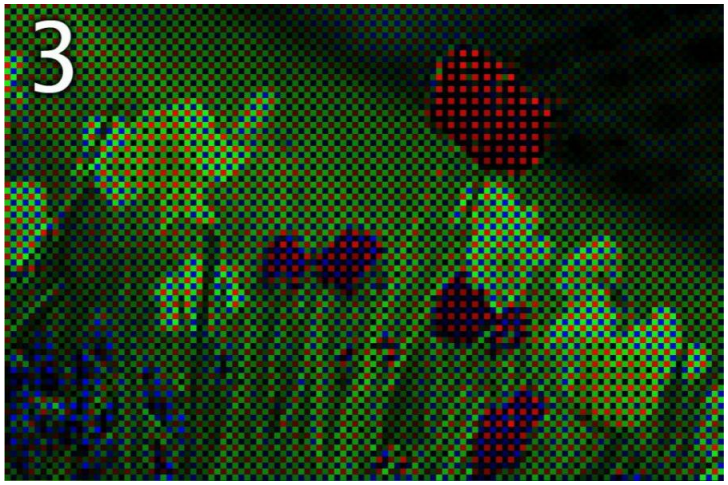


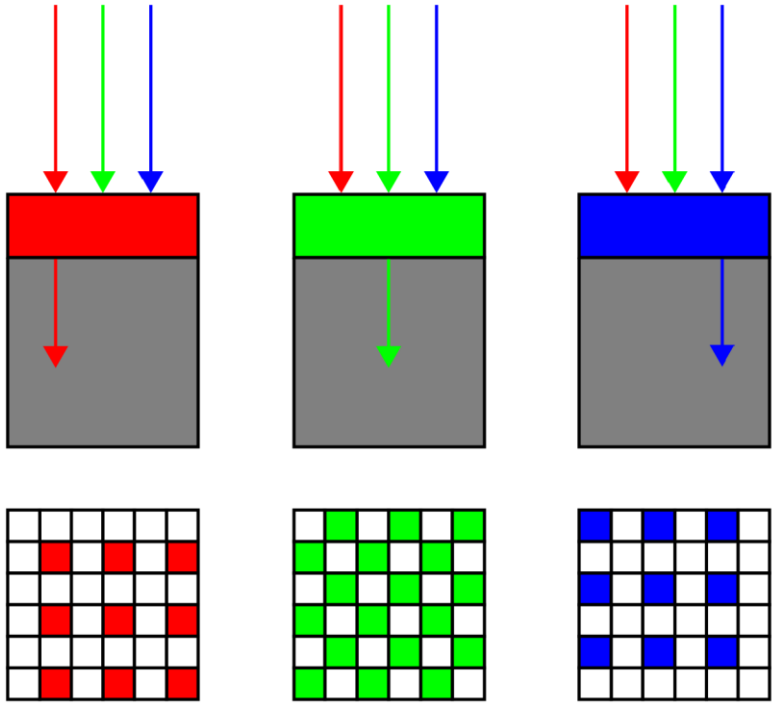
色彩模板



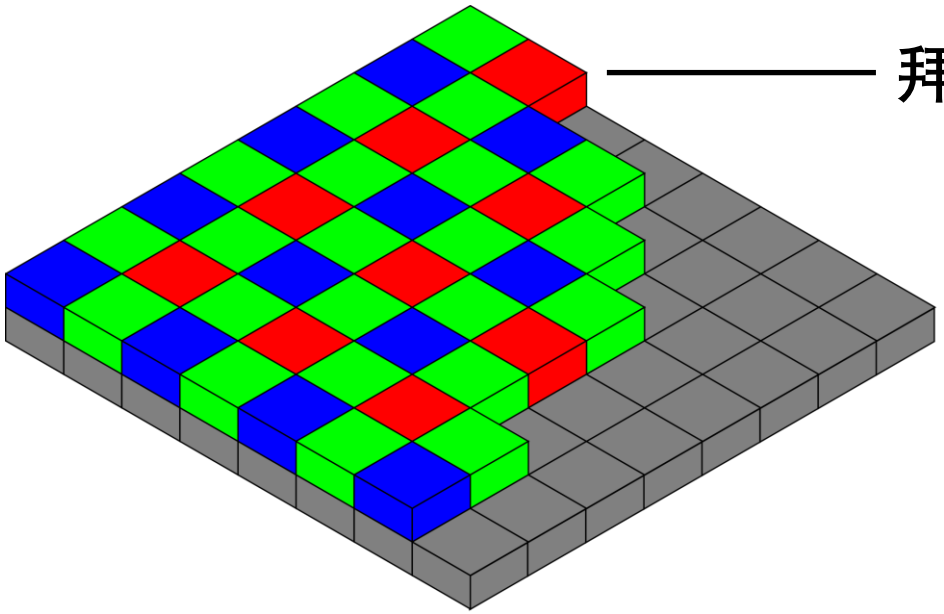
拜尔滤色镜

感光耦合原件



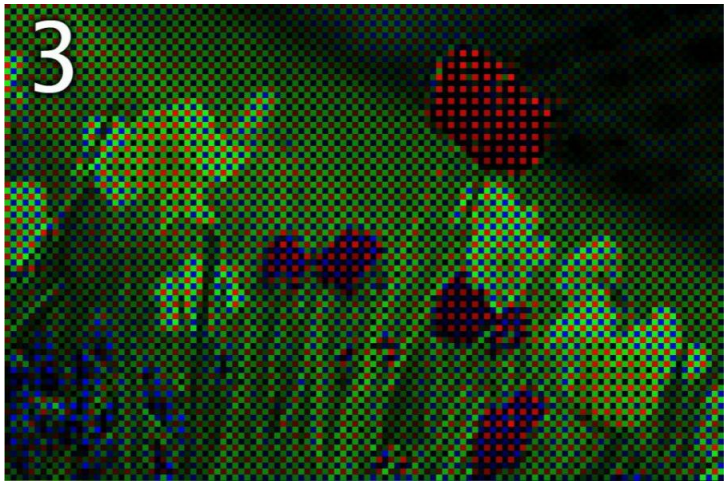


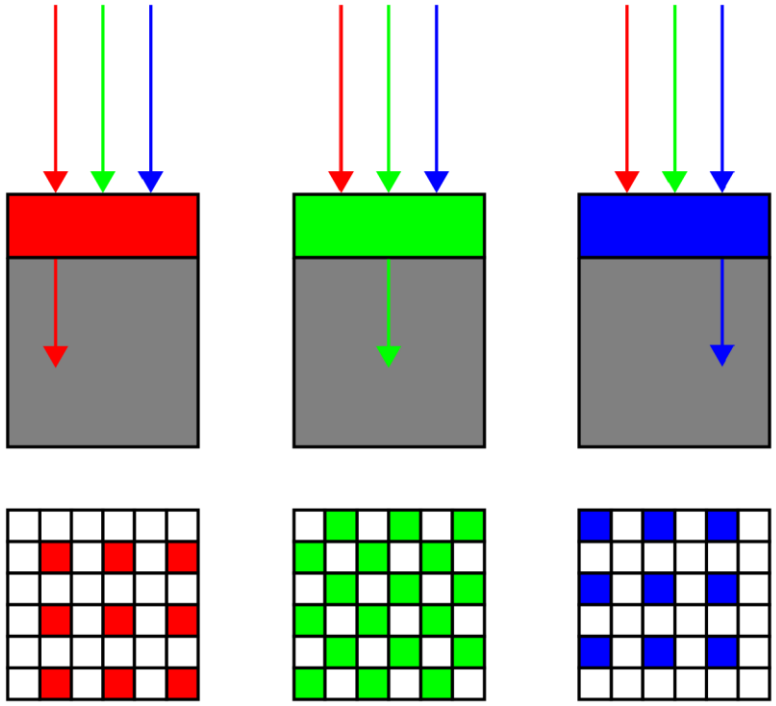
色彩模板

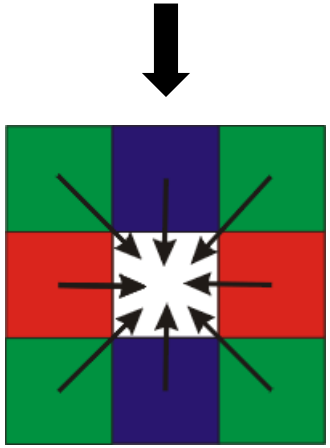


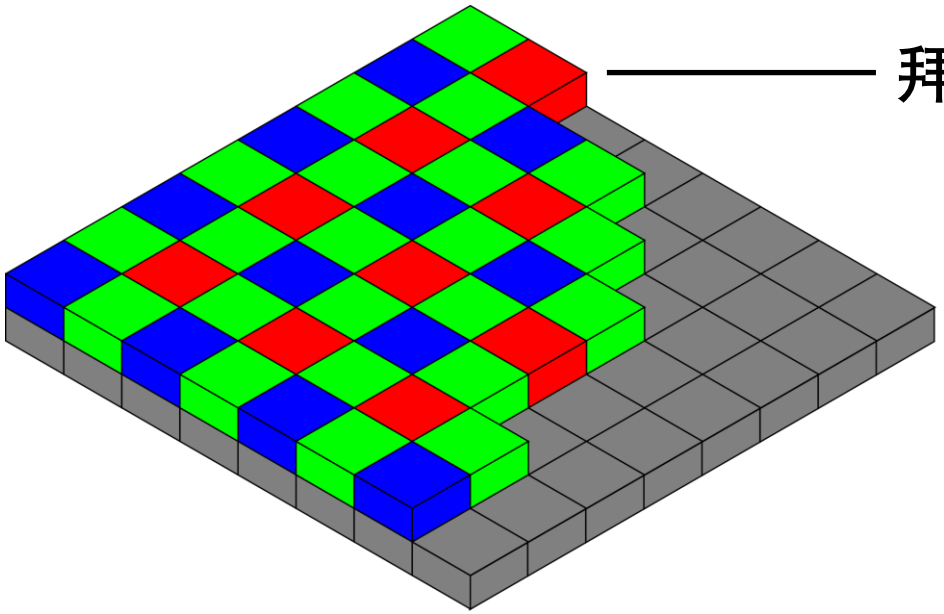
拜尔滤色镜

感光耦合原件



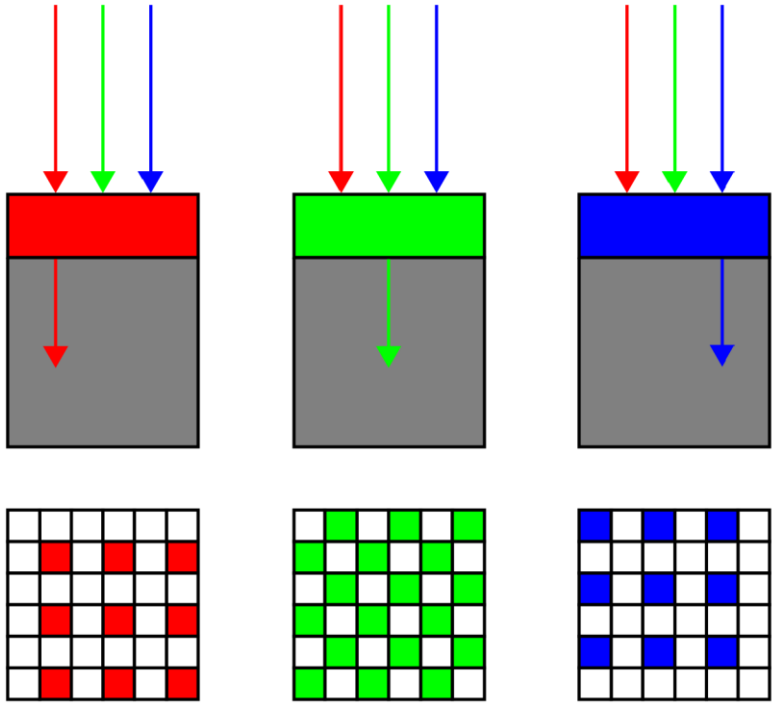






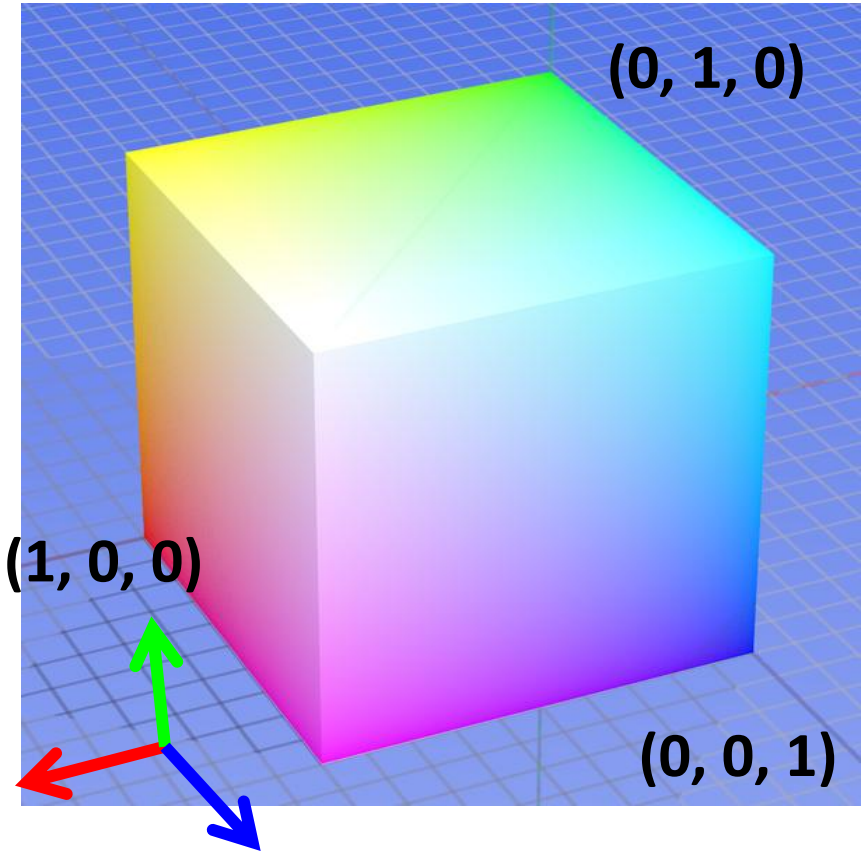
拜尔滤色镜

感光耦合原件




如何表示颜色？

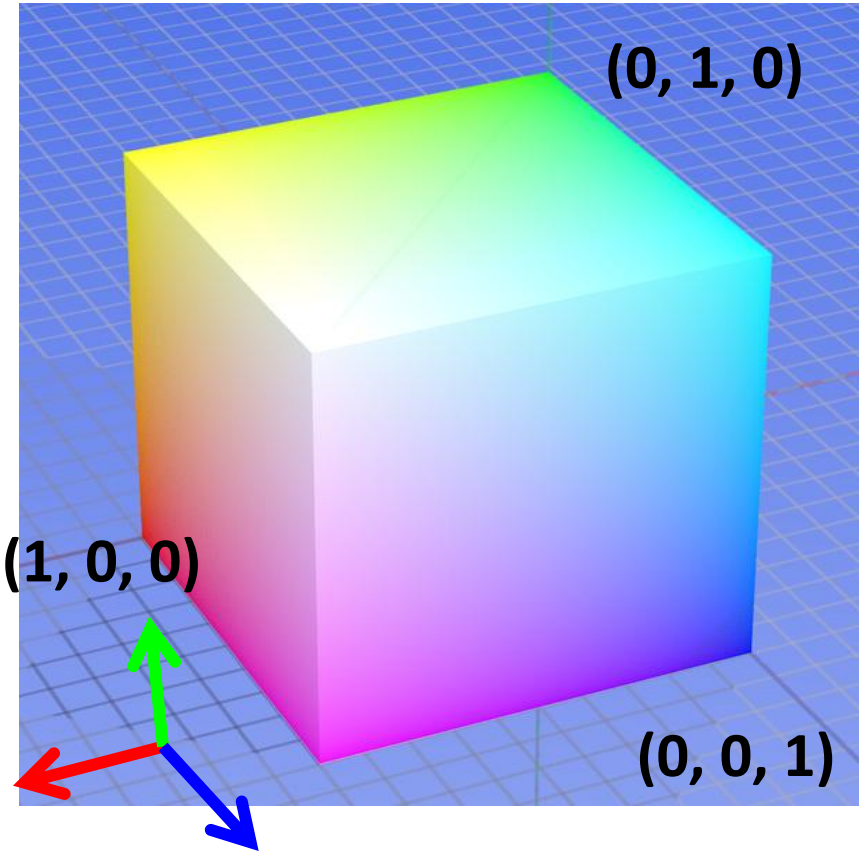
色彩空间
RGB



$$8\ \text{bit} \times 3 = 24\ \text{bit}$$

$$256 \times 256 \times 256$$
$$\approx 1677\ \text{万色}$$

色彩空间
RGB

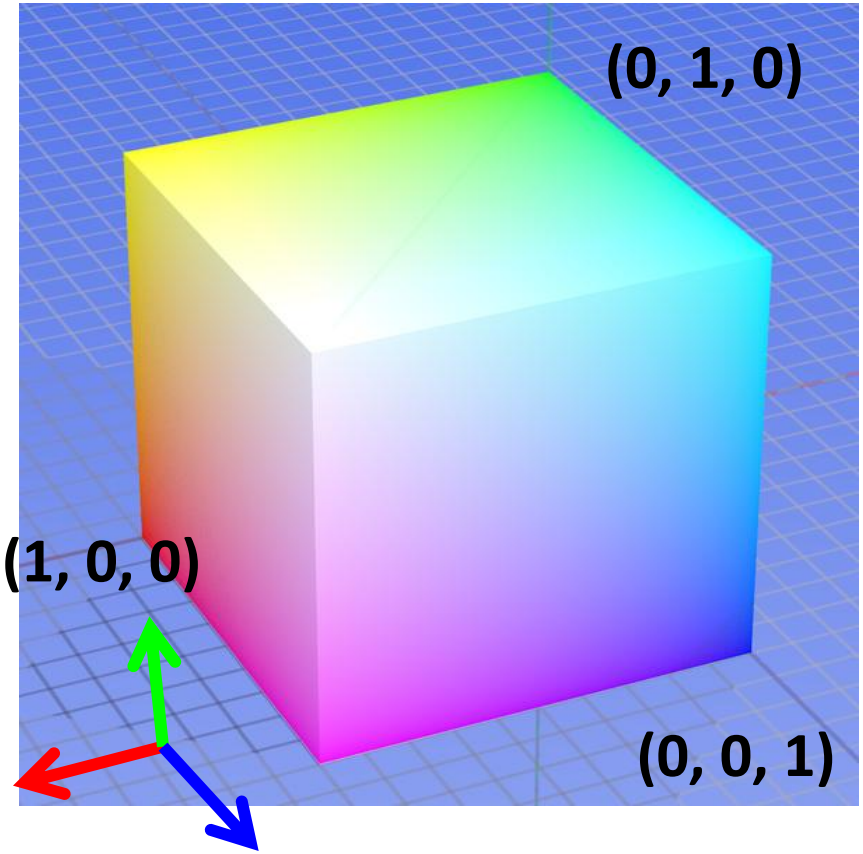


$$8\ \text{bit} \times 3 = 24\ \text{bit}$$

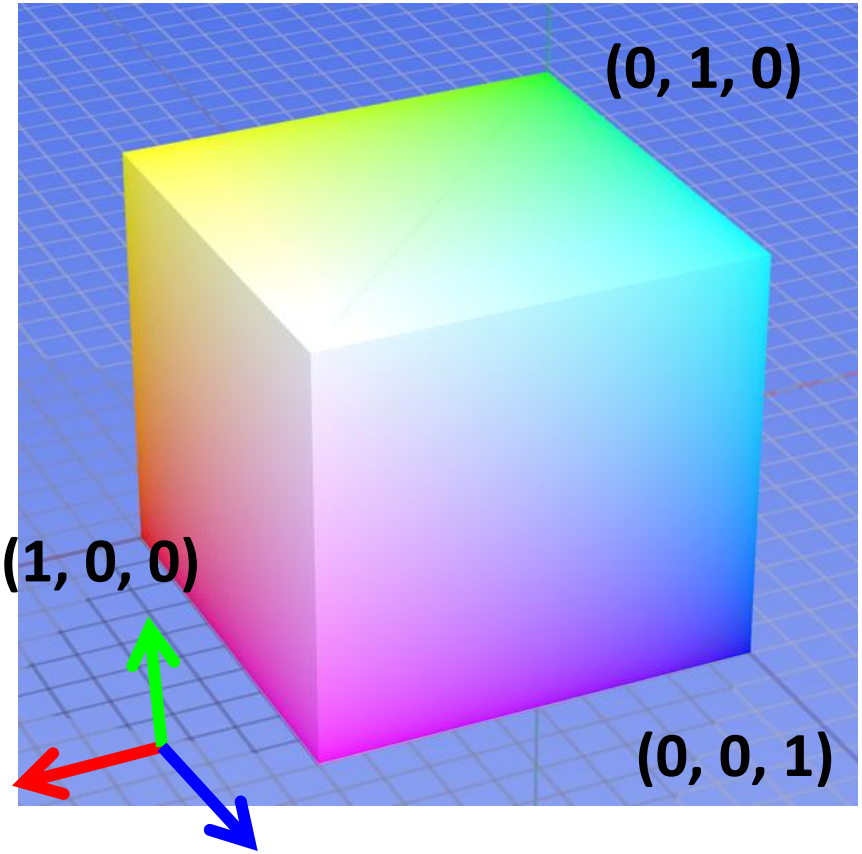
$$256 \times 256 \times 256$$
$$\approx 1677\ \text{万色}$$

24位真彩色

色彩空间
RGB



色彩空间 RGB

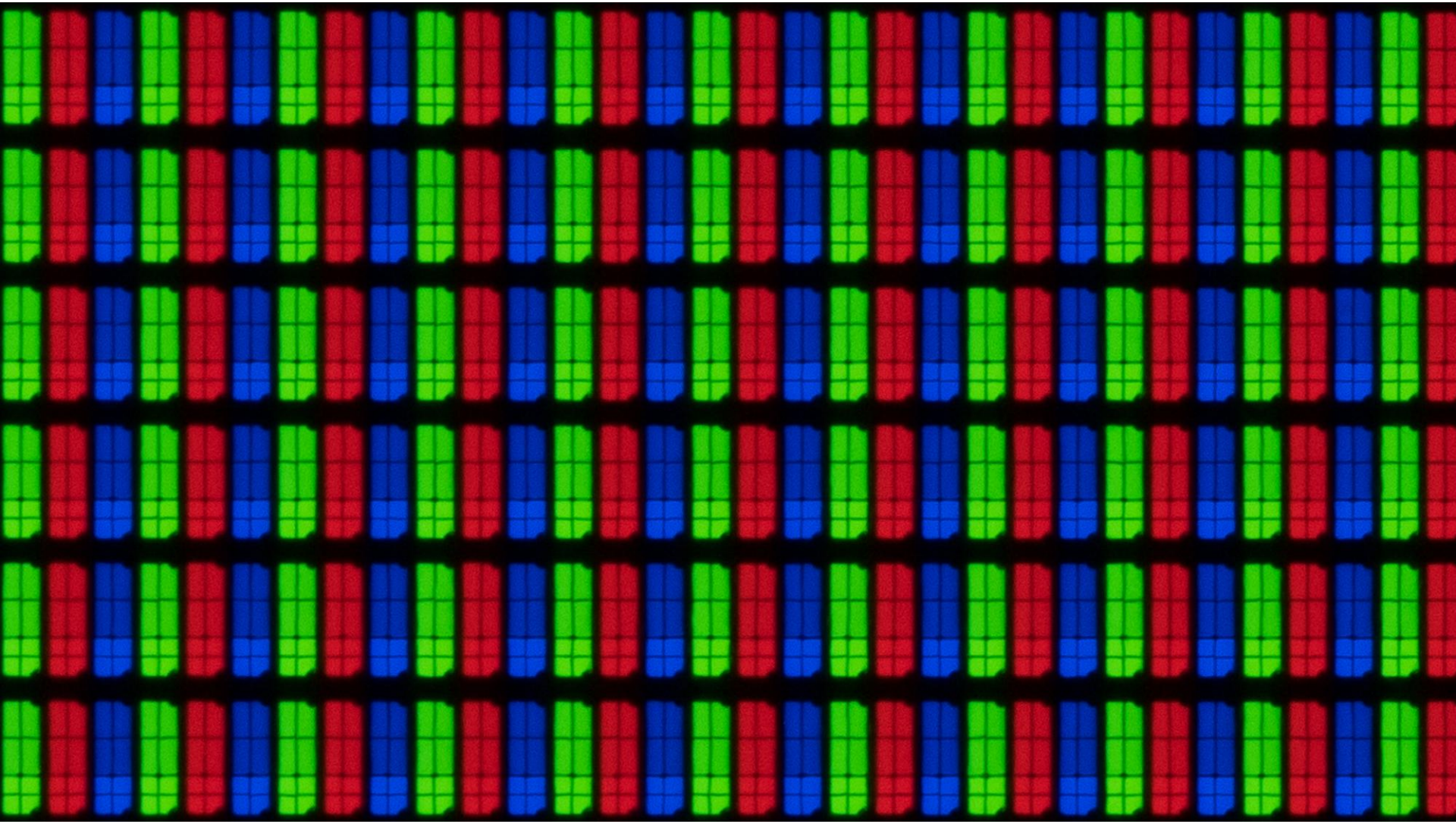


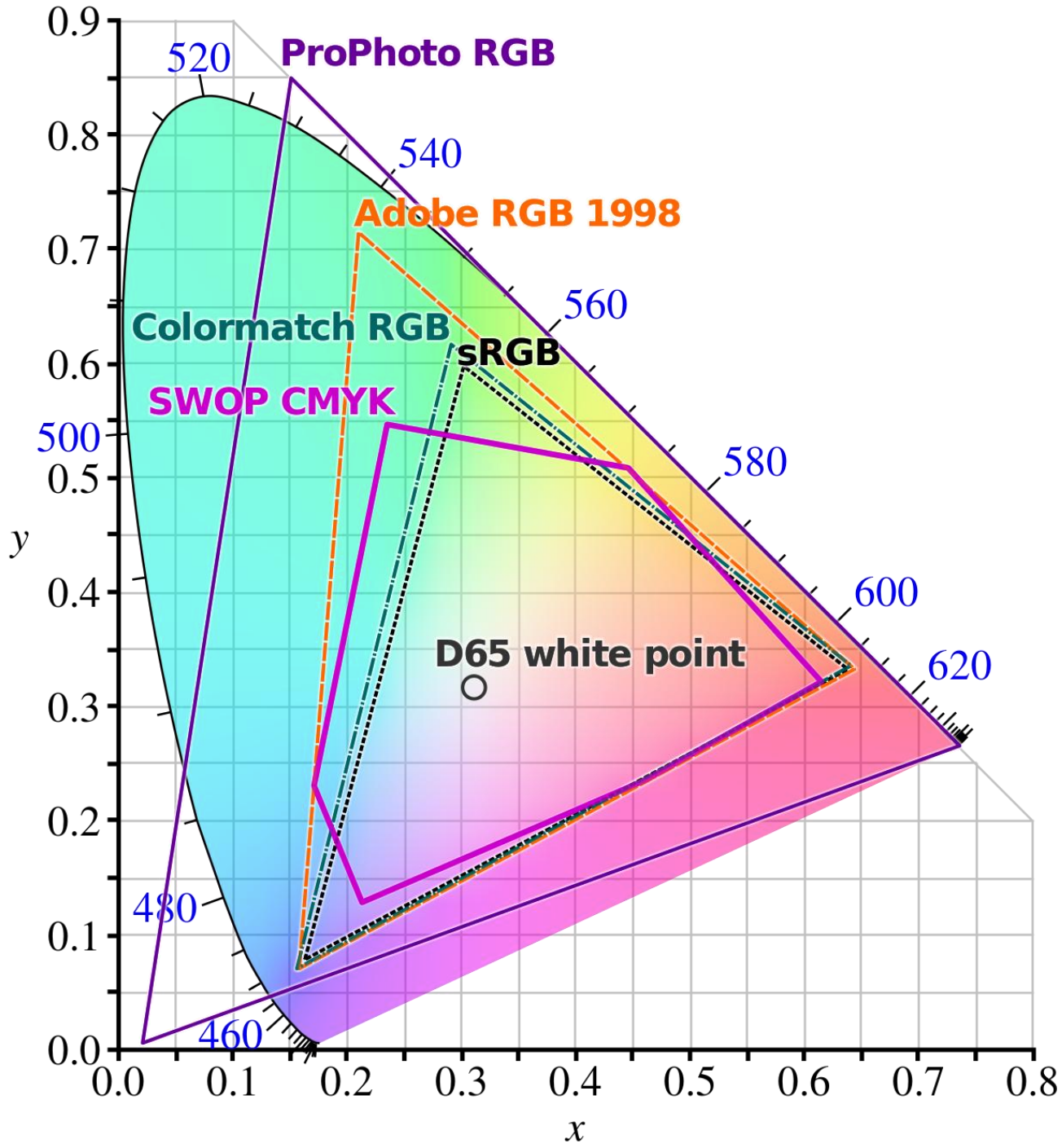
缺点：通道间相关性强
非感知

R
$(\mathbf{G}=0,\mathbf{B}=0)$

G
$(\mathbf{R}=0,\mathbf{B}=0)$

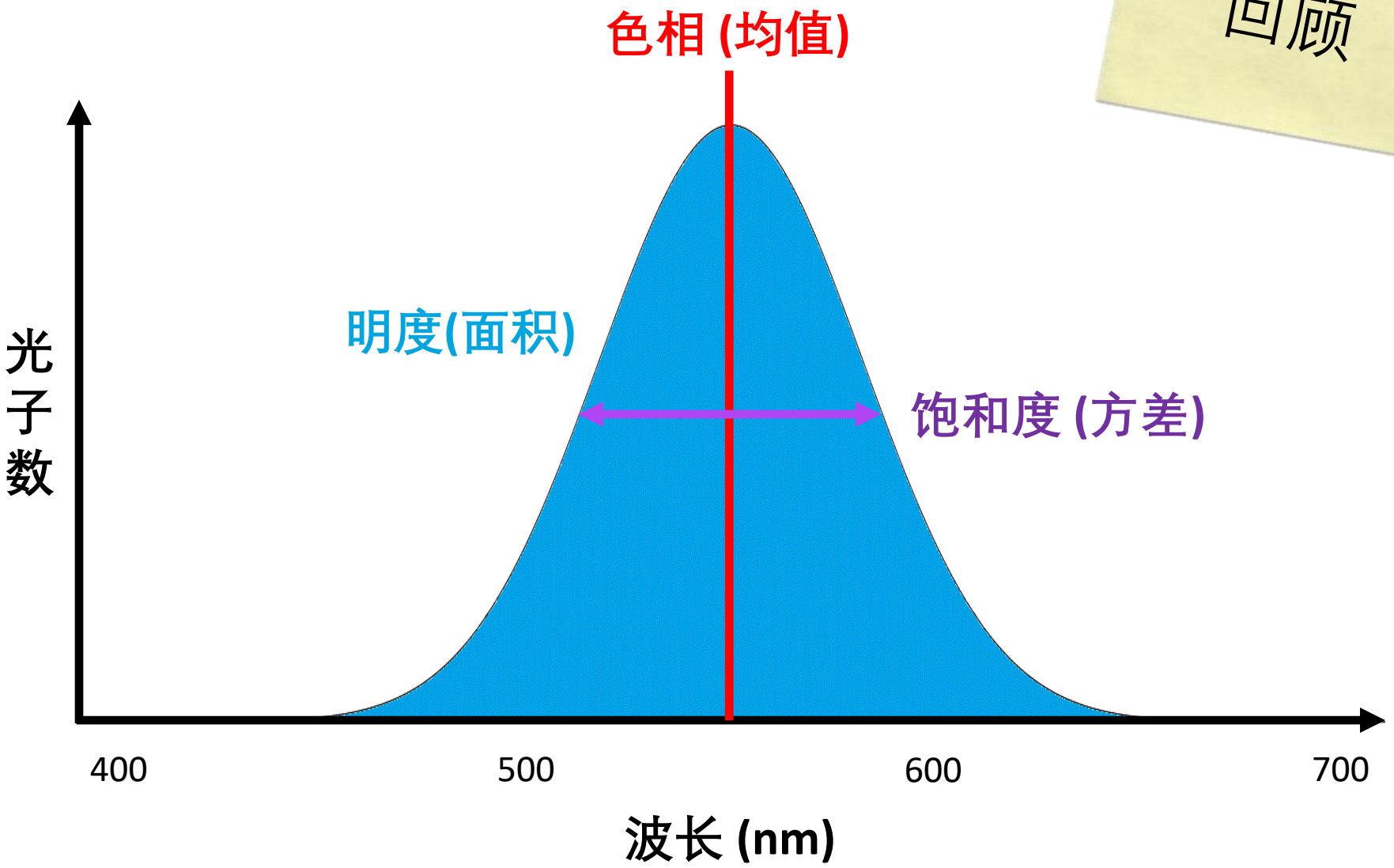
B
$(\mathbf{R}=0,\mathbf{G}=0)$




ProPhoto RGB
Adobe RGB 1998
Colormatch RGB
sRGB
SWOP CMYK
D65 white point
520
540
560
580
600
620
500
480
460
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
x

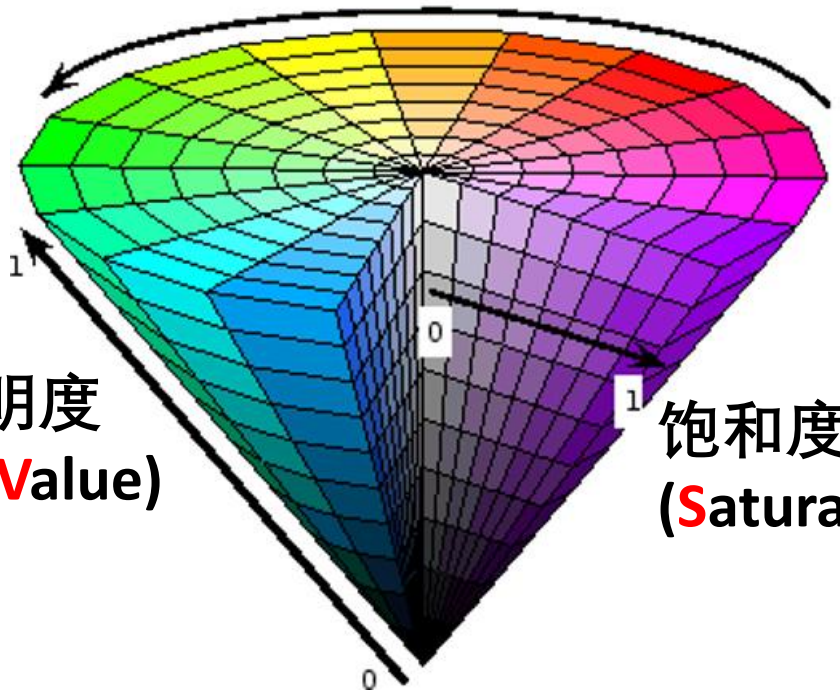
RGB色域

回顾



色彩空间 HSV

色相 (Hue)

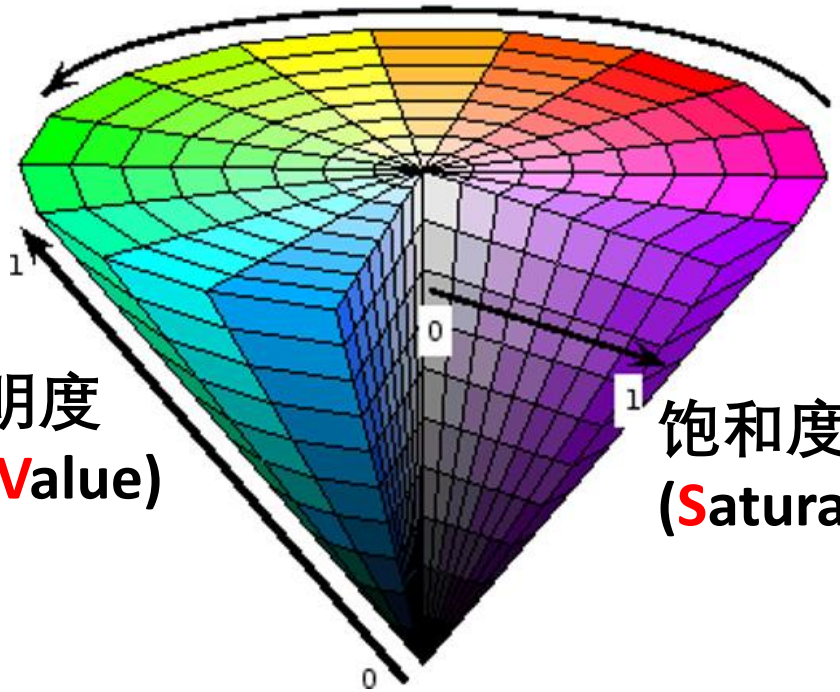


明度
(Value)

饱和度
(Saturation)

色彩空间 HSV

色相 (Hue)

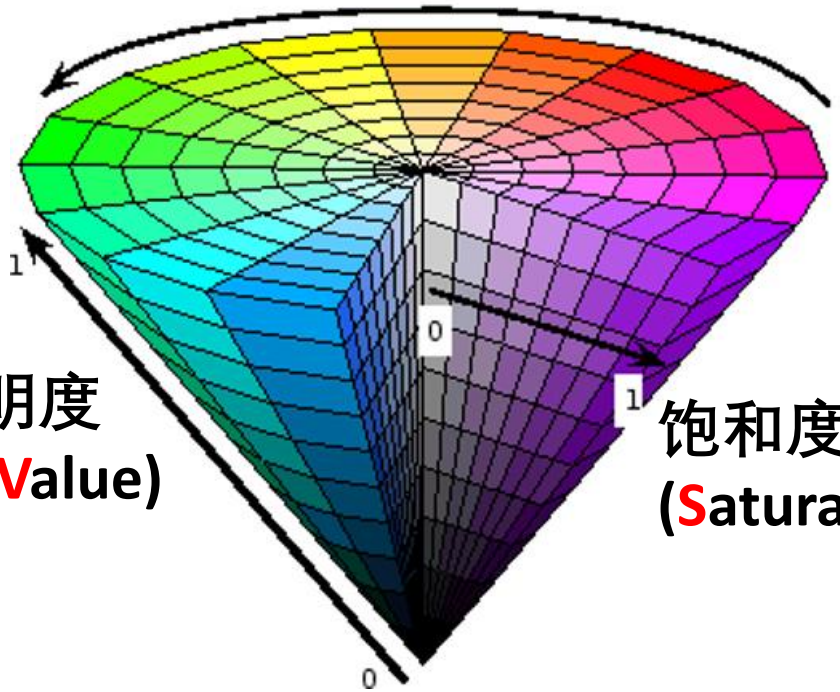


明度
(Value)

饱和度
(Saturation)

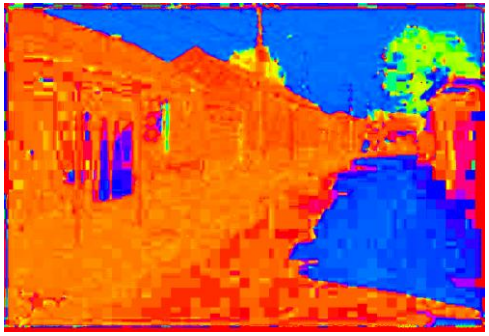
色彩空间 HSV

色相 (Hue)



明度 (Value)

饱和度 (Saturation)



H

$(\mathbf{S}=1,\mathbf{V}=1)$

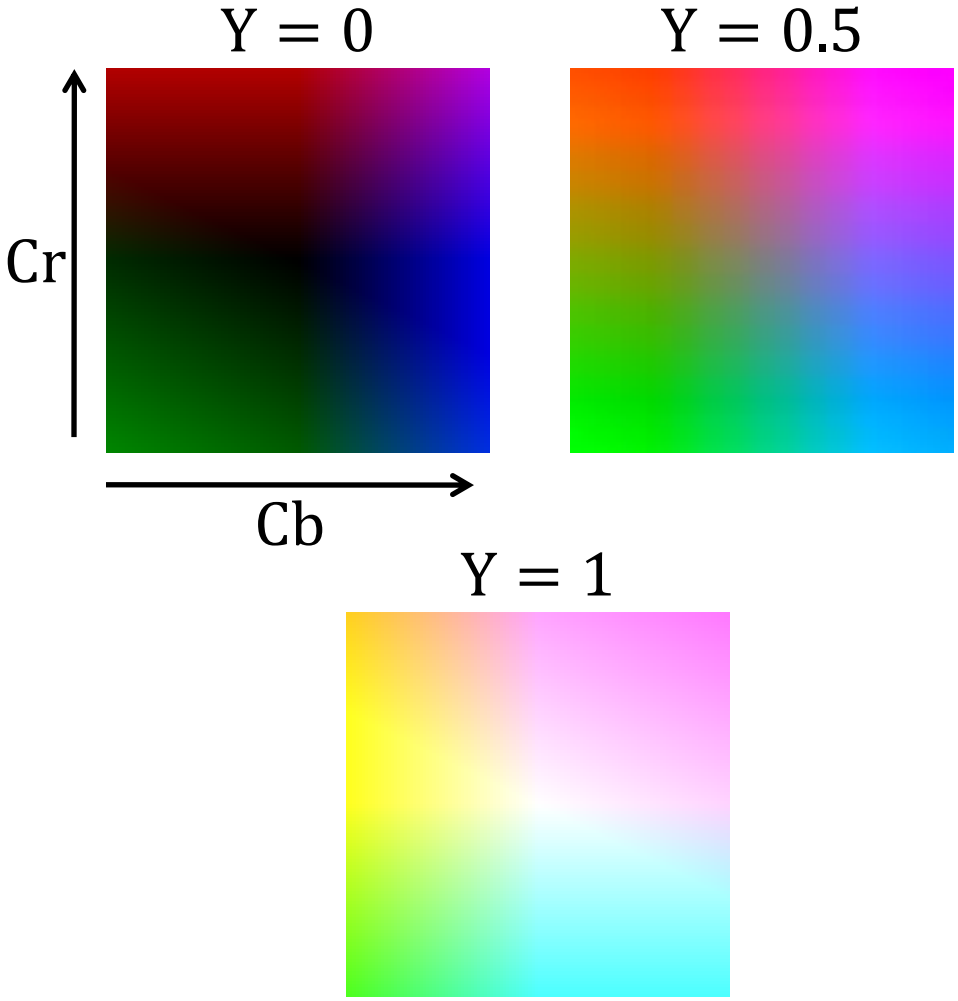
S

$(\mathbf{H}=1,\mathbf{V}=1)$

V

$(\mathbf{H}=1,\mathbf{S}=0)$

色彩空间 YCrCb

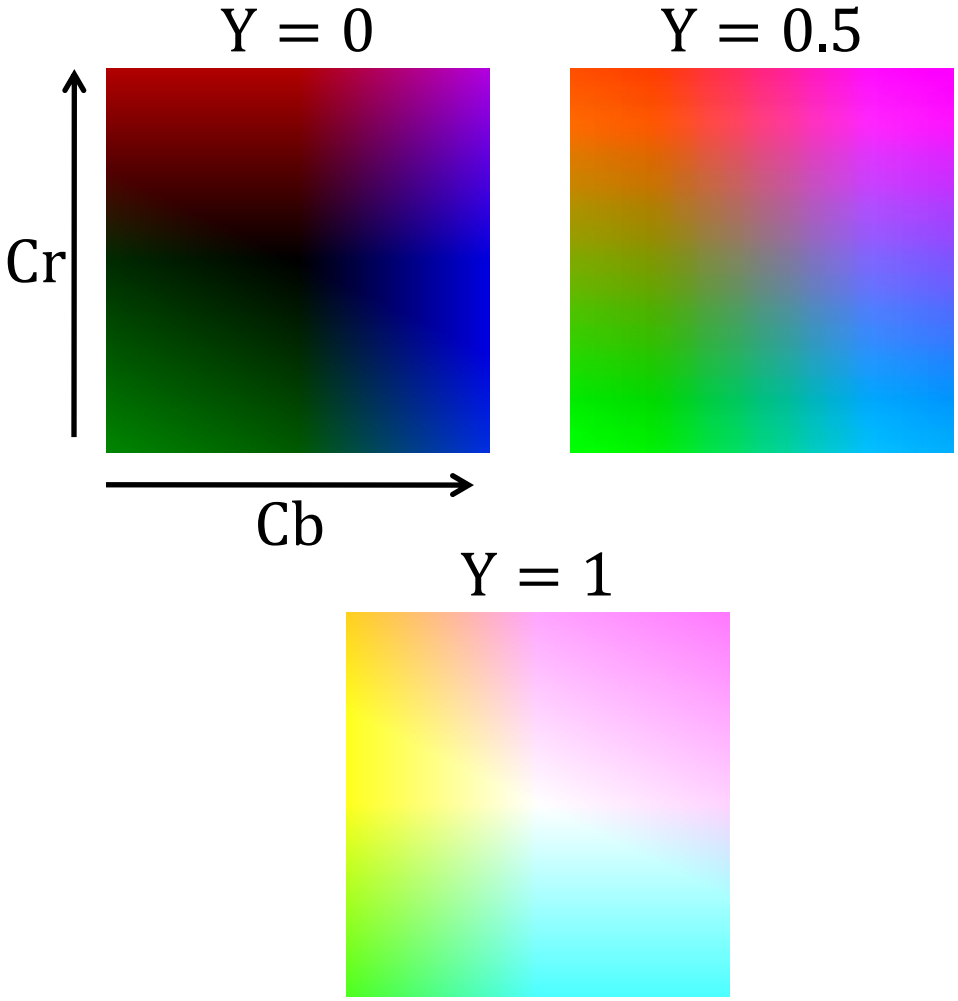


Y：明流，表示光的浓度且非线性

Cr：红色浓度偏移成分

Cb：蓝色浓度偏移成分

色彩空间 YCrCb



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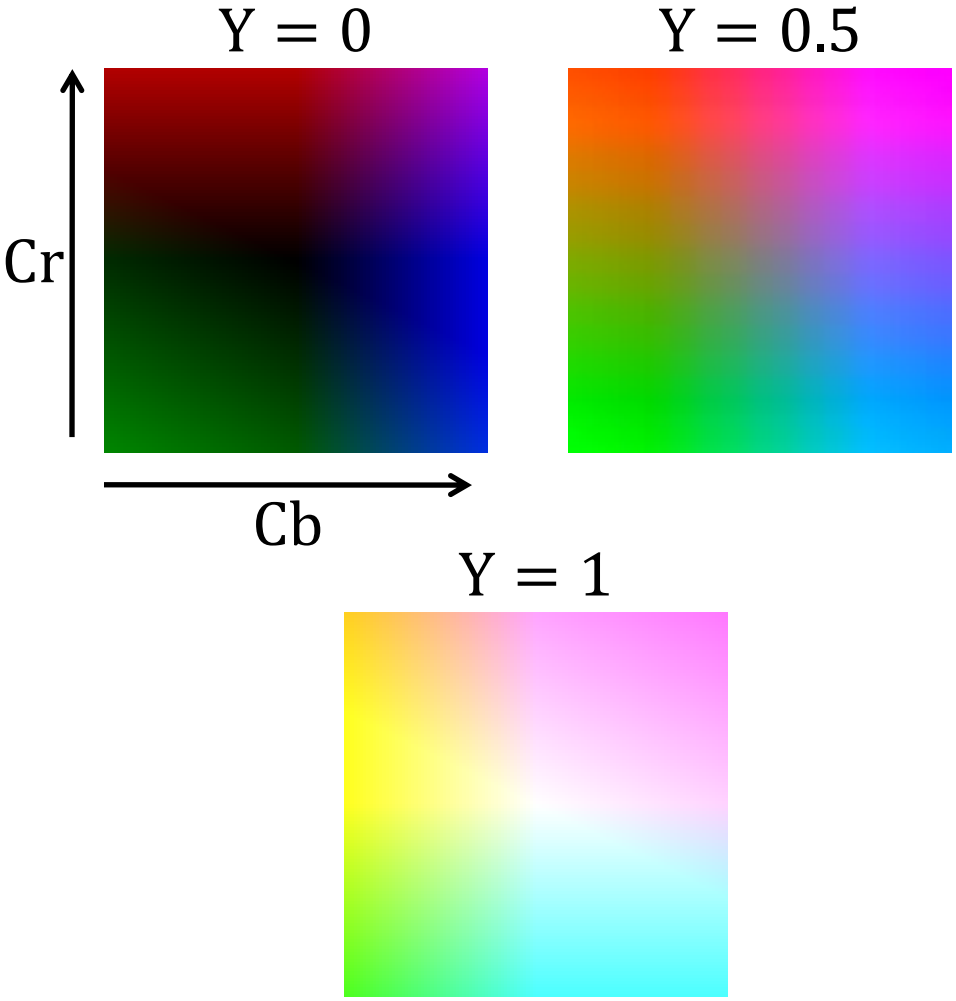
色彩空间 YCrCb



Y

$(\mathbf{Cr} = 0.5, \mathbf{Cb} = 0.5)$

Cb

$(\mathbf{Y} = 0.5, \mathbf{Cr} = 0.5)$

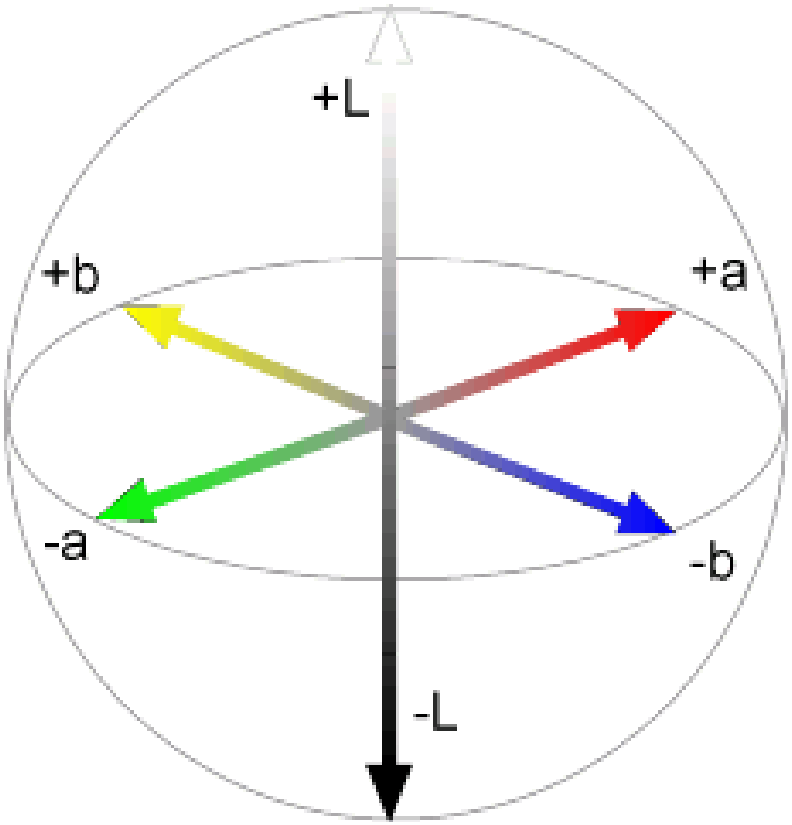
Cr

$(\mathbf{Y} = 0.5, \mathbf{Cb} = 0.5)$

Y：明流，表示光的浓度且非线性
Cr：红色浓度偏移成分
Cb：蓝色浓度偏移成分

色彩空间 CIELAB

“感知均匀”

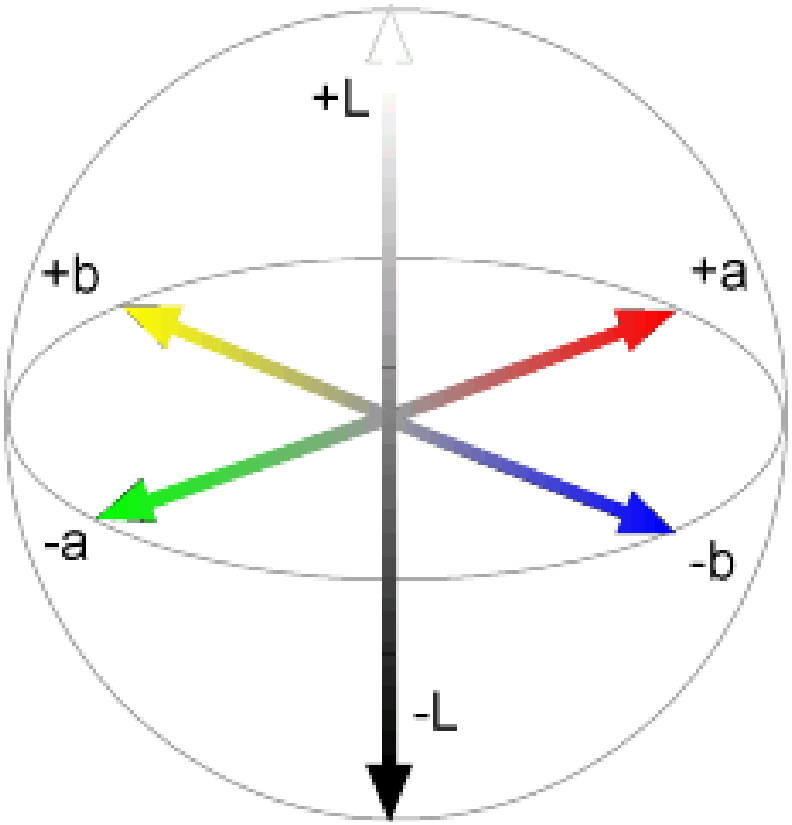


$$16\ \text{bit} \times 3 = 48\ \text{bit}$$

$$65536 \times 65536 \times 65536$$
$$\approx 281\ \text{千亿色}$$

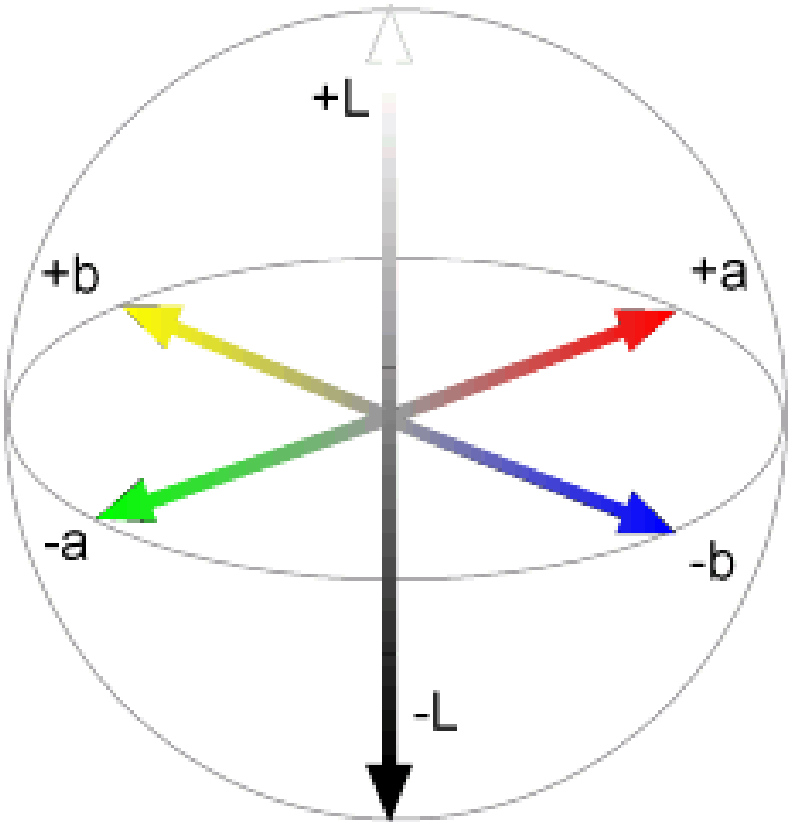
色彩空间 CIELAB

“感知均匀”



色彩空间 CIELAB

“感知均匀”



L

$(\mathbf{a} = 0, \mathbf{b} = 0)$

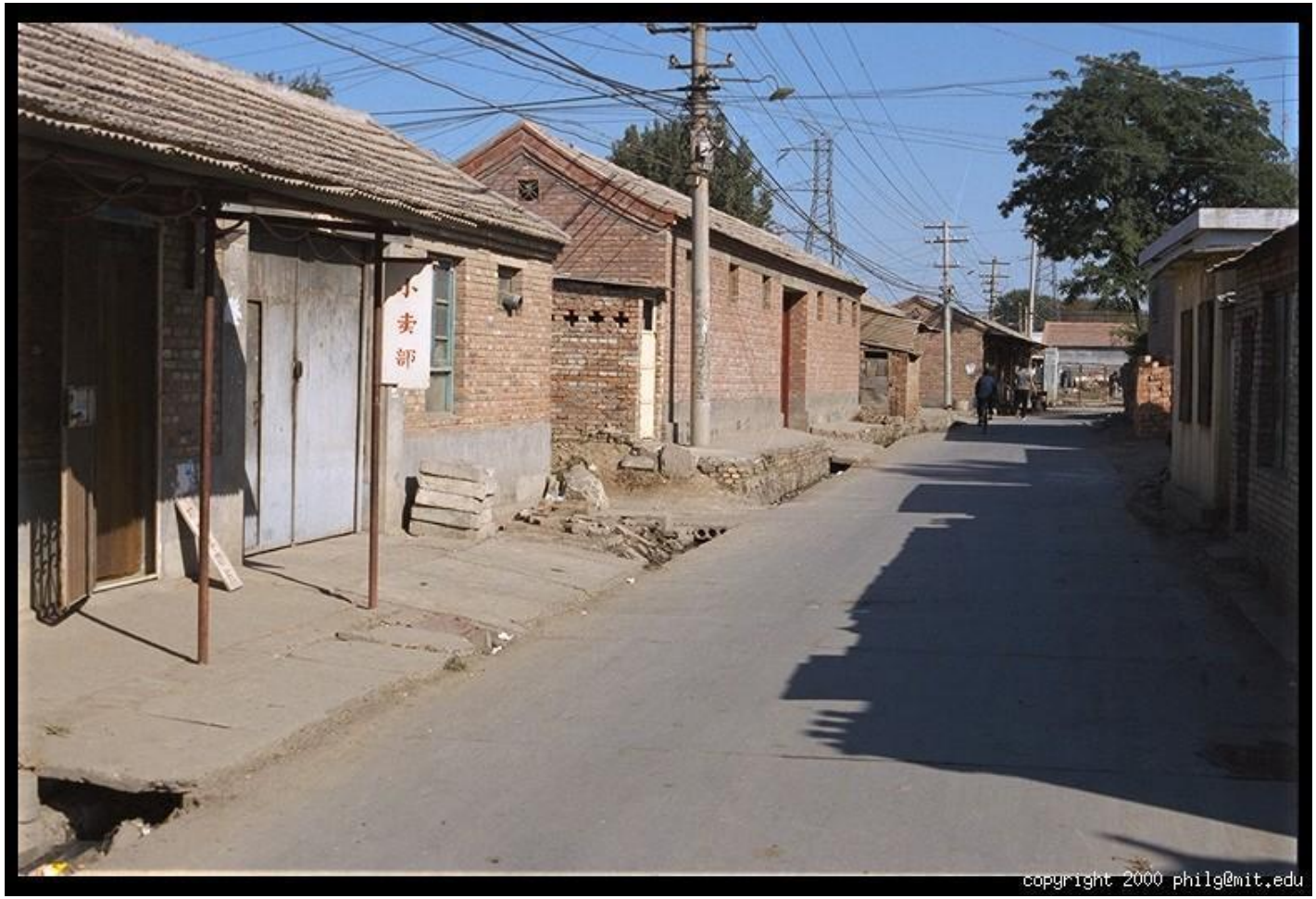
a

$(\mathbf{L} = 65, \mathbf{b} = 0)$

b

$(\mathbf{L} = 65, \mathbf{a} = 0)$

如果你不得不选择，你宁愿放弃亮度还是色度呢?



原图

仅显示颜色——恒定强度

仅显示强度——恒定色彩

因此在绝大多数计算机视觉任务中只使用灰度图

仅显示强度——恒定色彩

图像滤波

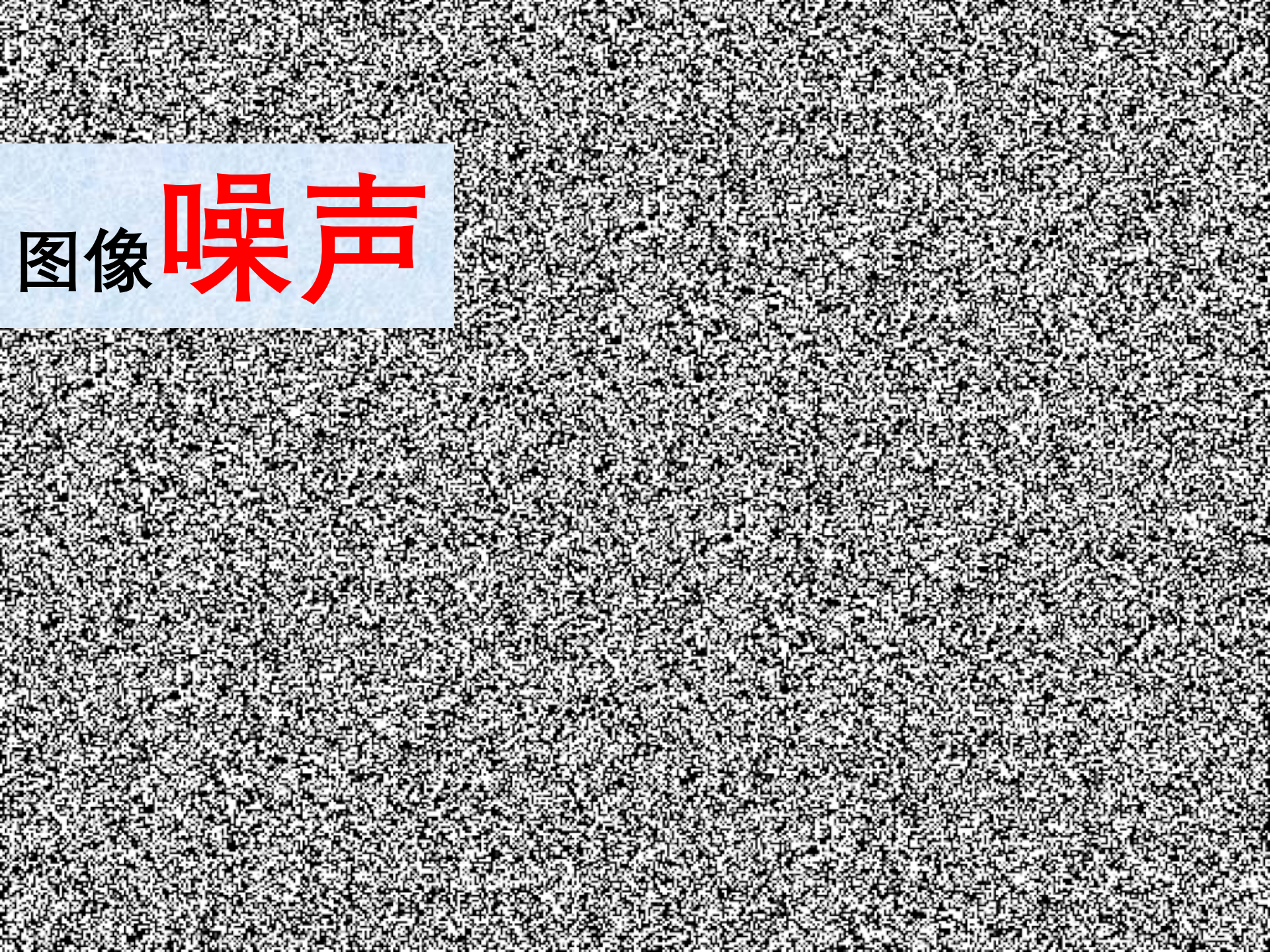
滤波器

滤波器

I

图像


滤波器
I
图像
ϕ
$\phi(I)$
图像


图像噪声


原始图像


椒盐噪声


高斯噪声

图像噪声

$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

图像噪声

$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

其中

$$\eta(x,y) \sim \mathcal{N}(\mu = 0, \sigma)$$

图像噪声

$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

其中

$$\eta(x,y) \sim \mathcal{N}(\mu = 0, \sigma)$$

假设噪声独立同分布

图像噪声

$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

其中

$$\eta(x,y) \sim \mathcal{N}(\mu = 0, \sigma)$$

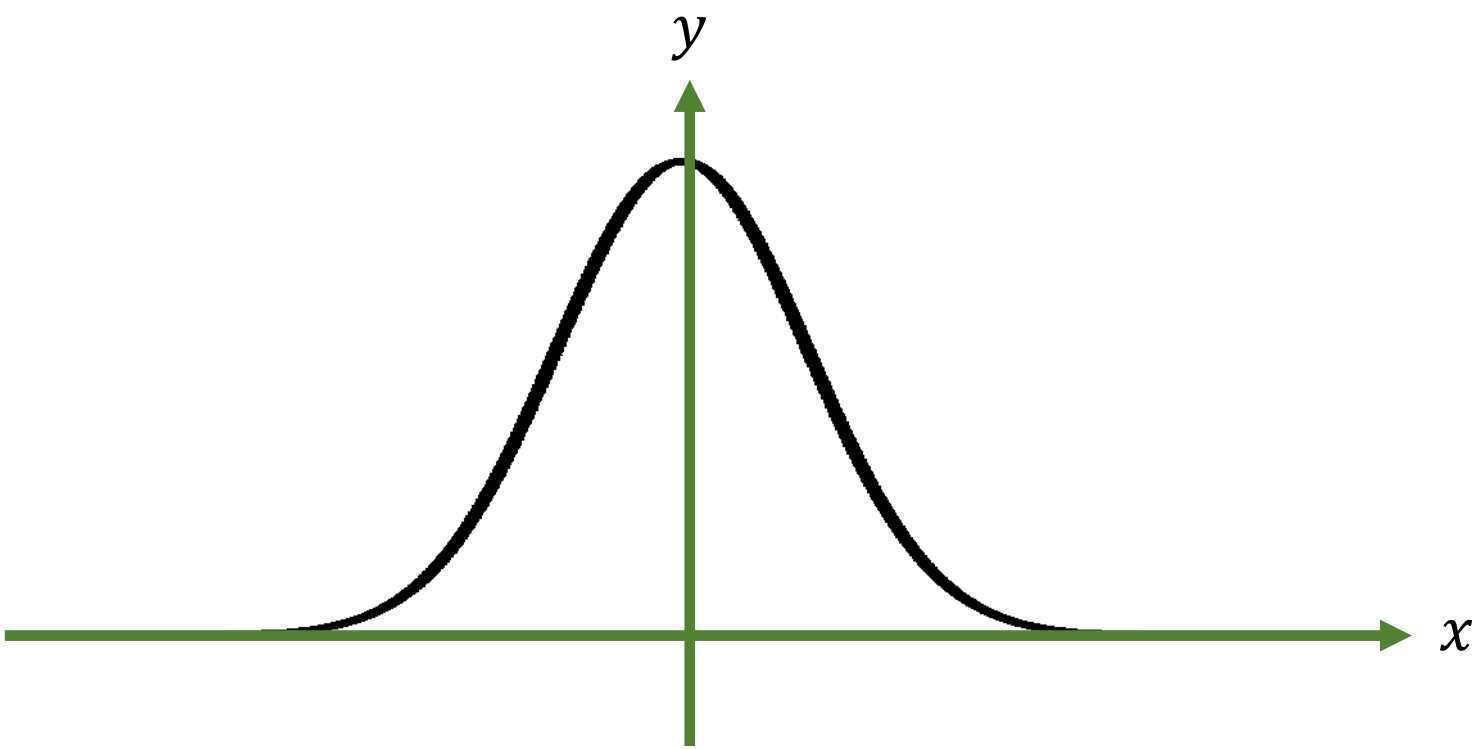
假设噪声独立同分布

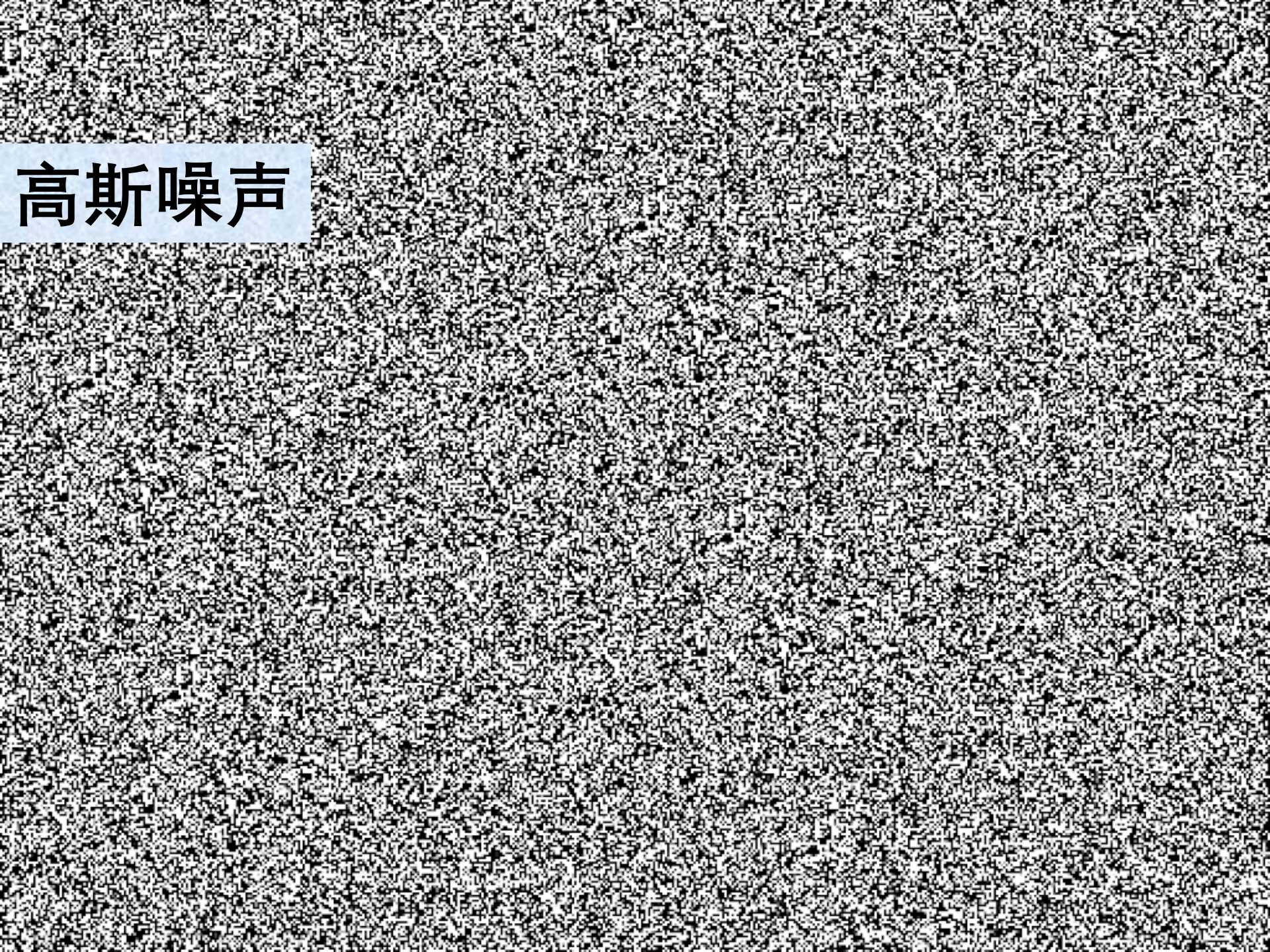
I.I.D

$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

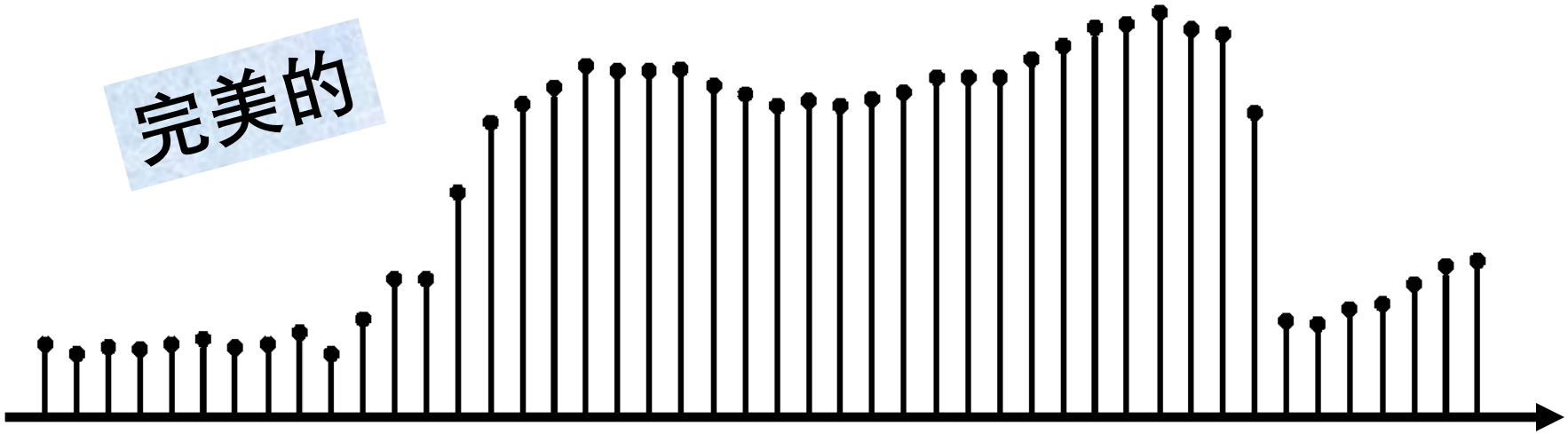
其中

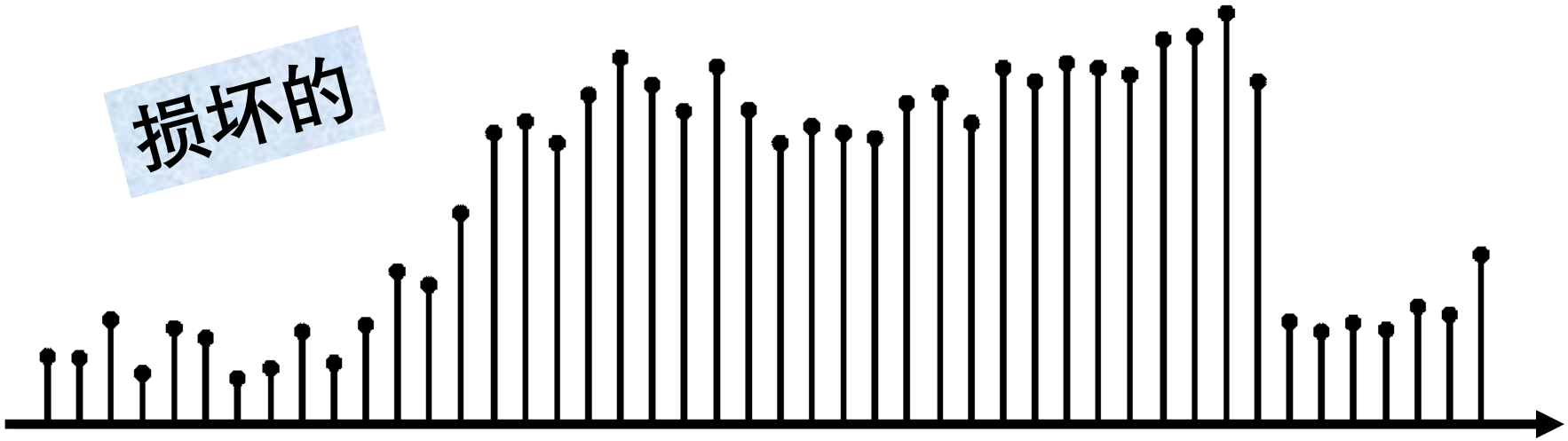
$$\eta(x,y) \sim \mathcal{N}(\mu = 0, \sigma)$$

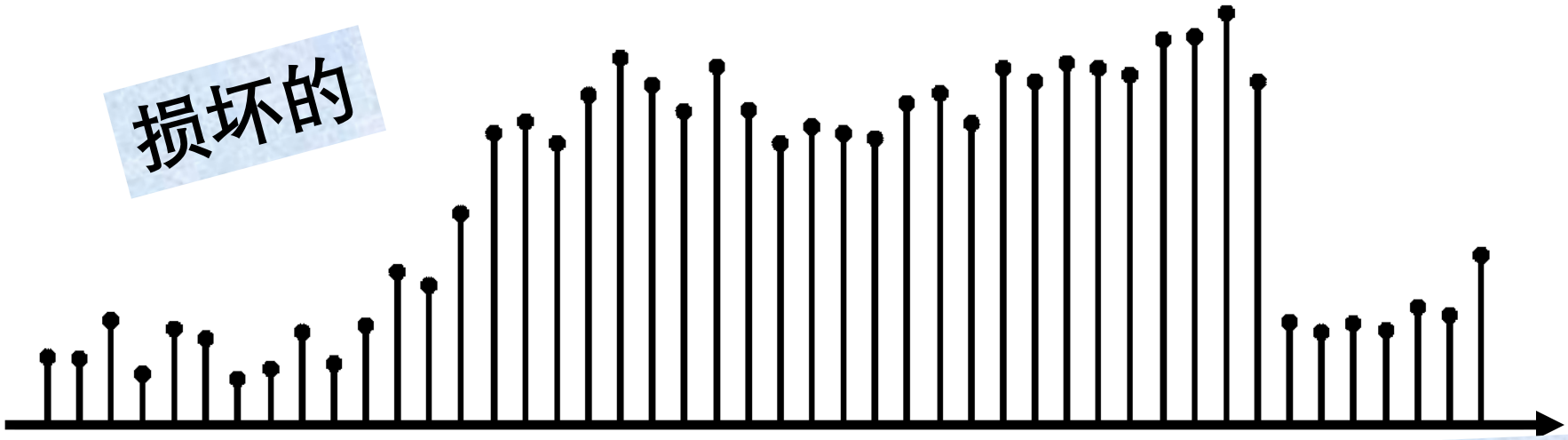



高斯噪声

完美的

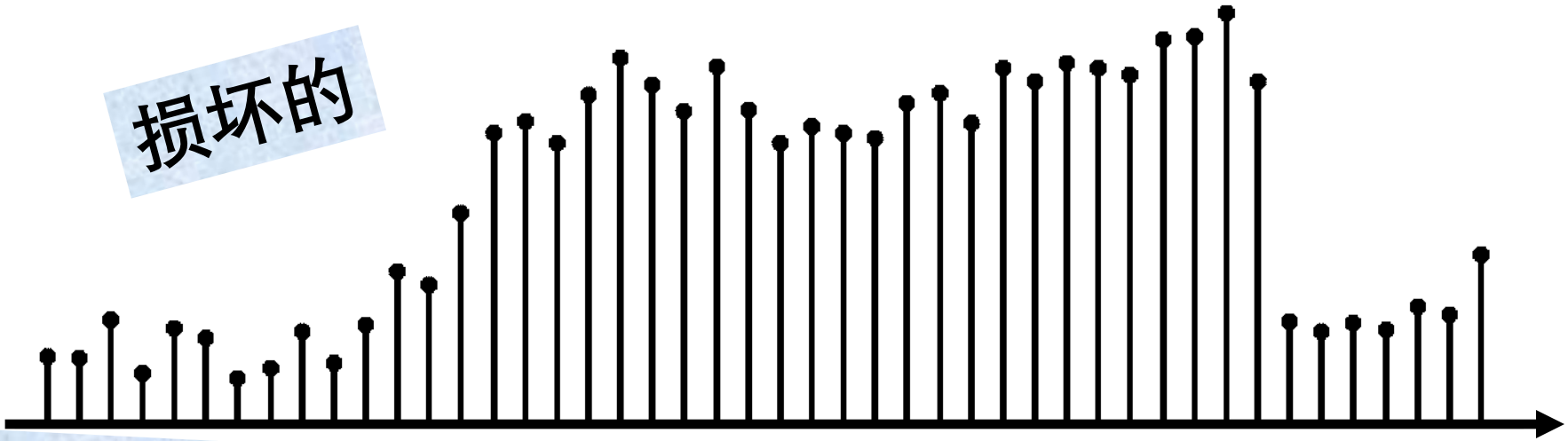



损坏的

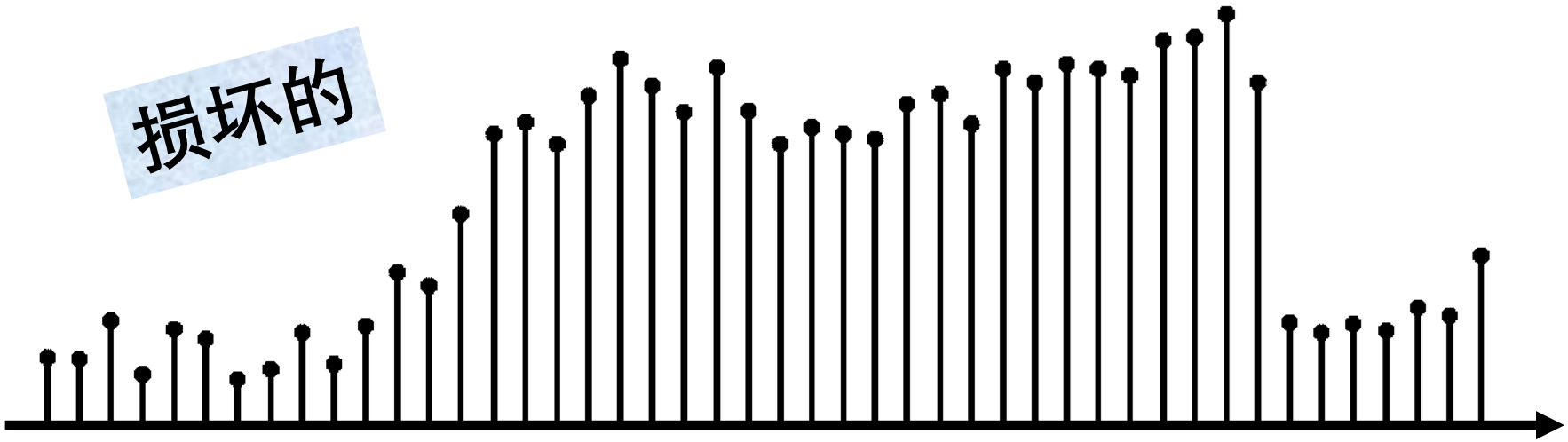


怎样才能消除噪声？

损坏的



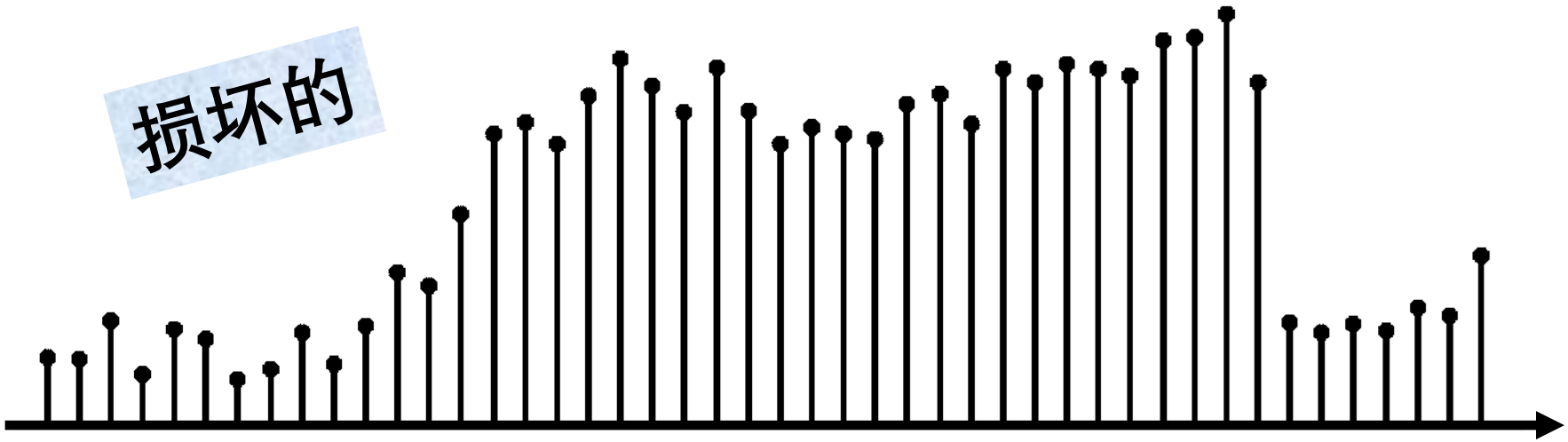
假设噪声是I.I.D且相邻像素是相似的



$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

其中

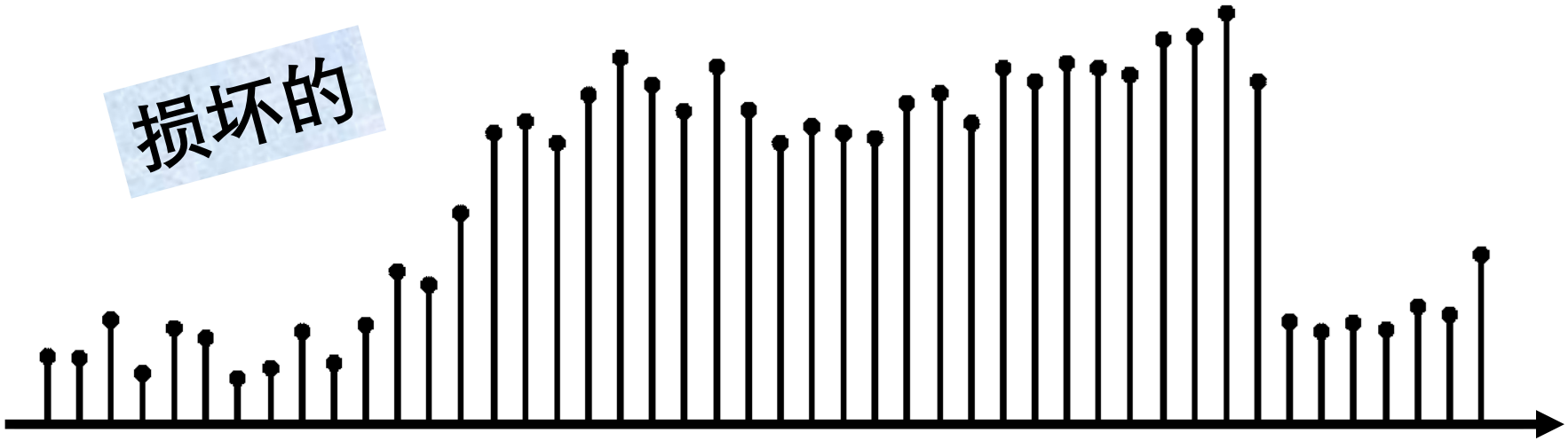
$$\eta(x,y) \sim \mathcal{N}(\mu = 0, \sigma)$$



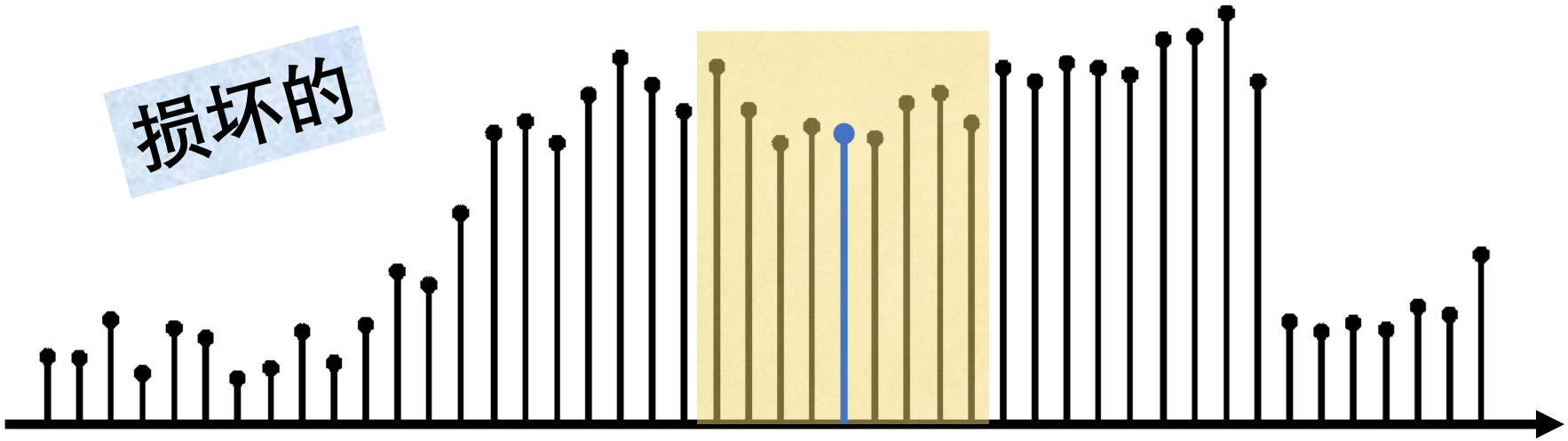
$$I(x,y) = I_{\text{ideal}}(x,y) + \eta(x,y)$$

其中

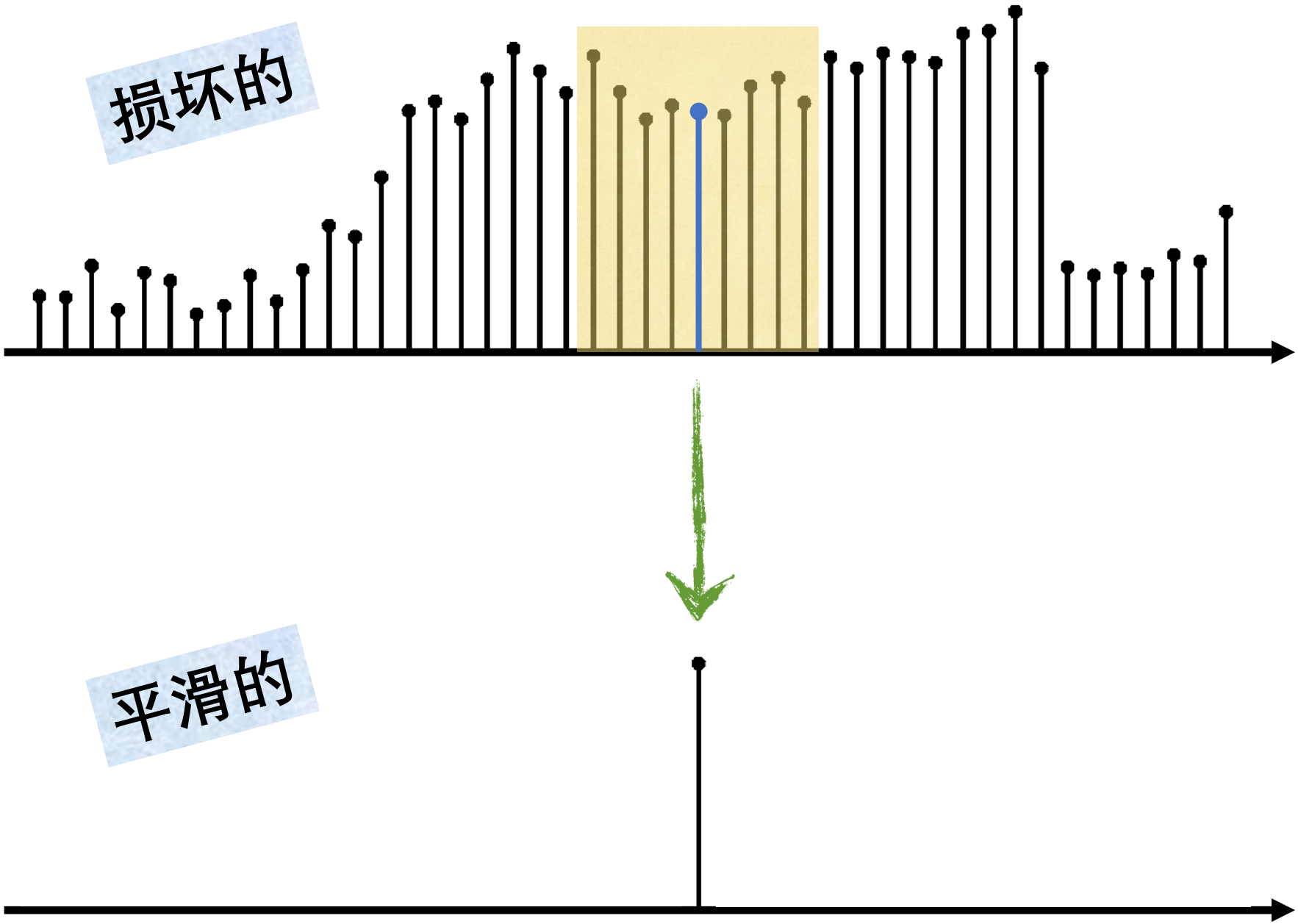
$$\eta(x,y) \sim \mathcal{N}(\mu = 0, \sigma)$$

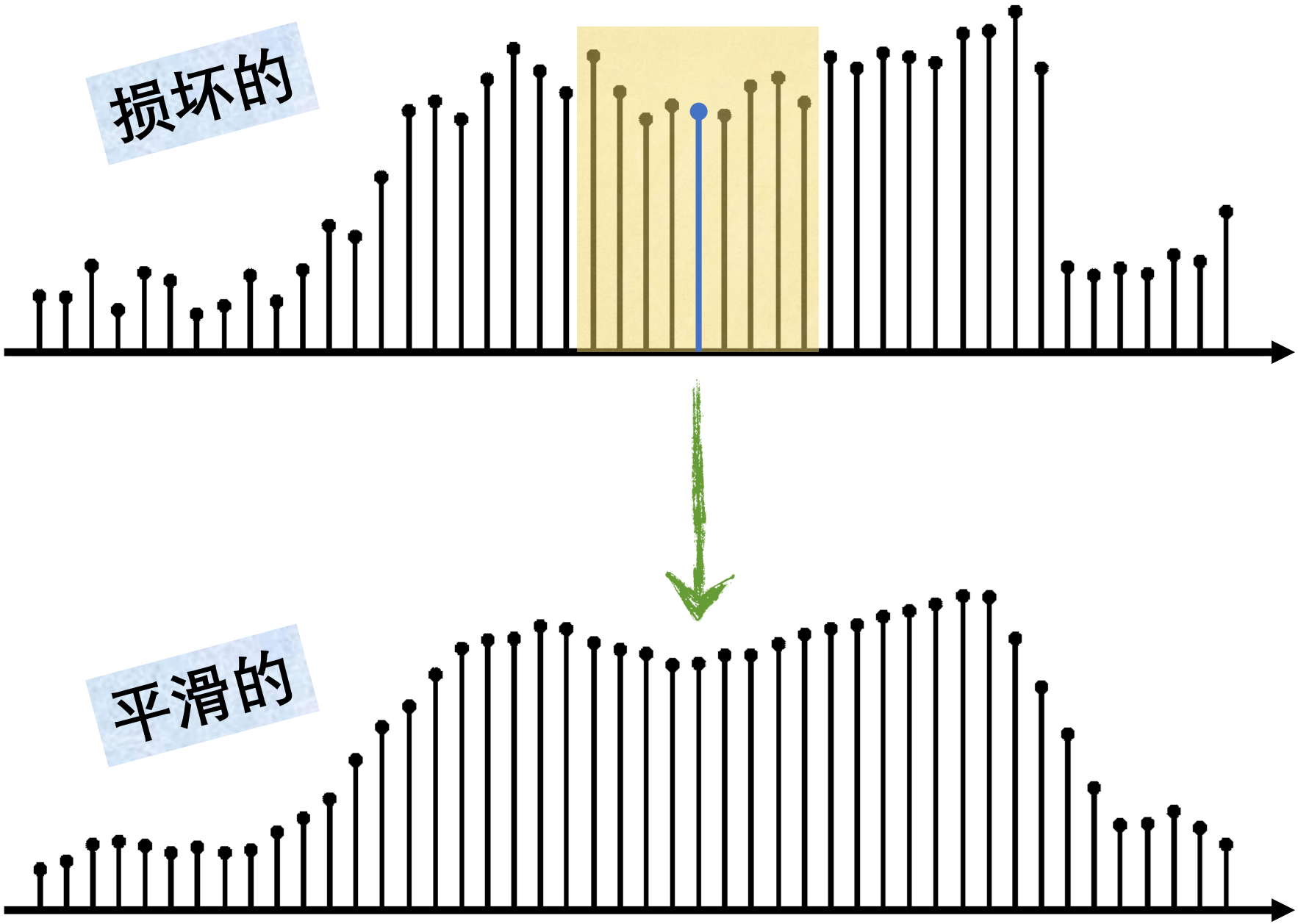


让我们将每个像素替换为其相邻像素的平均值

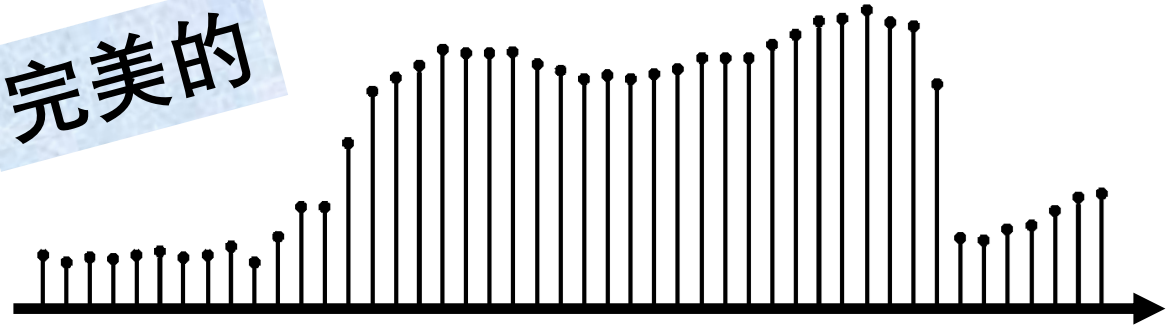


让我们将每个像素替换为其相邻像素的平均值

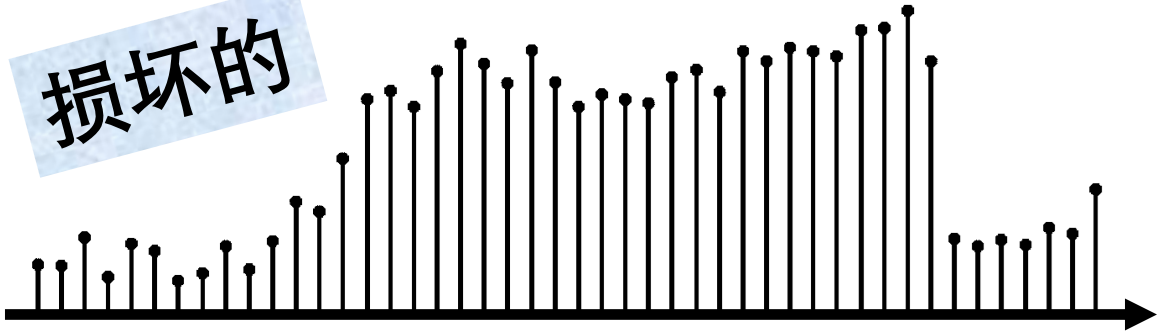

损坏的
平滑的


损坏的
平滑的

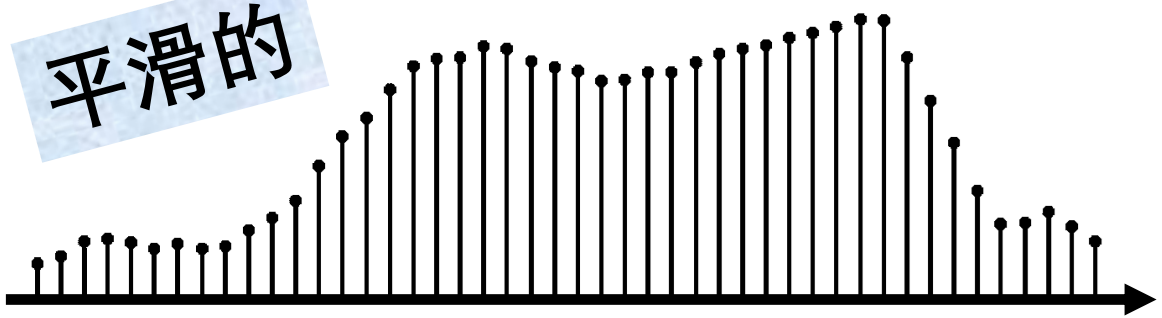
完美的



损坏的



平滑的

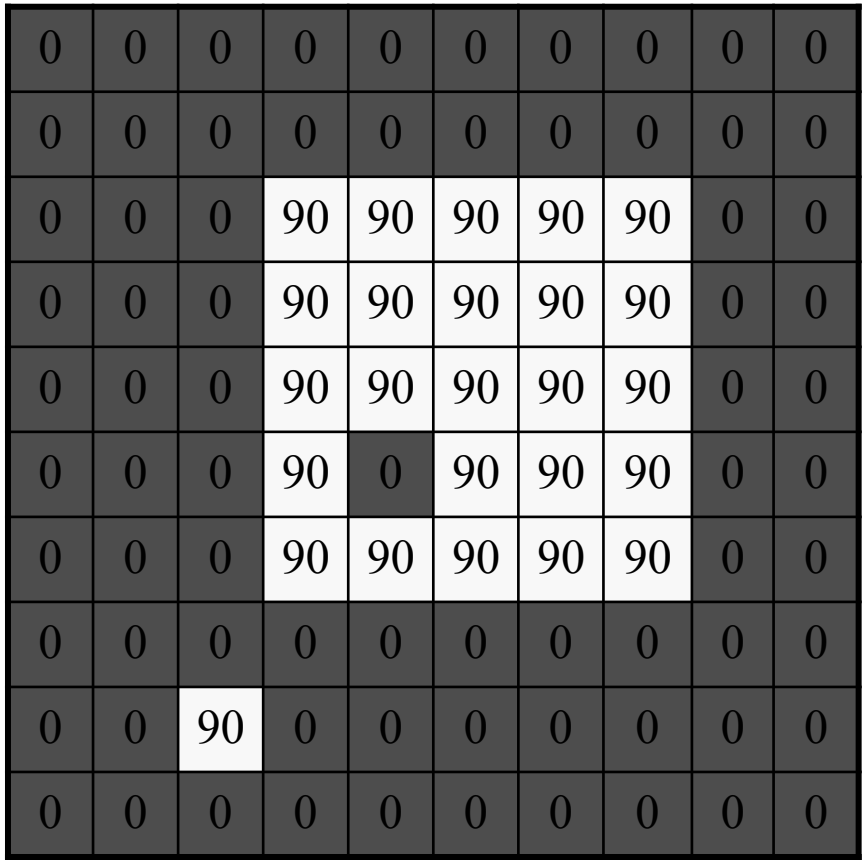


2D 滑动平均

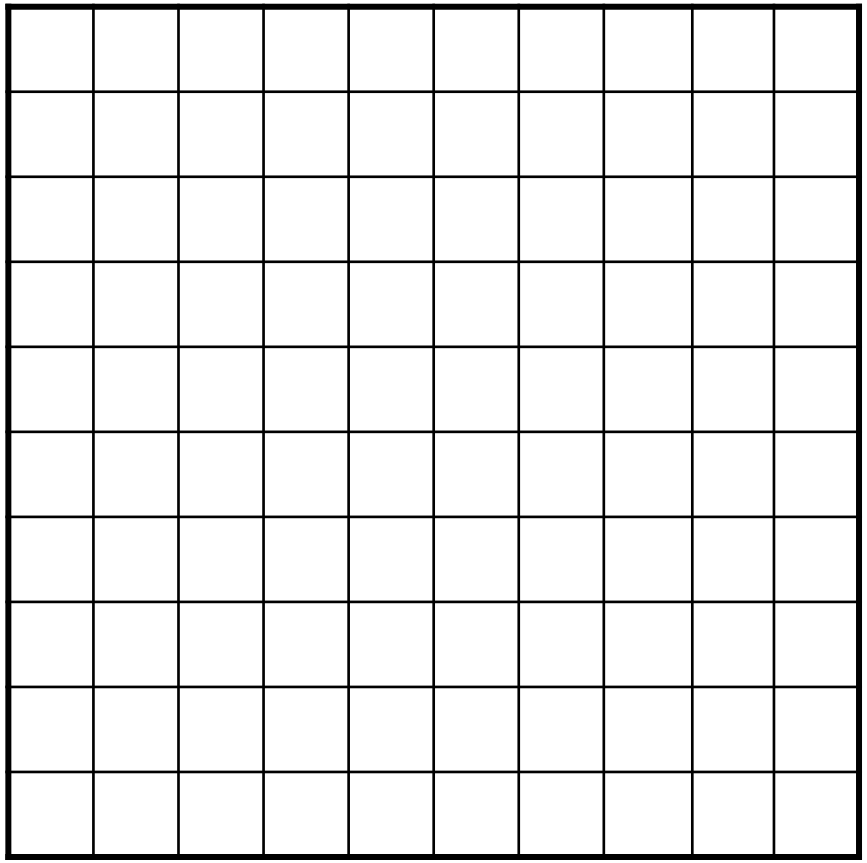
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

输入

2D 滑动平均

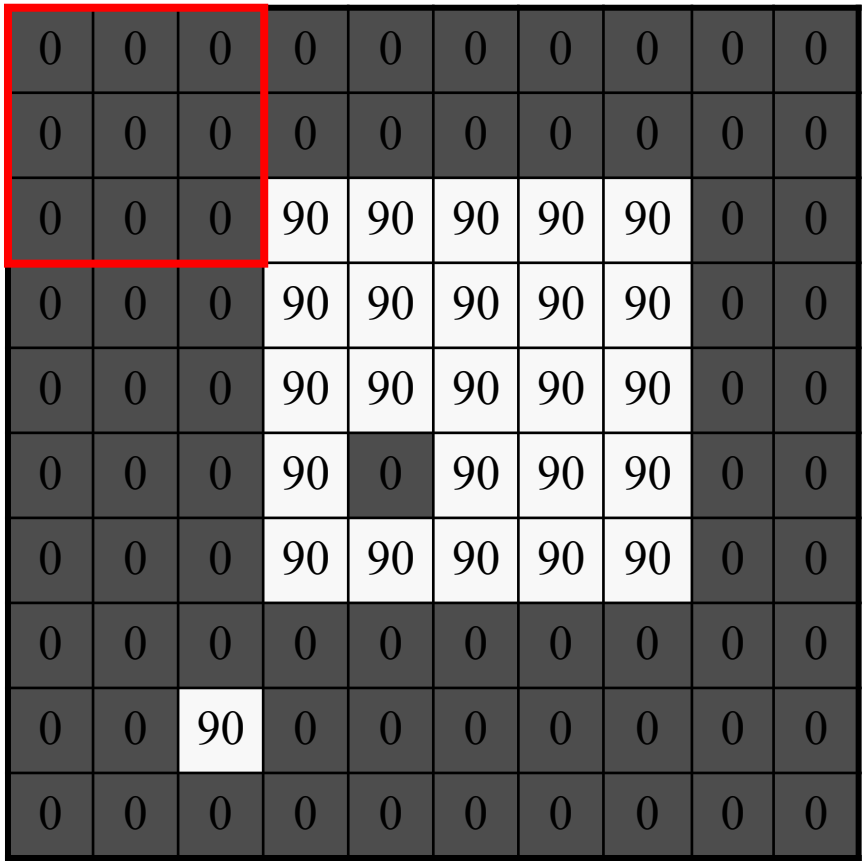


输入

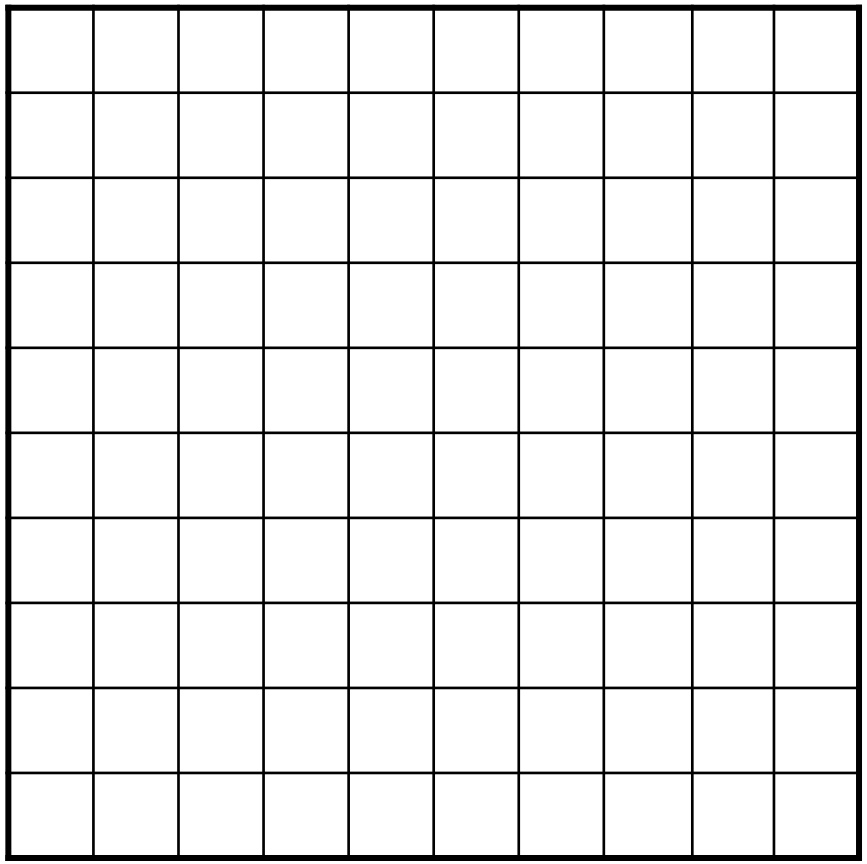


平滑的

2D 滑动平均

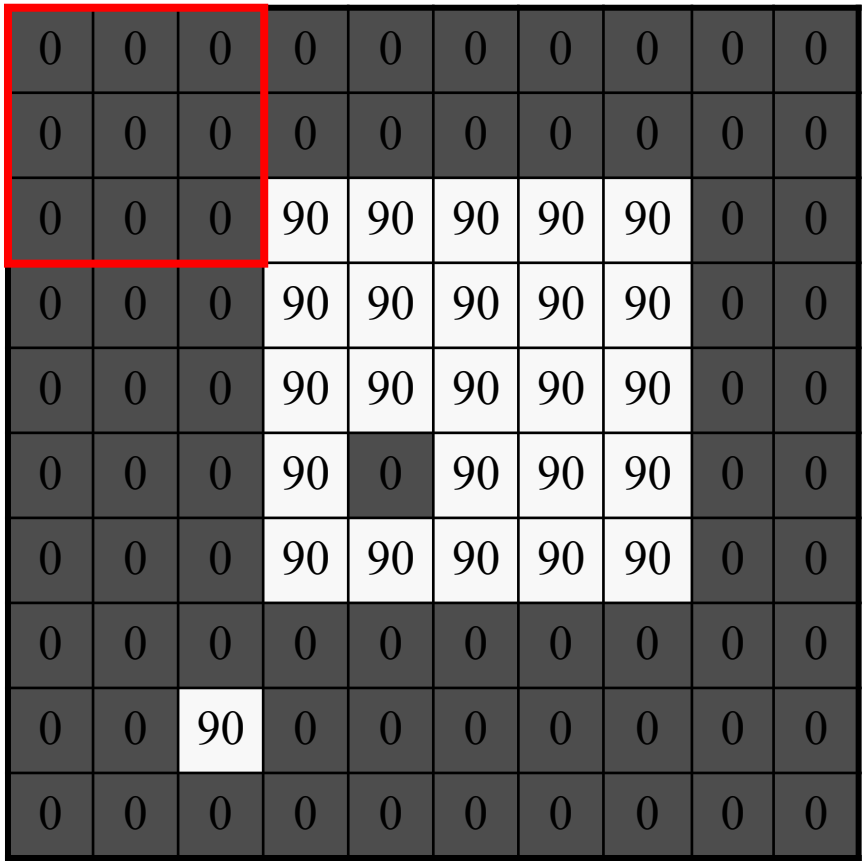


输入

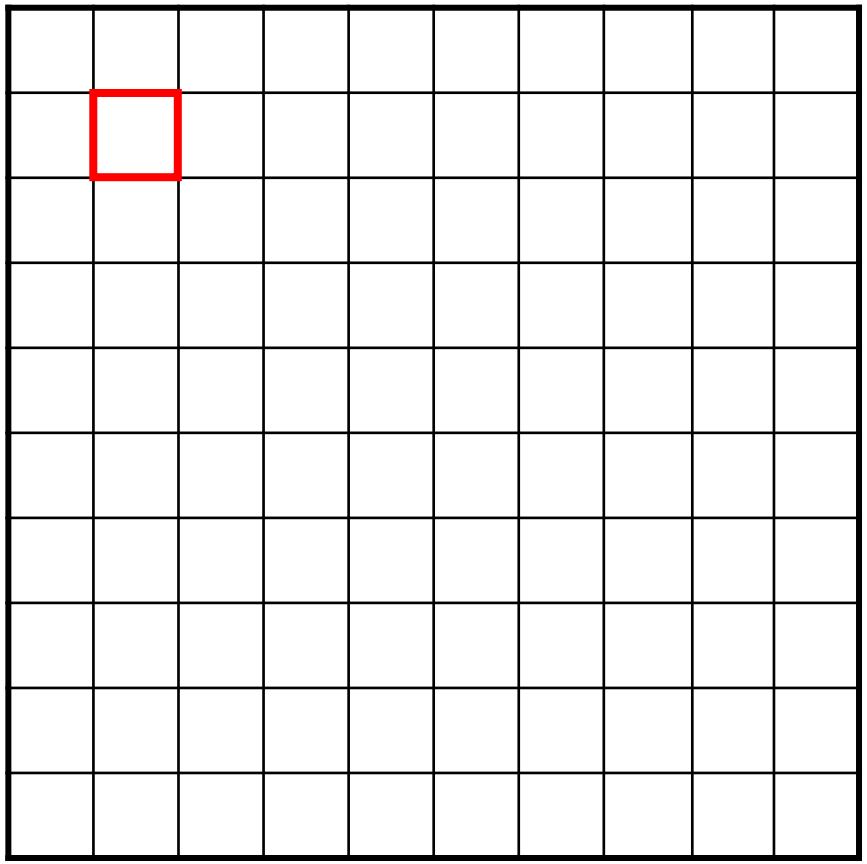


平滑的

2D 滑动平均

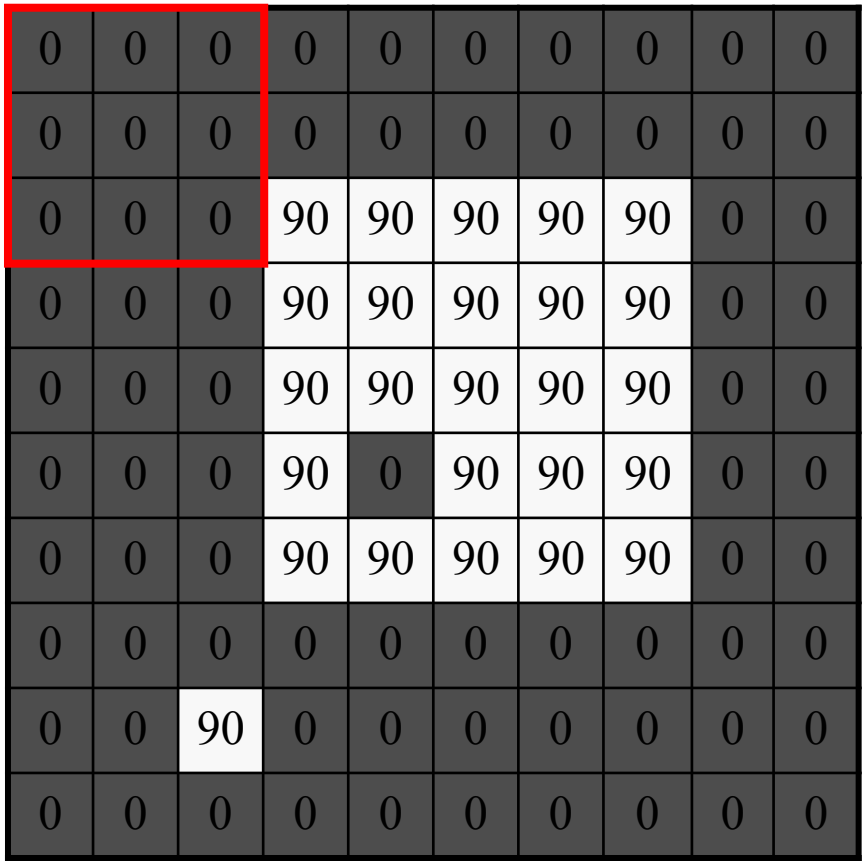


输入

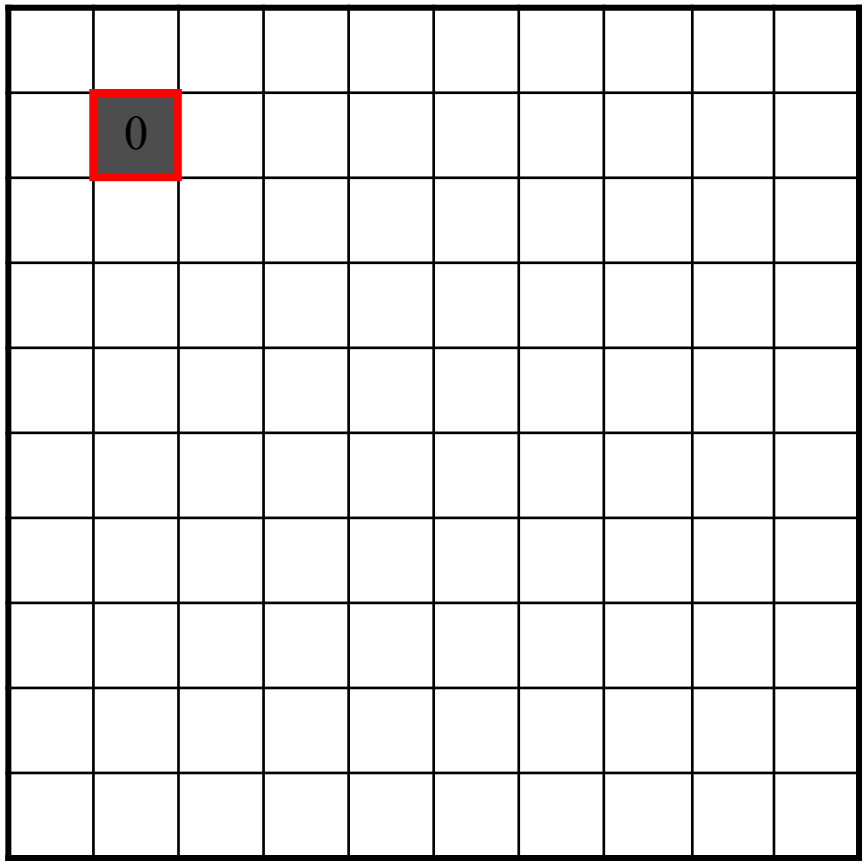


平滑的

2D 滑动平均

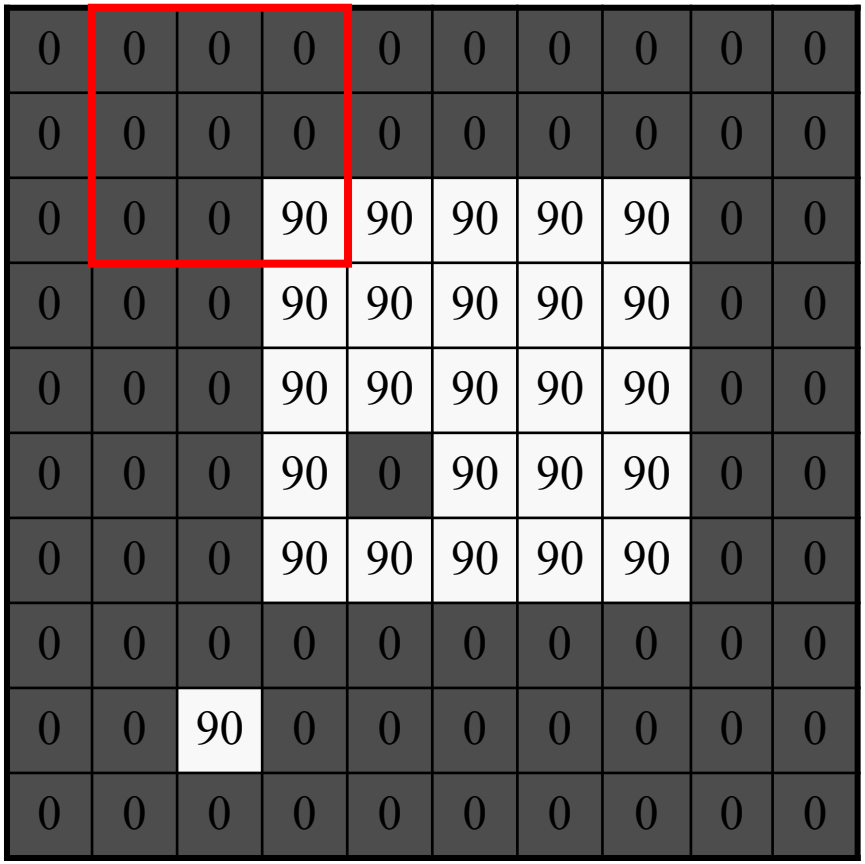


输入



平滑的

2D 滑动平均



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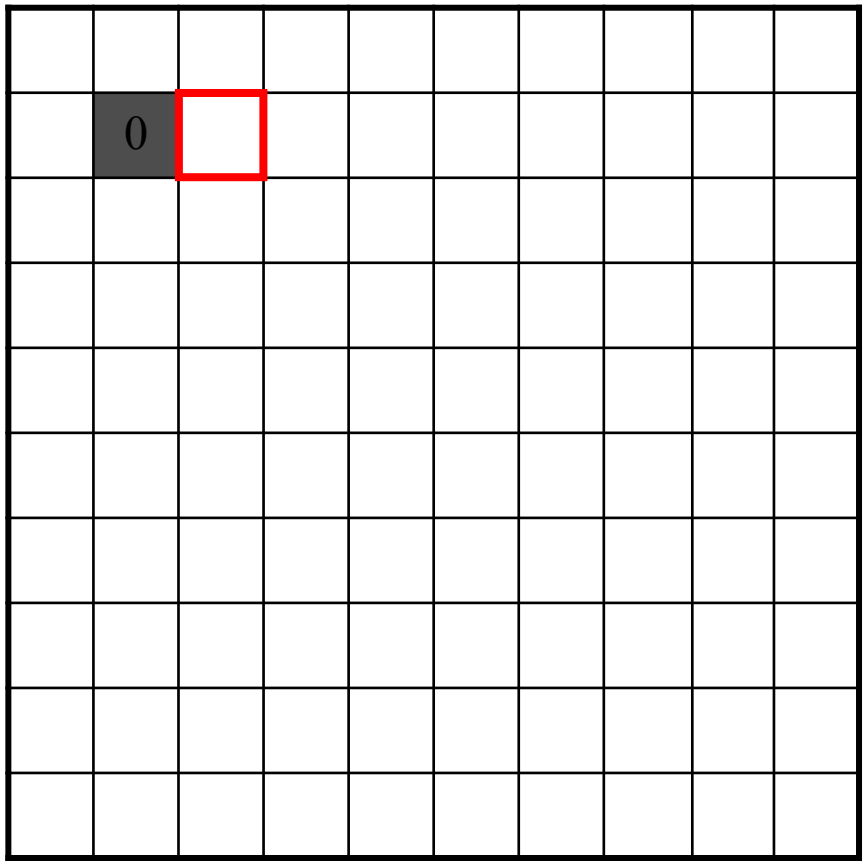
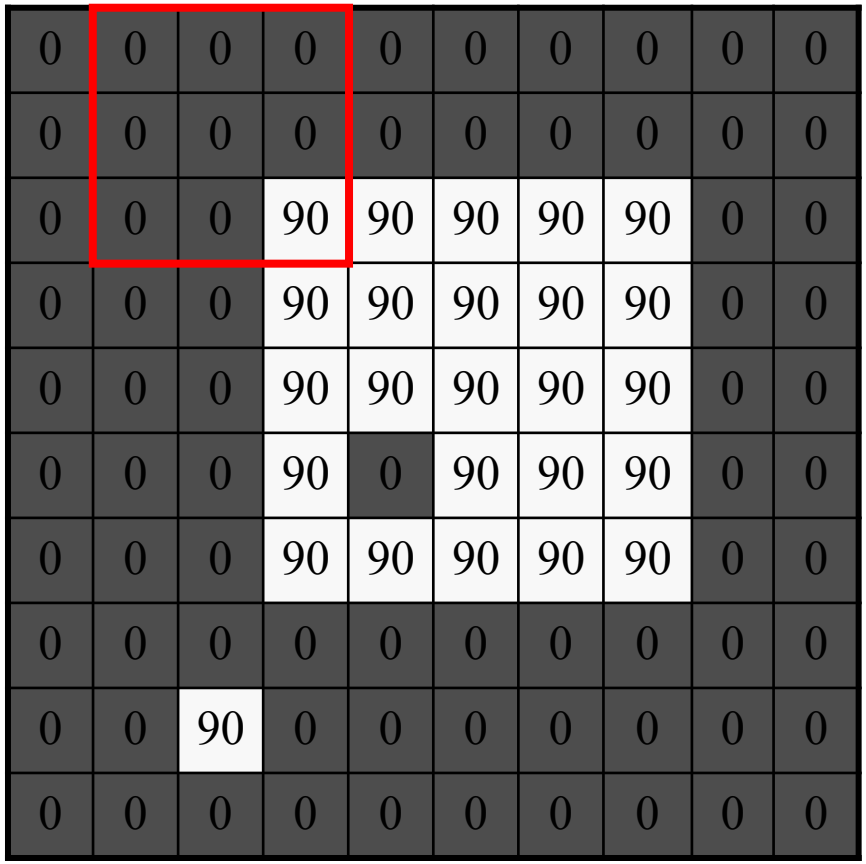


平滑的

2D 滑动平均



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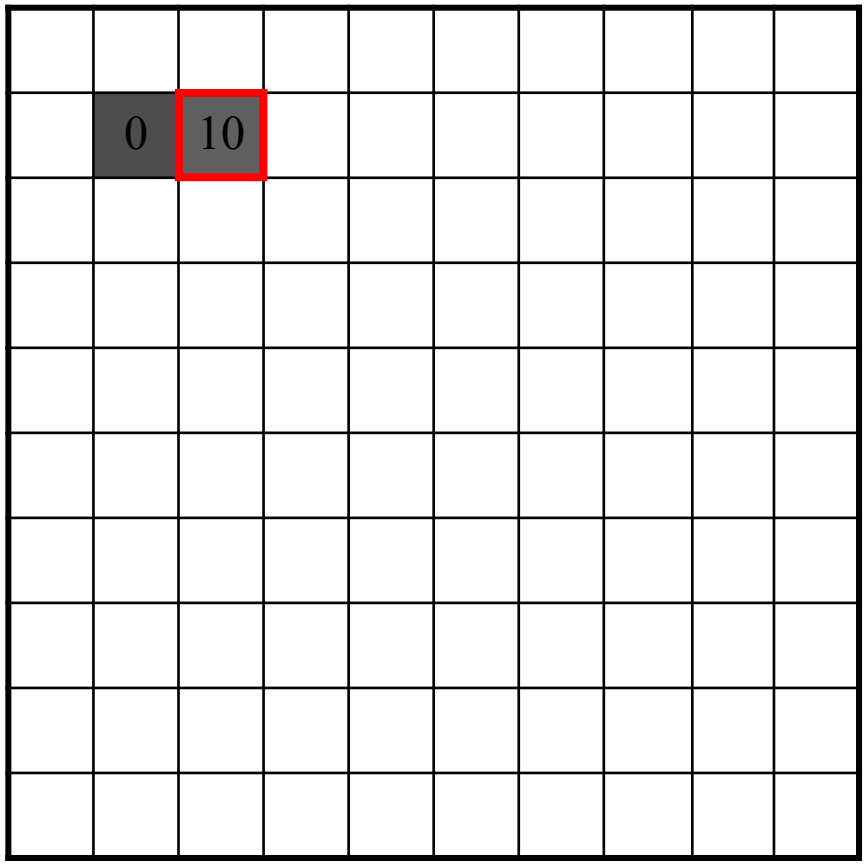
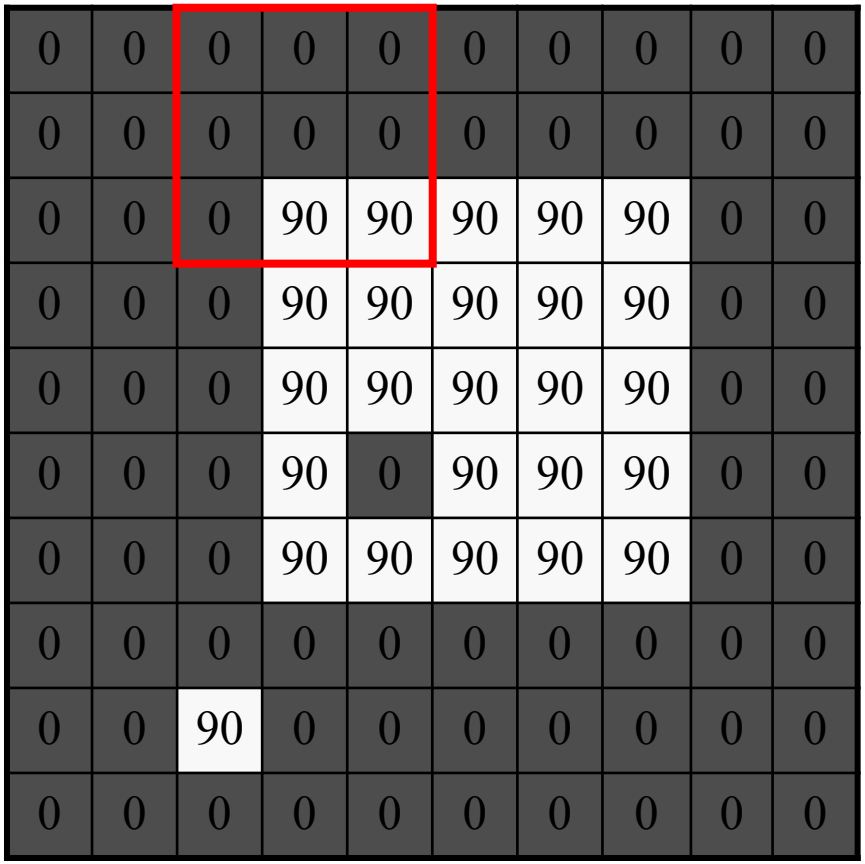
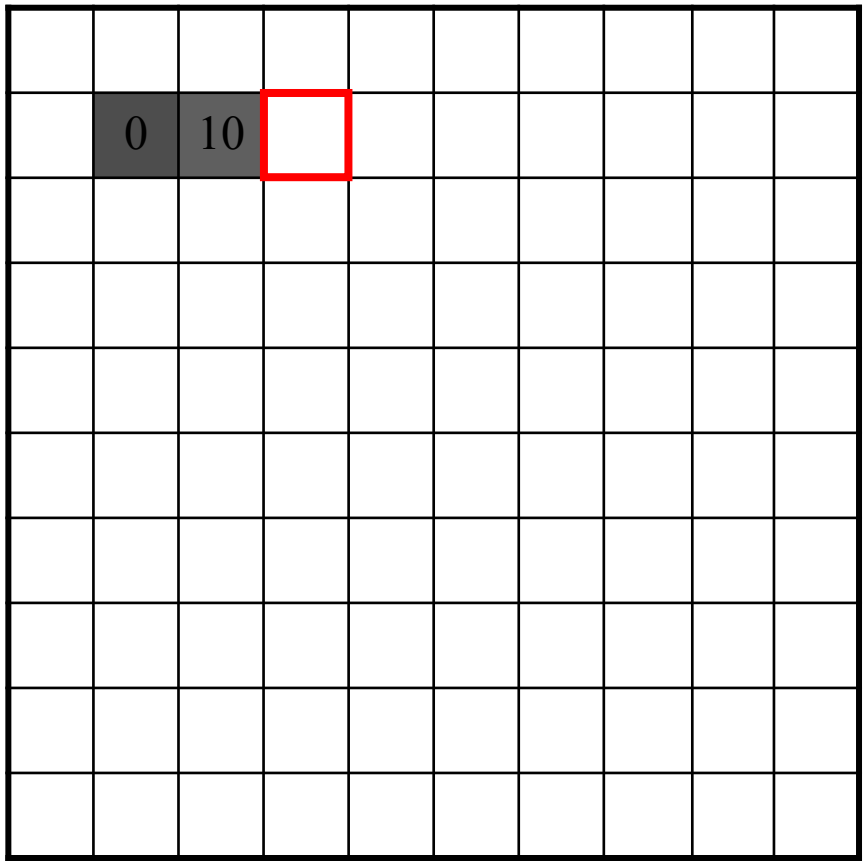


平滑的

2D 滑动平均

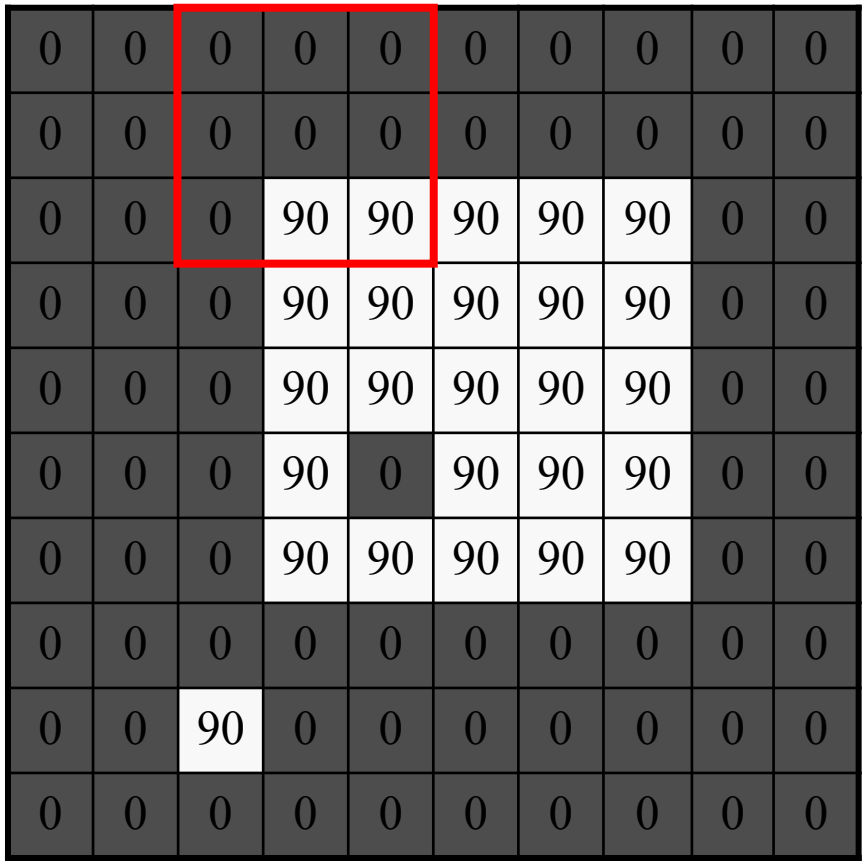


输入

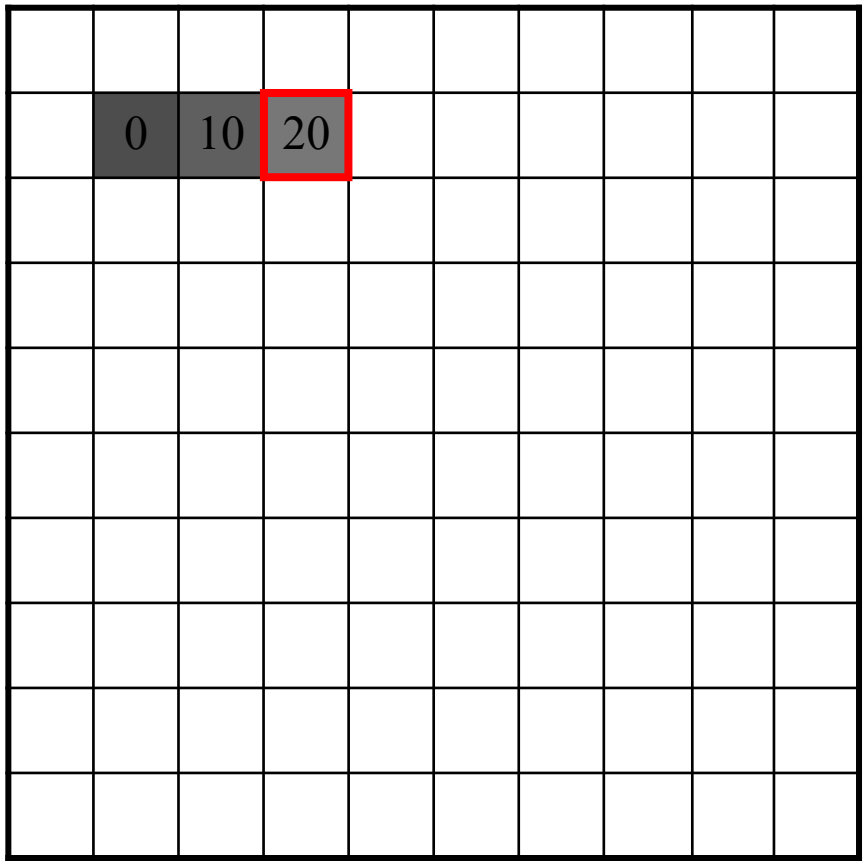


平滑的

2D 滑动平均

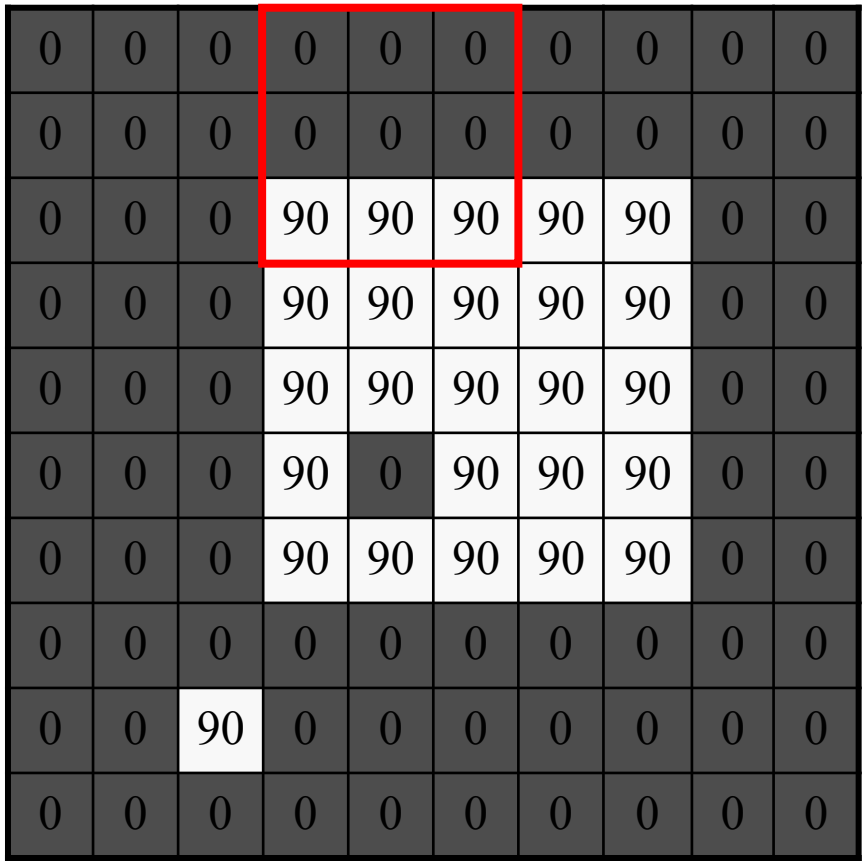


输入

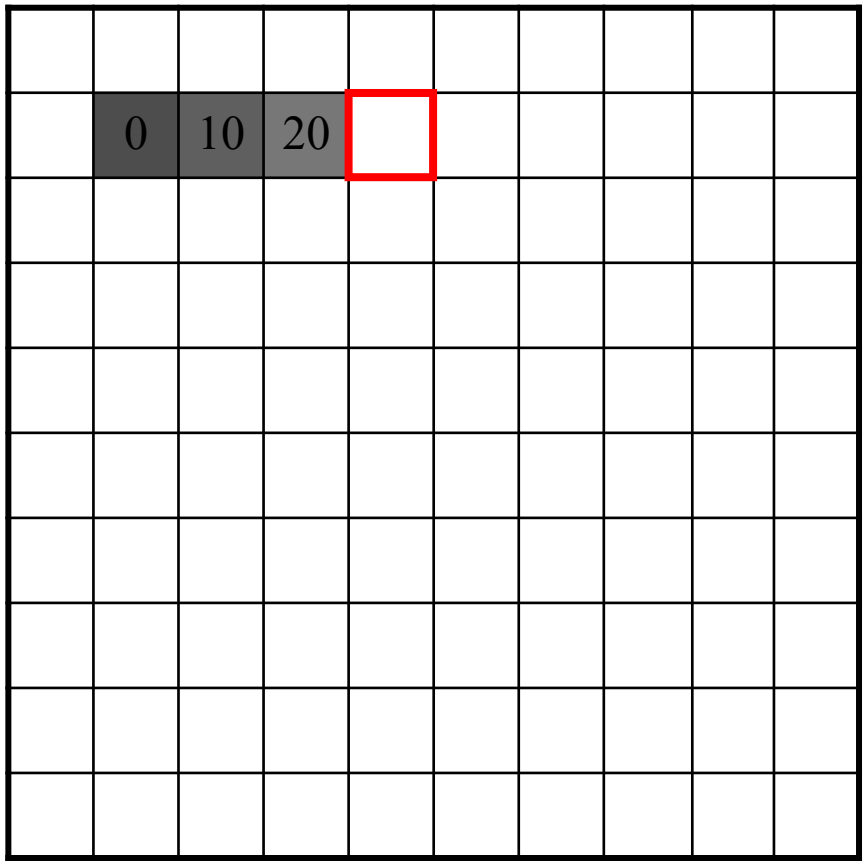


平滑的

2D 滑动平均

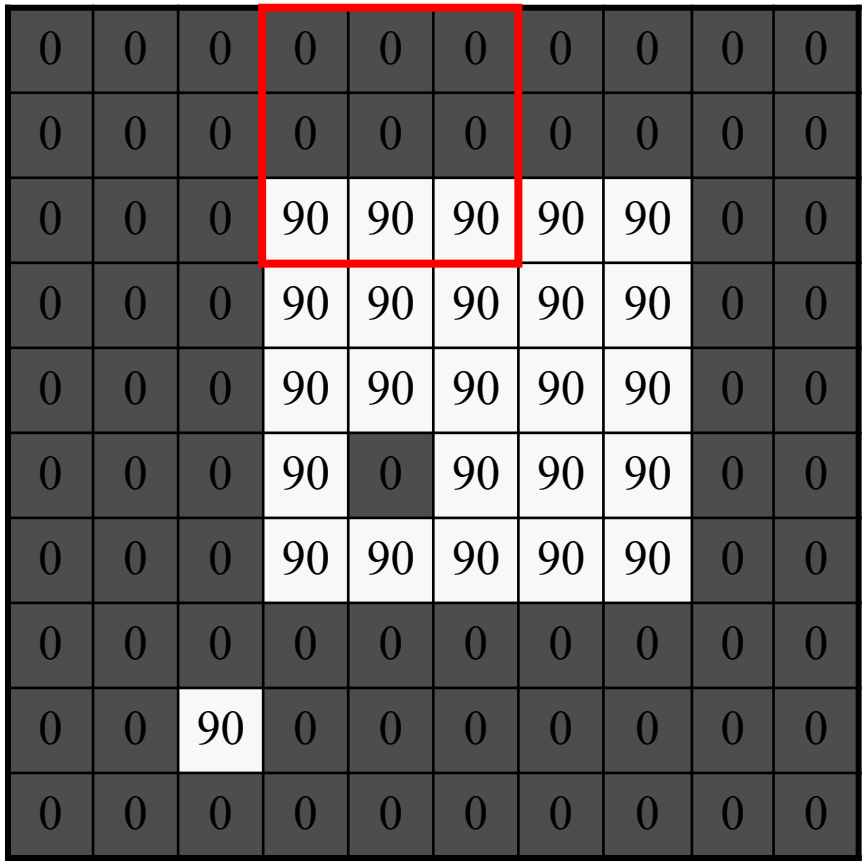


输入

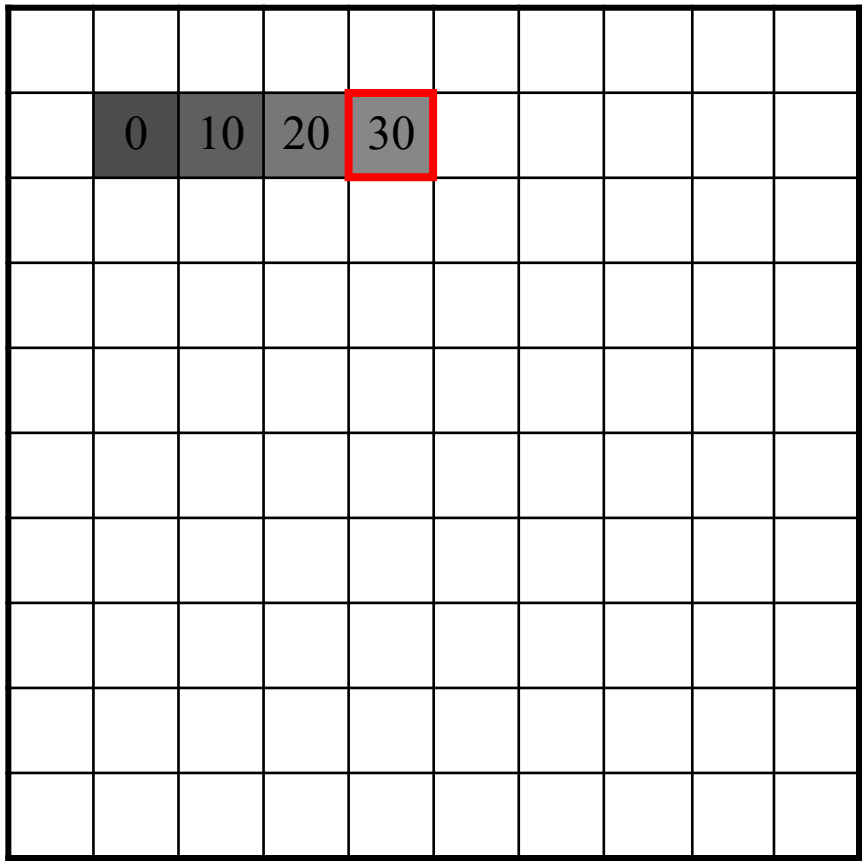


平滑的

2D 滑动平均

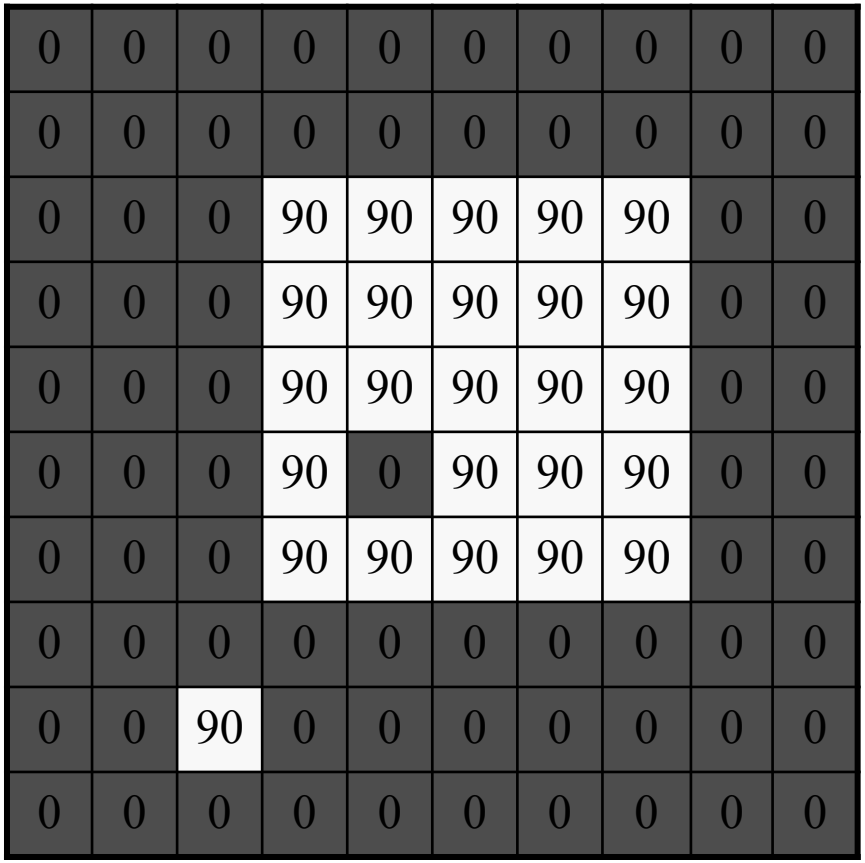


输入



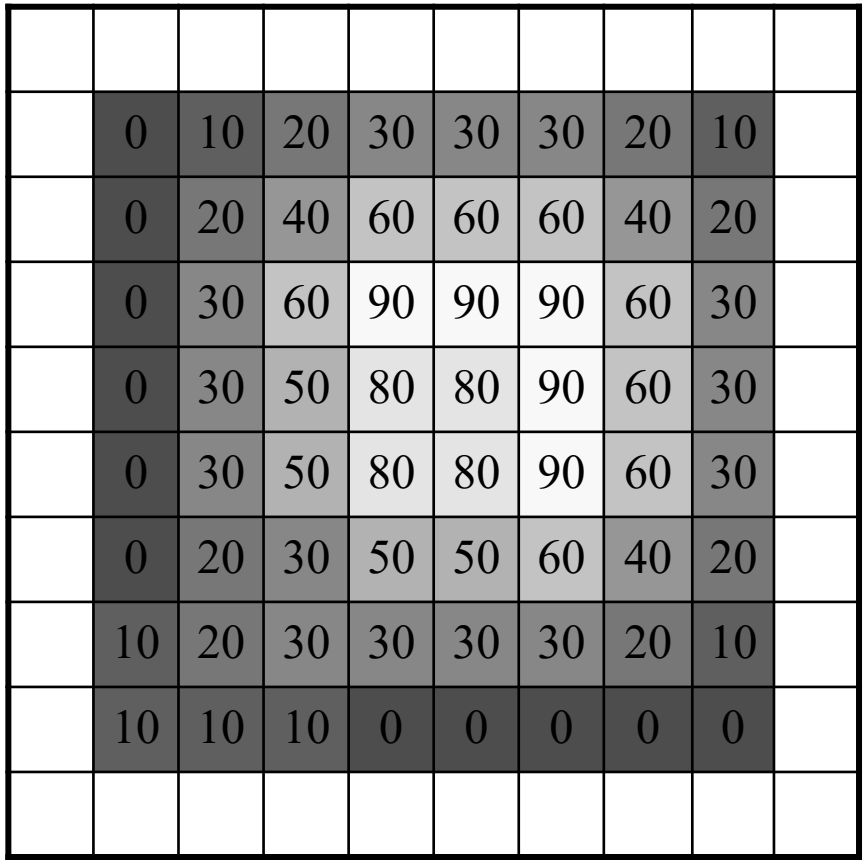
平滑的

2D 滑动平均



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

输入



	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

平滑的

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\frac{1}{(2K+1)^2}\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]$$

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\frac{1}{(2K+1)^2}\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]$$

输出

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\frac{1}{(2K+1)^2}\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]$$

输入

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\frac{1}{(2K+1)^2}\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]$$

在邻域内像素中循环

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\frac{1}{(2K+1)^2}\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]$$

权重

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\frac{1}{(2K+1)^2}\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]$$

如何根据像素在邻域内的位置来设置不同的权重？

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]G[u,v]$$

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]G[u,v]$$

掩膜 (mask)、核函数 (kernel) 或滤波器 (filter)

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]G[u,v]$$

该式叫作互相关，记作$H=F\otimes G$

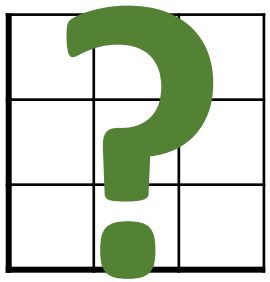
令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]G[u,v]$$

该式叫作互相关，记作$H=F\otimes G$

将每个像素替换成其相邻像素的线性组合

均值滤波



$G[u,v]$

$\otimes$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x,y]$

均值滤波

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$G[u,v]$

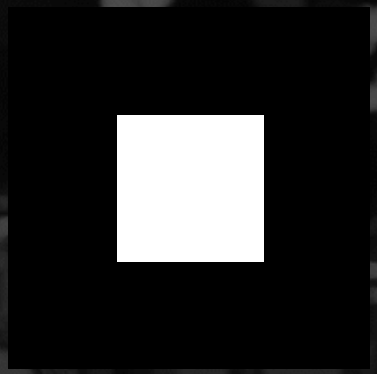
$\otimes$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x,y]$

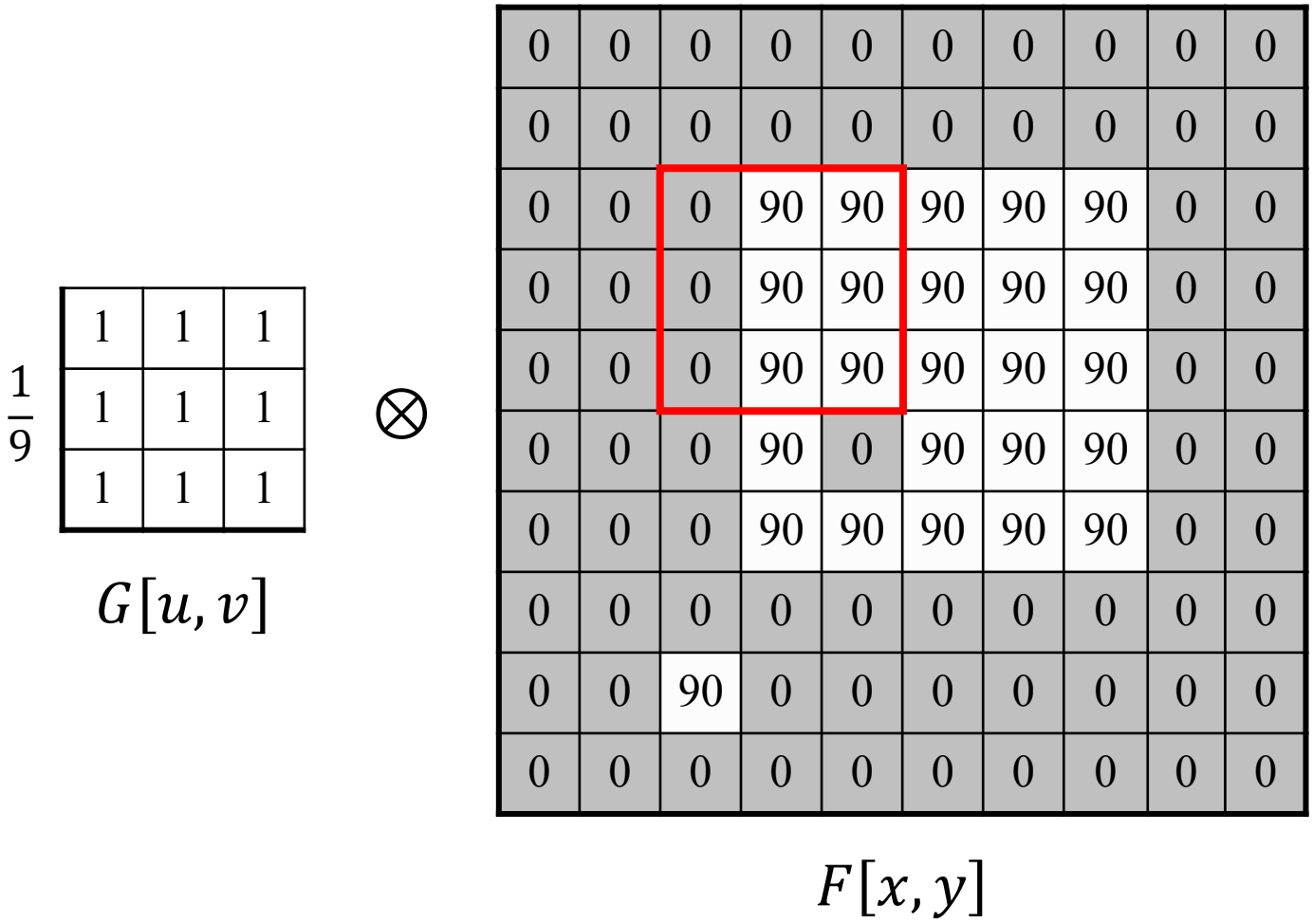

原始图像

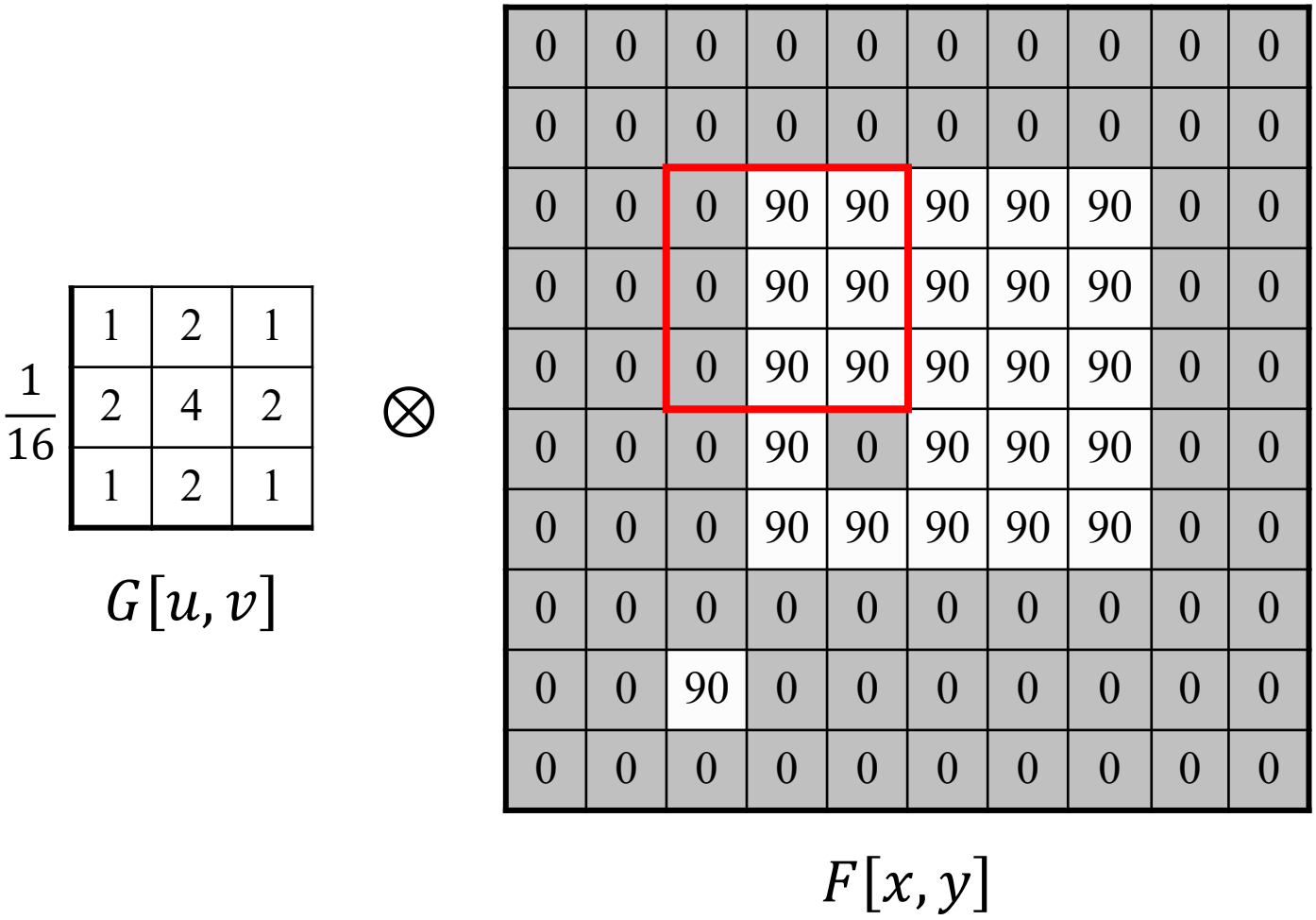
原始图像



方框滤波器

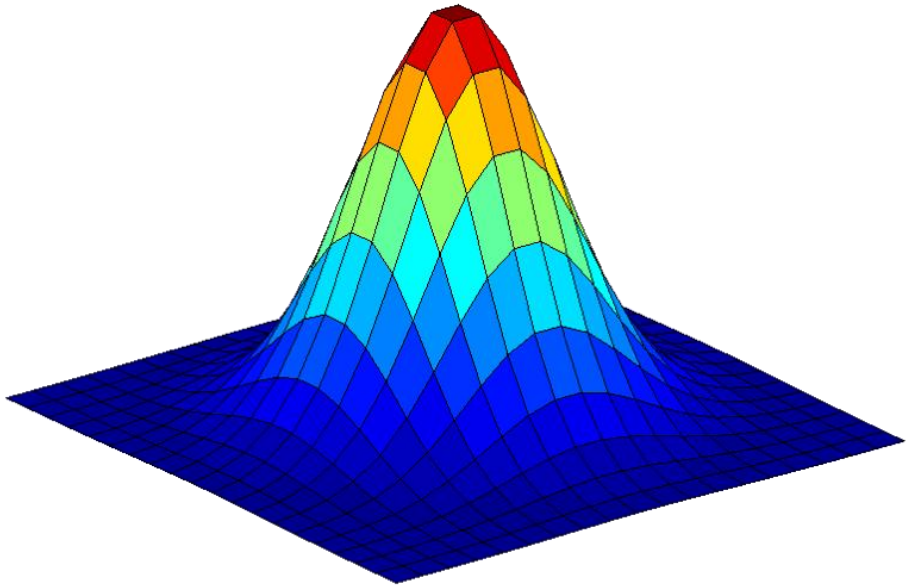

平滑图像


1/9
1 1 1
1 1 1
1 1 1
G[u, v]
⊗
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 90 0 90 90 90 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 0 0 0 0 0 0 0
0 0 90 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
F[x, y]


1/16
1 2 1
2 4 2
1 2 1
G[u, v]
⊗
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 90 0 90 90 90 0 0
0 0 0 90 90 90 90 90 0 0
0 0 0 0 0 0 0 0 0 0
0 0 90 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
F[x, y]

高斯滤波

$$\frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \approx$$



$$G[u,v]$$

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

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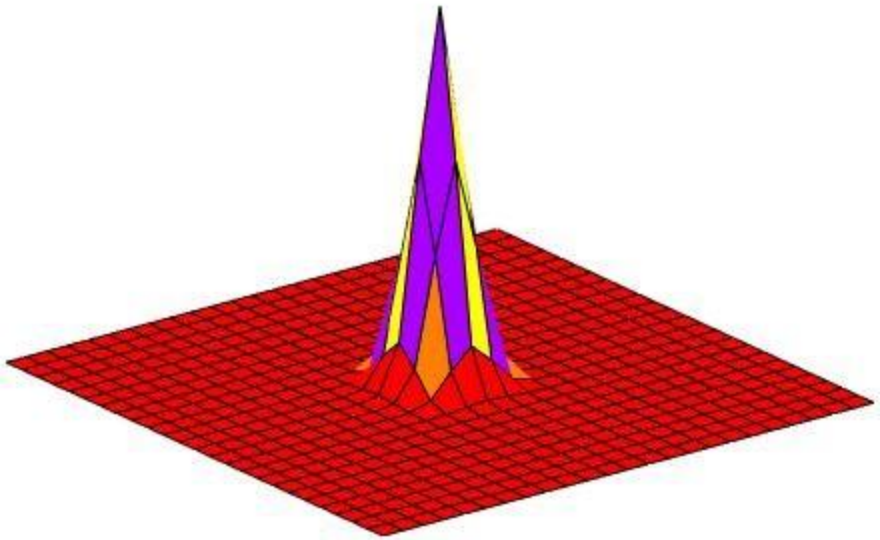
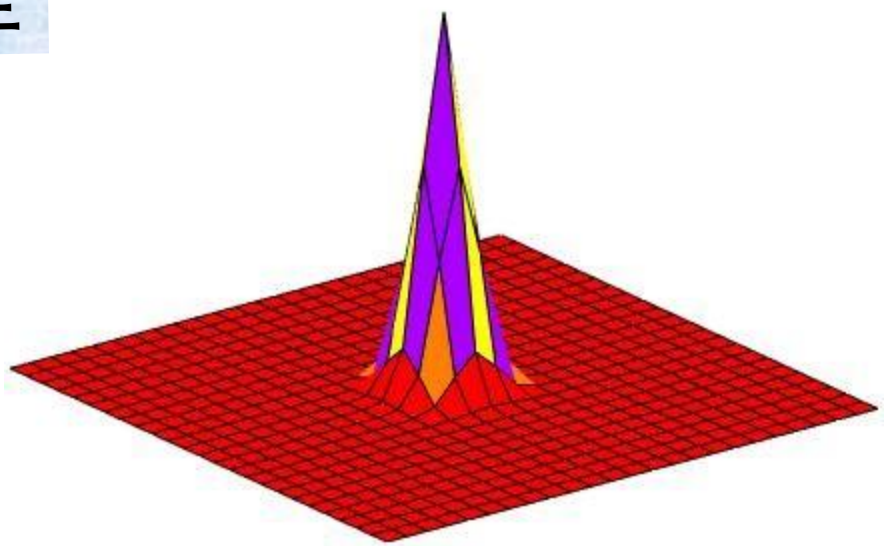


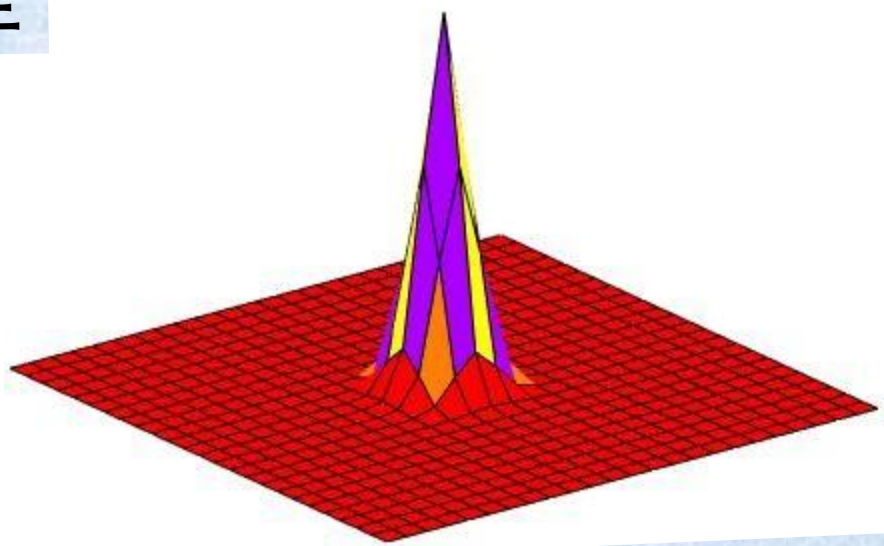
$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

标准差



$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

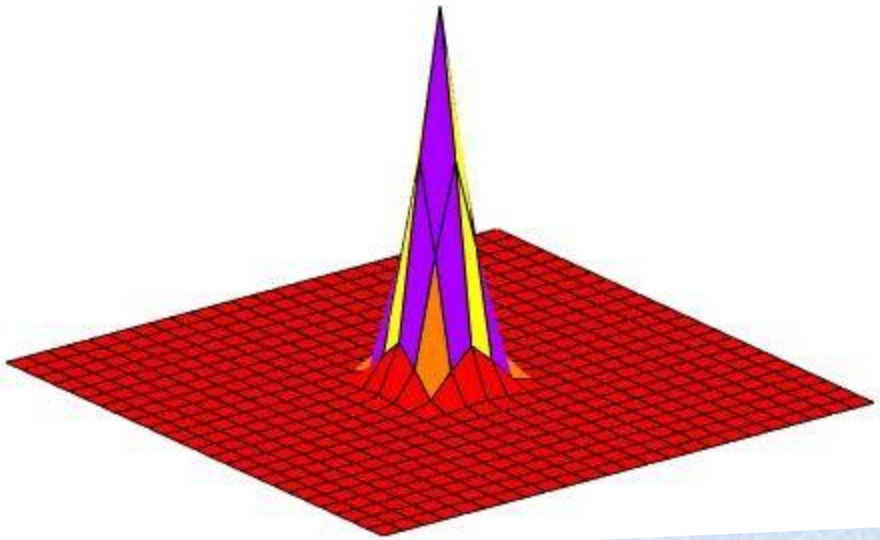
标准差



高斯函数是如何随σ变化的?

归一化

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$



高斯函数是如何随σ变化的?

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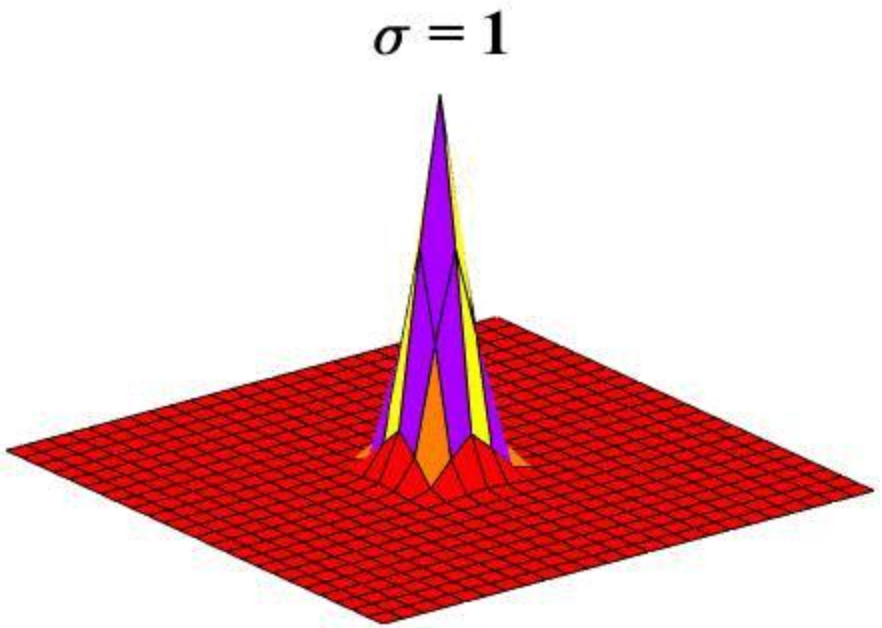
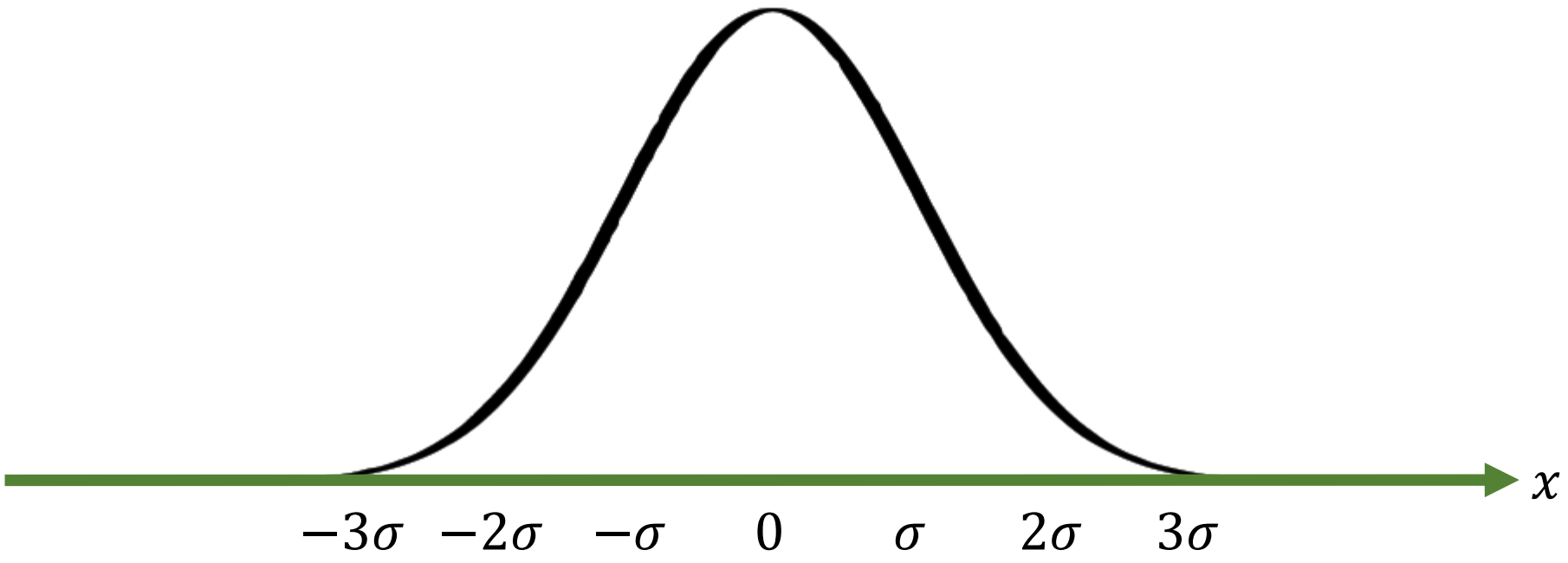


高斯滤波器有无限长的支撑

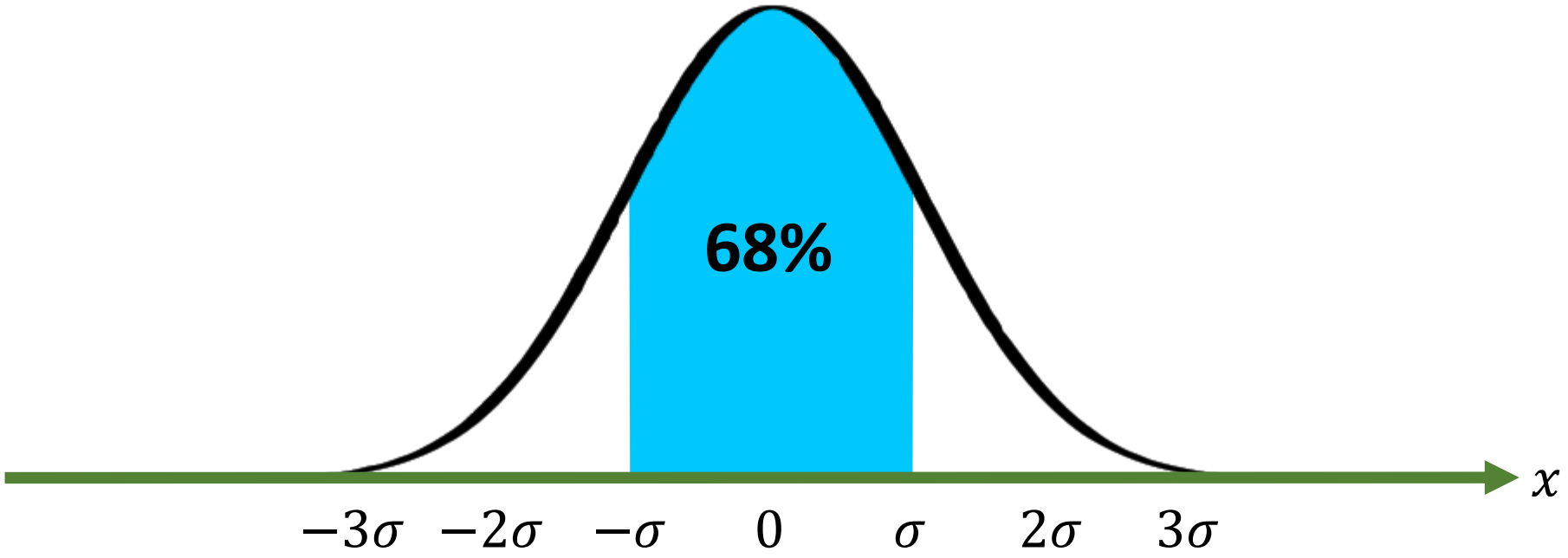
高斯滤波器有无限长的支撑

离散滤波器使用有限大小的核函数

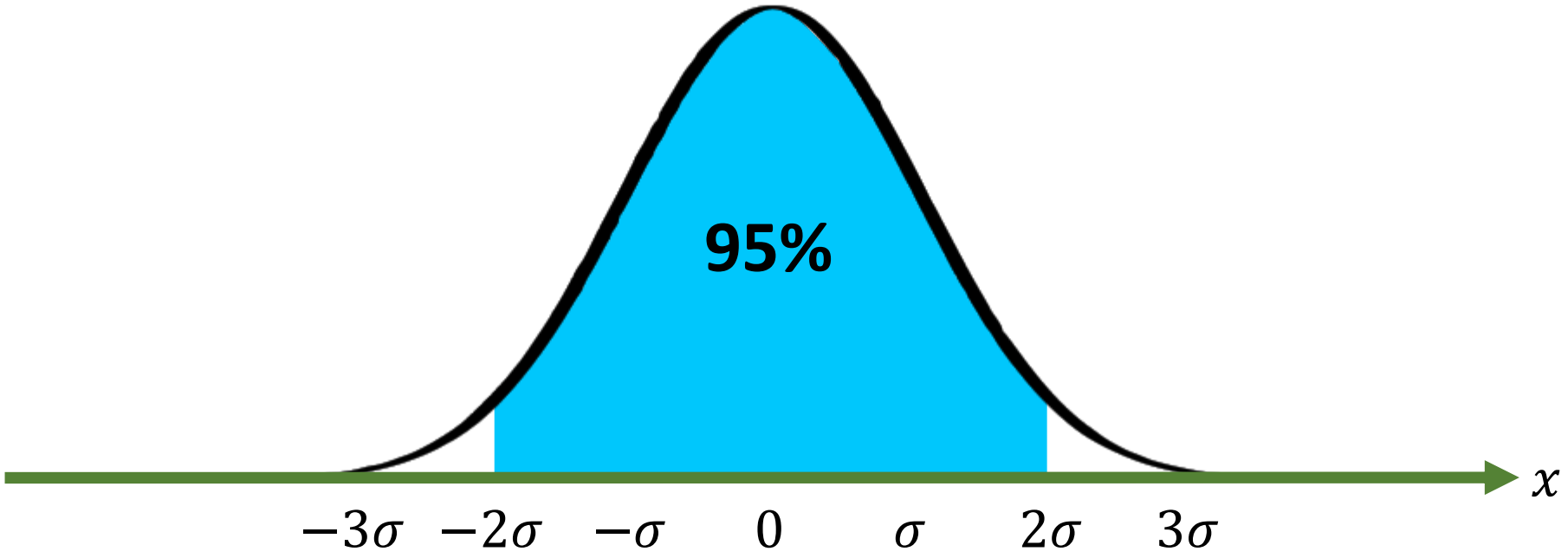
3-σ法则



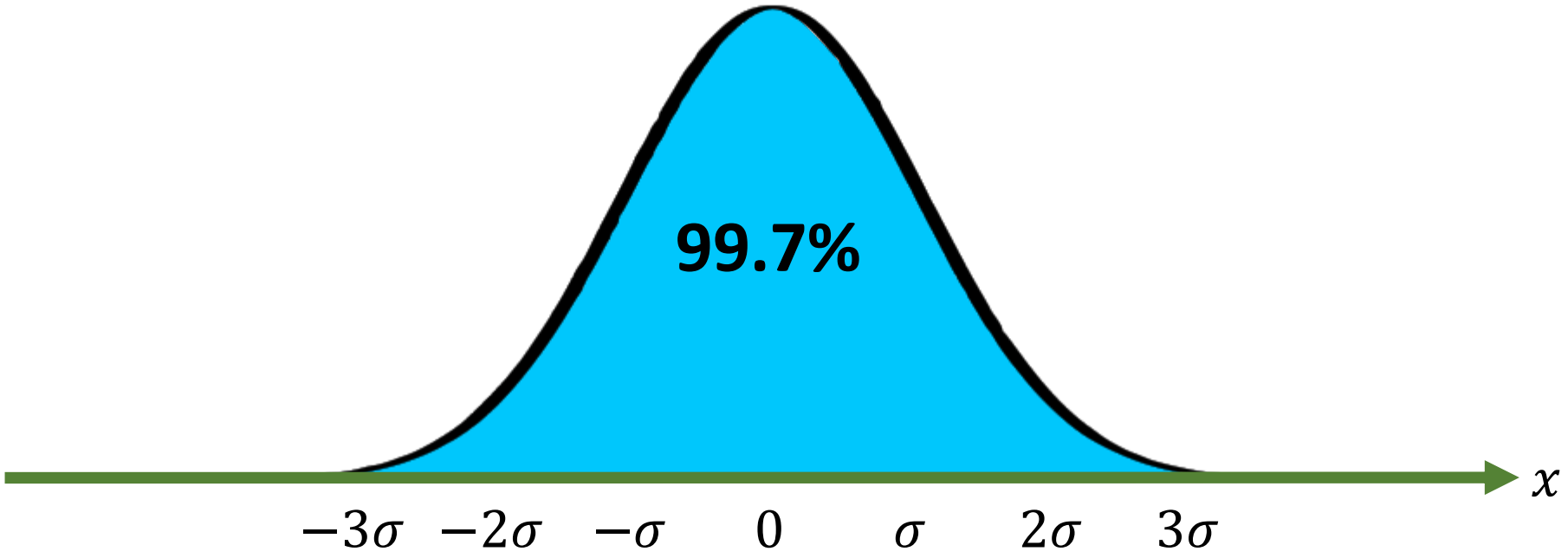
3-σ法则



3-σ法则

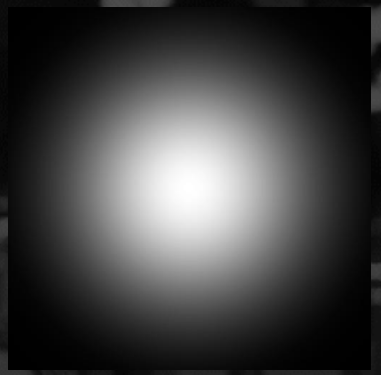


3-σ法则




原始图像

原始图像



高斯滤波器


平滑图像

Python时间

```
>> sigma = 16
>> noise = np.random.randn(im.shape[0], im.shape[1]) * sigma
>> I = im + noise
>> I = (I – I.min()) / (I.max() - I.min())
>> cv2.imshow('noised', I)
>> cv2.waitKey(0)
```

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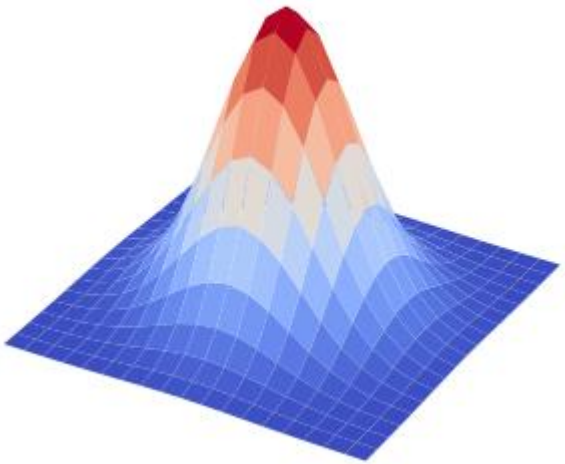
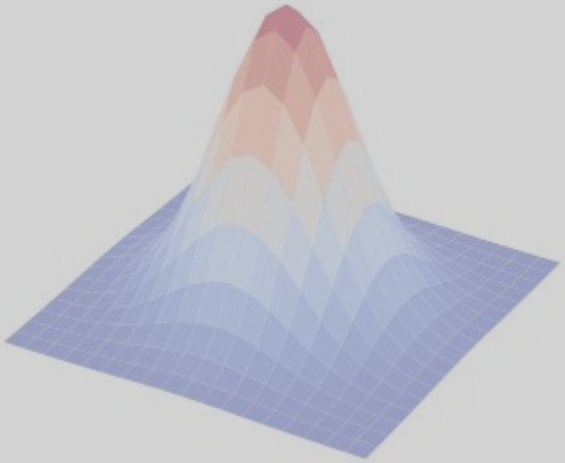


```
>>> from matplotlib import pyplot, cm
>>> width, sigma = 19, 3
>>> h = cv2.getGaussianKernel(width, sigma)
>>> Z = h @ h.T
>>> X = Y = np.arange(-width//2, width//2, 1)
>>> X, Y = np.meshgrid(X, Y)
>>> fig, ax = pyplot.subplots(subplot_kw={'projection': '3d'})
>>> surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm)
>>> ax.set_axis_off()
>>> pyplot.show()
```



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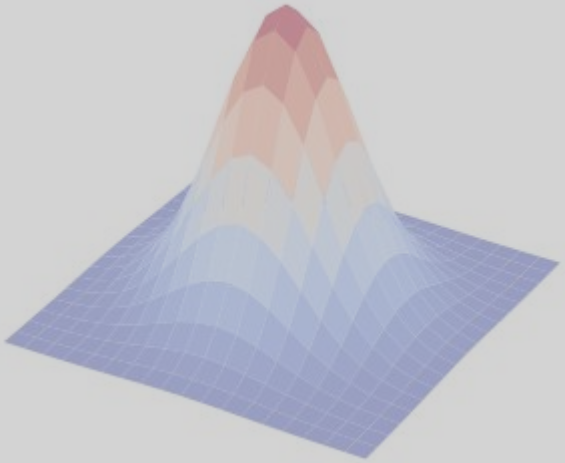

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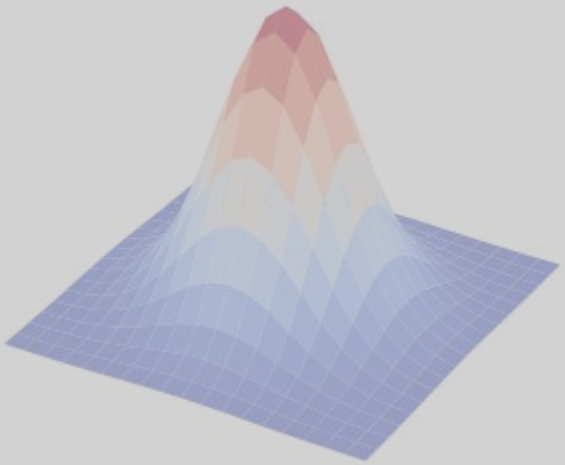

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>>> width, sigma = 19, 3
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>>> Z = h @ h.T
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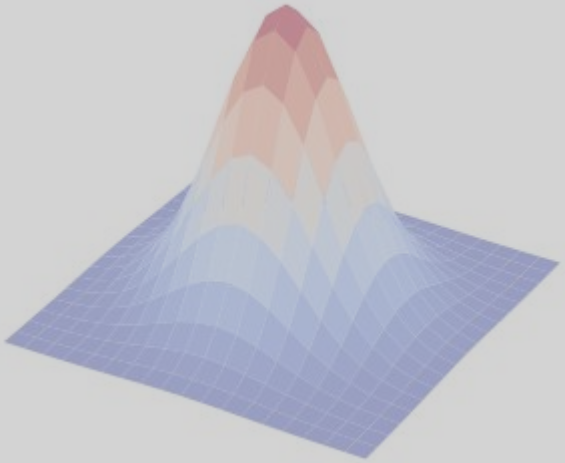
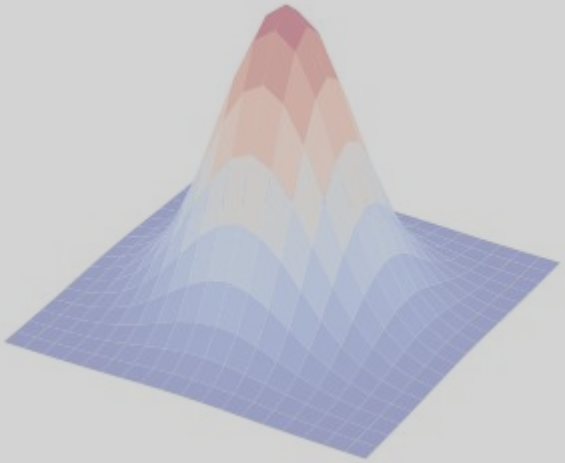
```



```
>>> from matplotlib import pyplot, cm
>>> width, sigma = 10, 2
>>> h = cv2.getGaussianKernel(width, sigma)
>>> Z = h @ h.T
>>> X = Y = np.arange(-width//2, width//2, 1)
>>> X, Y = np.meshgrid(X, Y)
>>> fig, ax = pyplot.subplots(subplot_kw={'projection': '3d'})
>>> surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm)
>>> ax.set_axis_off()
>>> pyplot.show()
```

转置



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>>> width, sigma = 10, 3
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>>> Z = h @ h.T
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>>> surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm)
>>> ax.set_axis_off()
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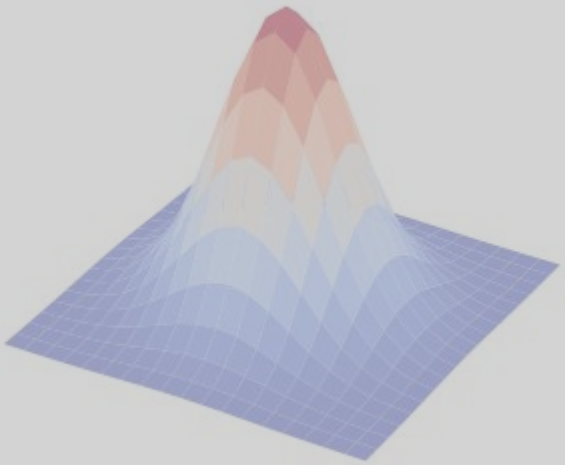
```

矩阵乘法



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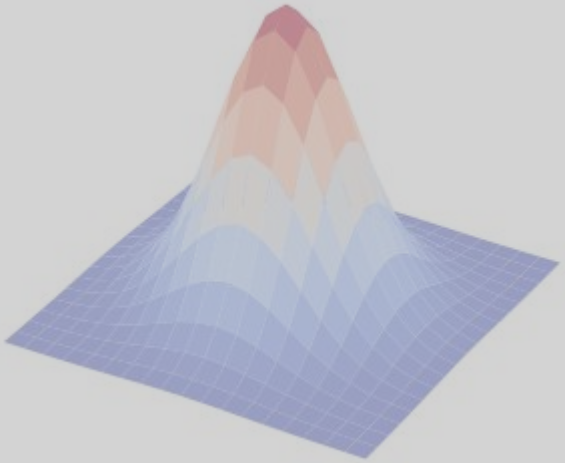

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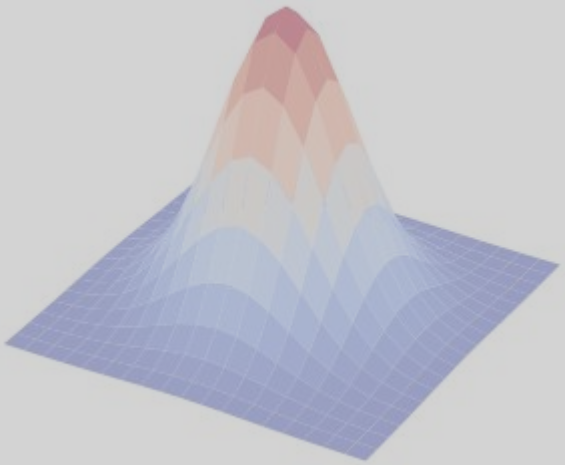

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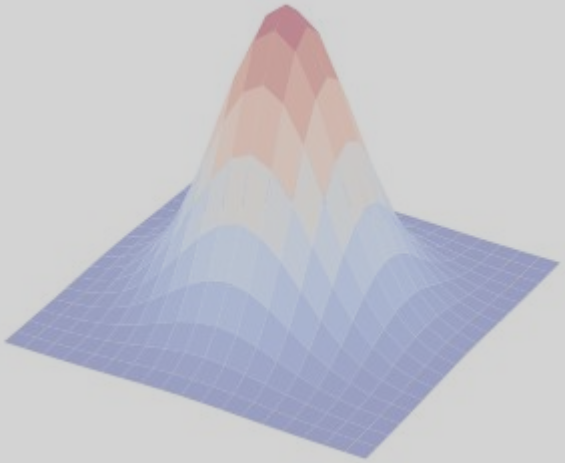
>>> surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm)
>>> ax.set_axis_off()
>>> pyplot.show()
```

向下取整除



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>>> from matplotlib import pyplot, cm
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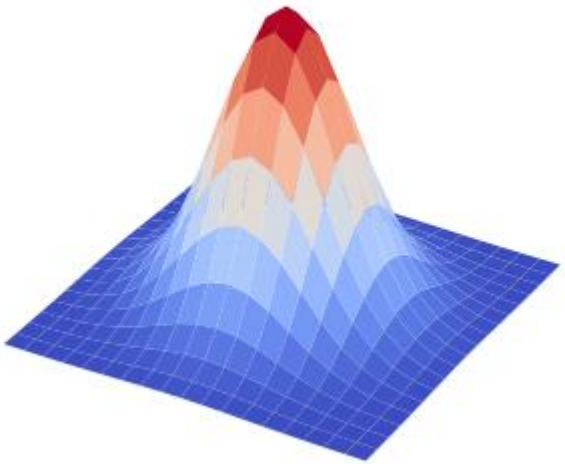
```



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>>> X, Y = np.meshgrid(X, Y)
>>> fig, ax = pyplot.subplots(subplot_kw={'projection': '3d'})
>>> surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm)
>>> ax.set_axis_off()
>>> pyplot.show()
```

```
>>> im = cv2.imread('Avengers.png', cv2.IMREAD_GRAYSCALE)
>>> I = cv2.filter2D(im, -1, g, cv2.BORDER_REFLECT)
>>> cv2.imshow('Avengers', I)
>>> cv2.waitKey(0)
```

```
>>> im = cv2.imread('Avengers.png', cv2.IMREAD_GRAYSCALE)
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>>> cv2.imshow('Avengers', I)
>>> cv2.waitKey(0)
```

输出数据类型与输入一致

```
>>> im = cv2.imread('[illegible].jpg', cv2.IMREAD_GRAYSCALE)
>>> I = cv2.filter2D(im, -1, g, cv2.BORDER_REFLECT)
>>> cv2.imshow('Avengers', I)
>>> cv2.waitKey(0)
```

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>>> im = cv2.imread('Avengers.png', cv2.IMREAD_GRAYSCALE)
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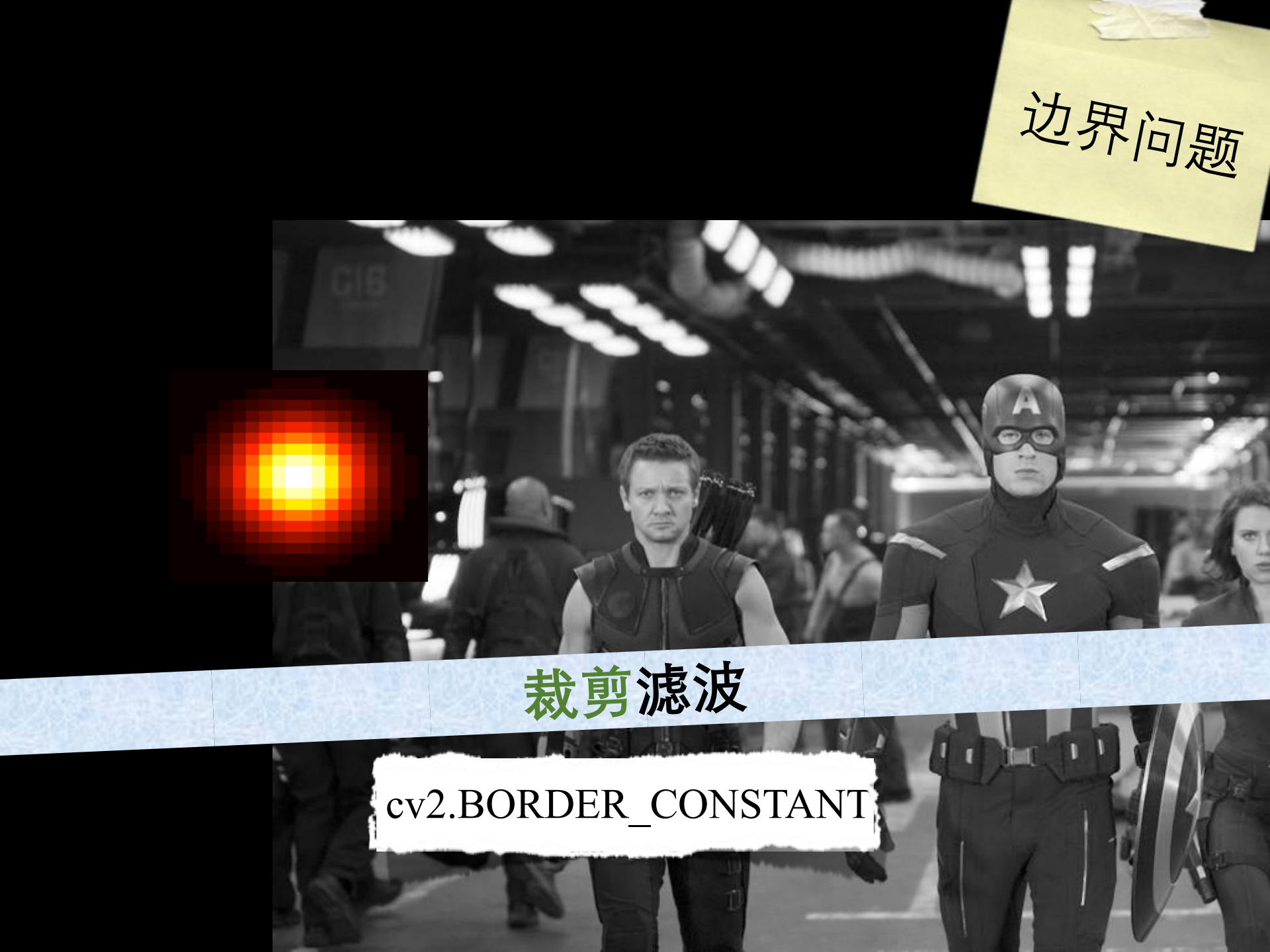
靠近图像边缘时怎么办？


边界问题

边界问题

边界问题

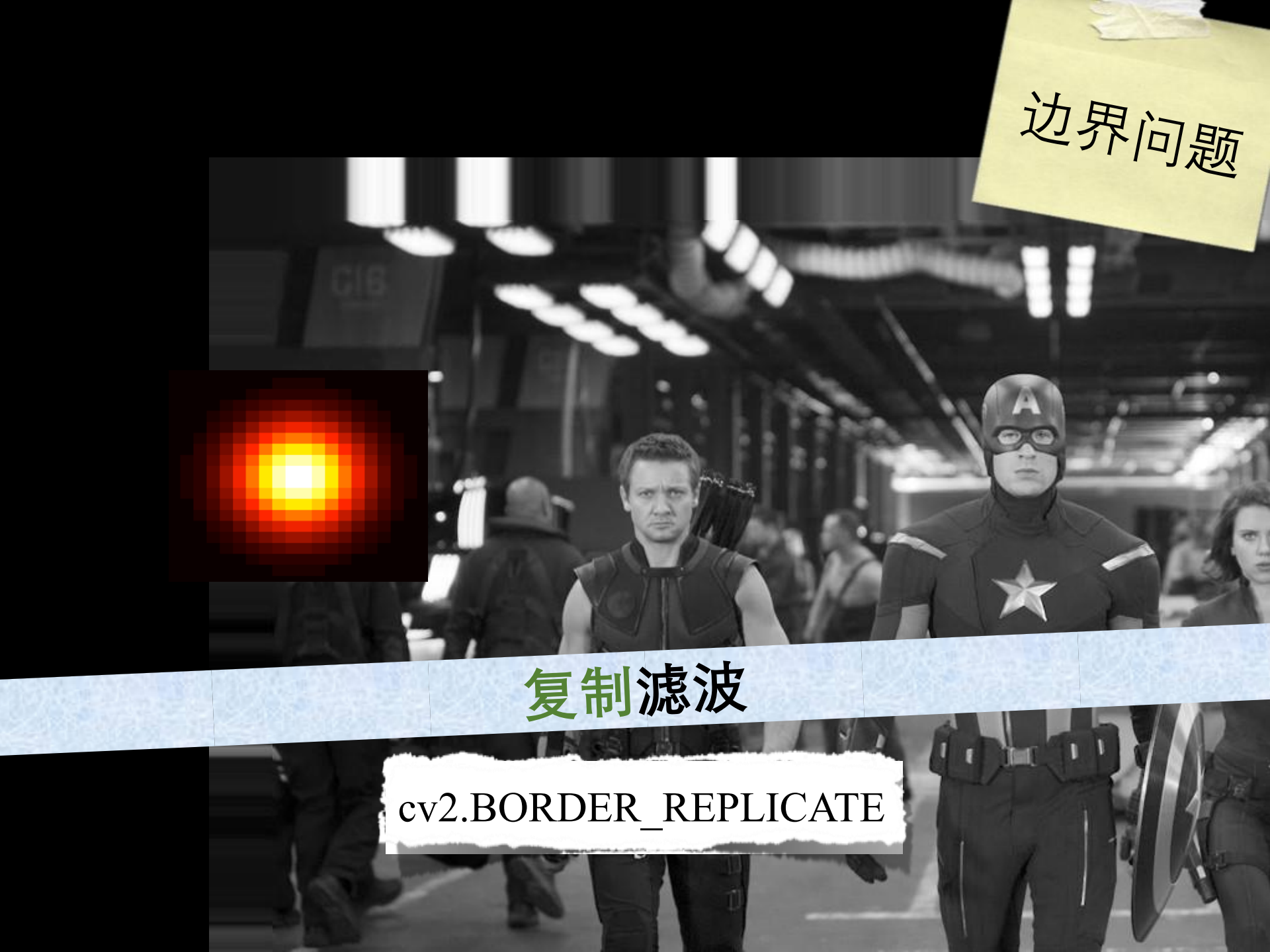

边界问题
裁剪滤波


边界问题
裁剪滤波
cv2.BORDER_CONSTANT


边界问题
循环滤波


边界问题
循环滤波
cv2.BORDER_WARP


边界问题
复制滤波

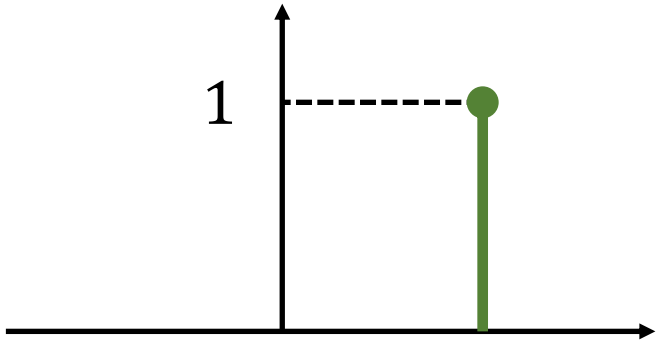

边界问题
复制滤波
cv2.BORDER_REPLICATE


边界问题
对称滤波


边界问题
对称滤波
cv2.BORDER_REFLECT

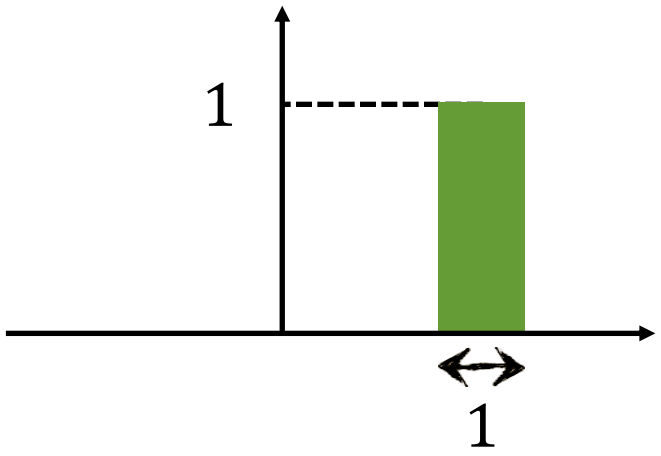
Python时间

离散冲击响应



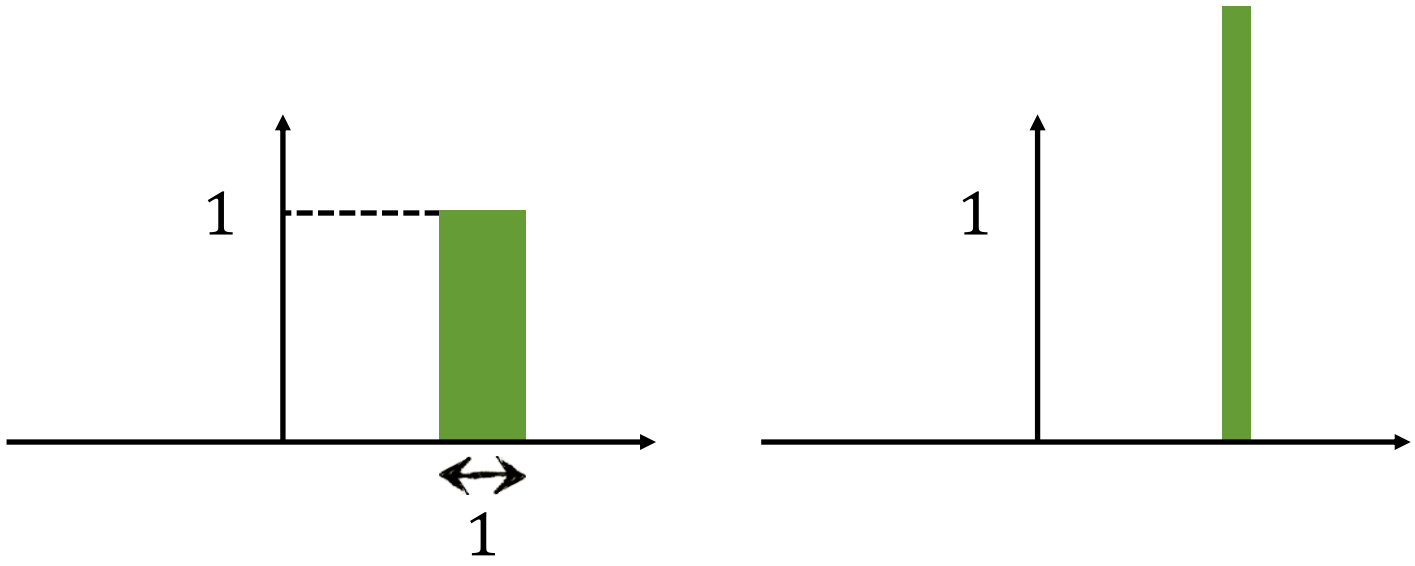
在单个位置上值为1的函数

离散
冲击响应



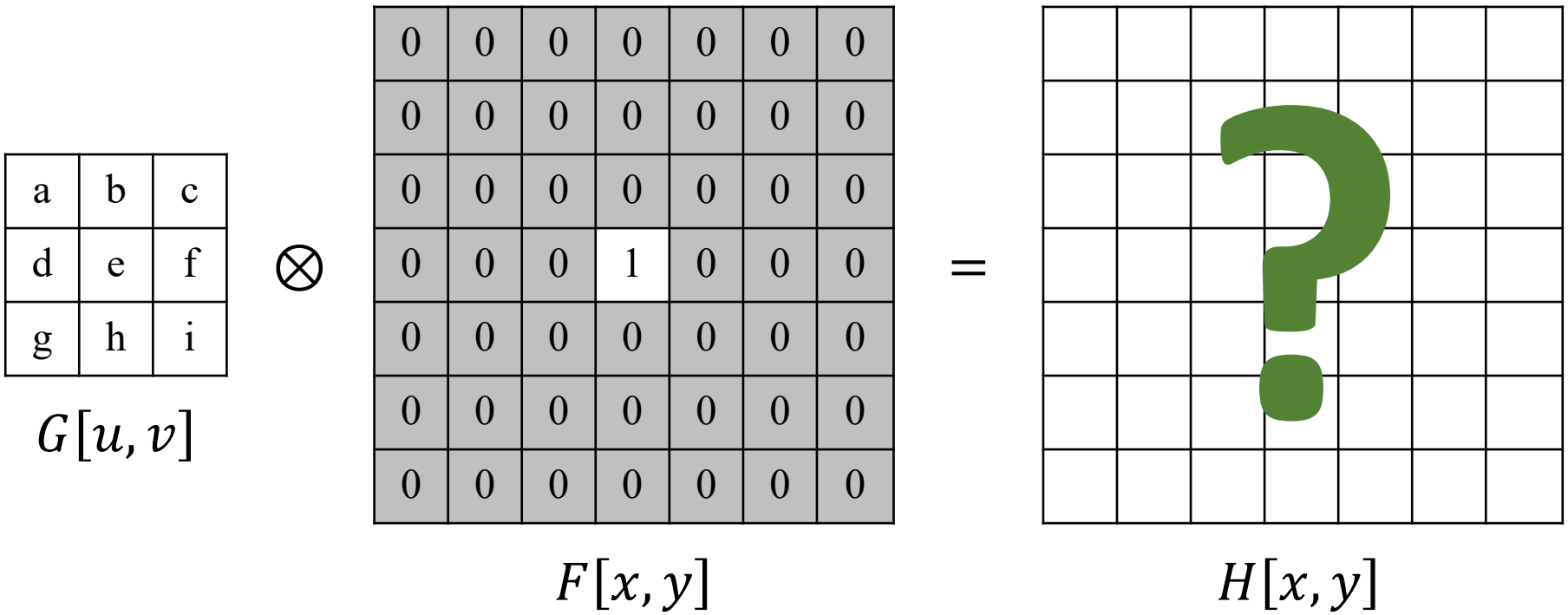
在单个位置上值为1的函数

连续
冲击响应



一个非常窄和非常高的函数，在极限处有一个单位面积

对冲击响应滤波


a b c
d e f
g h i
$G[u,v]$
⊗
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 1 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
$F[x,y]$
=
?
$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0						

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0					

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i				

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i				

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h			

$H[x,y]$

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h			

$H[x,y]$

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h	g		

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h	g	0	0
0	0					

$H[x,y]$

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h	g	0	0
0	0	f				

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h	g	0	0
0	0	f	e	d	0	0
0	0	c	b	a	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$H[x,y]$

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$F[x,y]$

$=$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	i	h	g	0	0
0	0	f	e	d	0	0
0	0	c	b	a	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$H[x,y]$

滤波后输出反转了！

对冲击响应滤波

a	b	c
d	e	f
g	h	i

$G[u,v]$

$\otimes$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

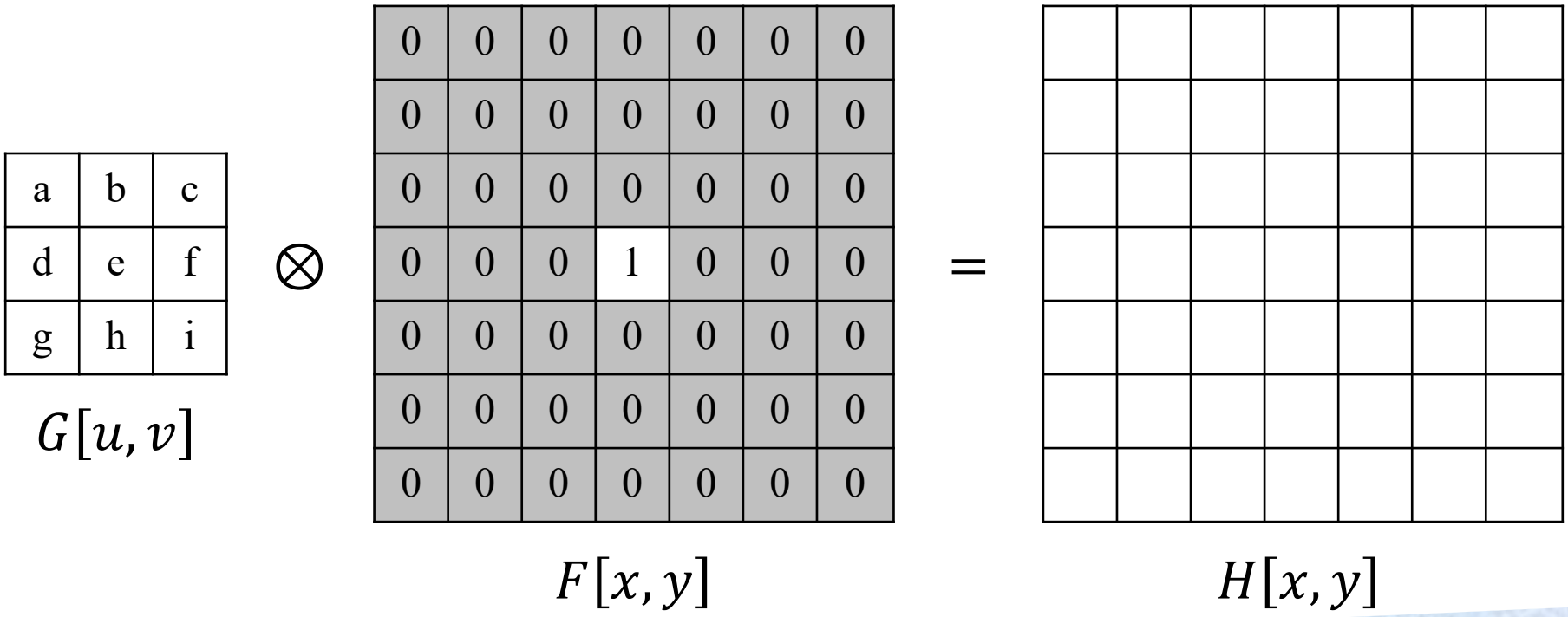
$F[x,y]$

$=$

$H[x,y]$

如何避免输出反转呢?

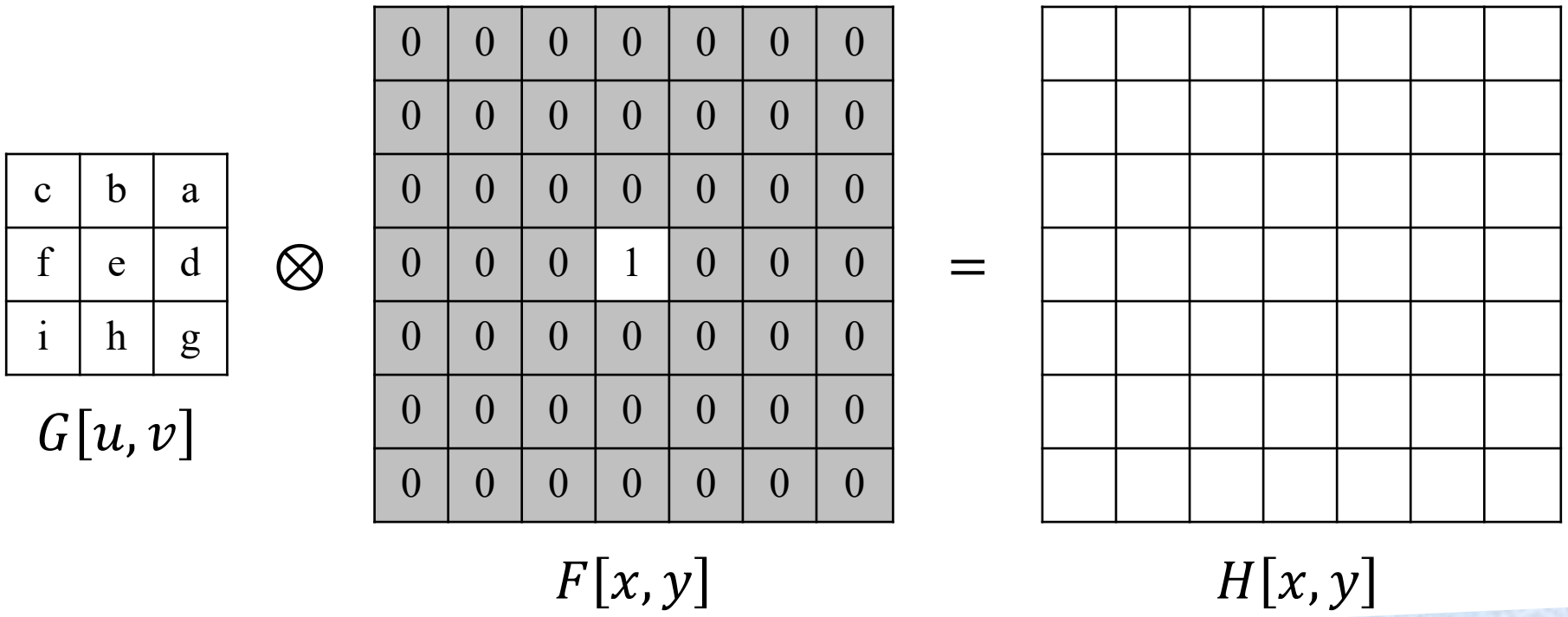
对冲击响应滤波



如何避免输出反转呢？

反转滤波器

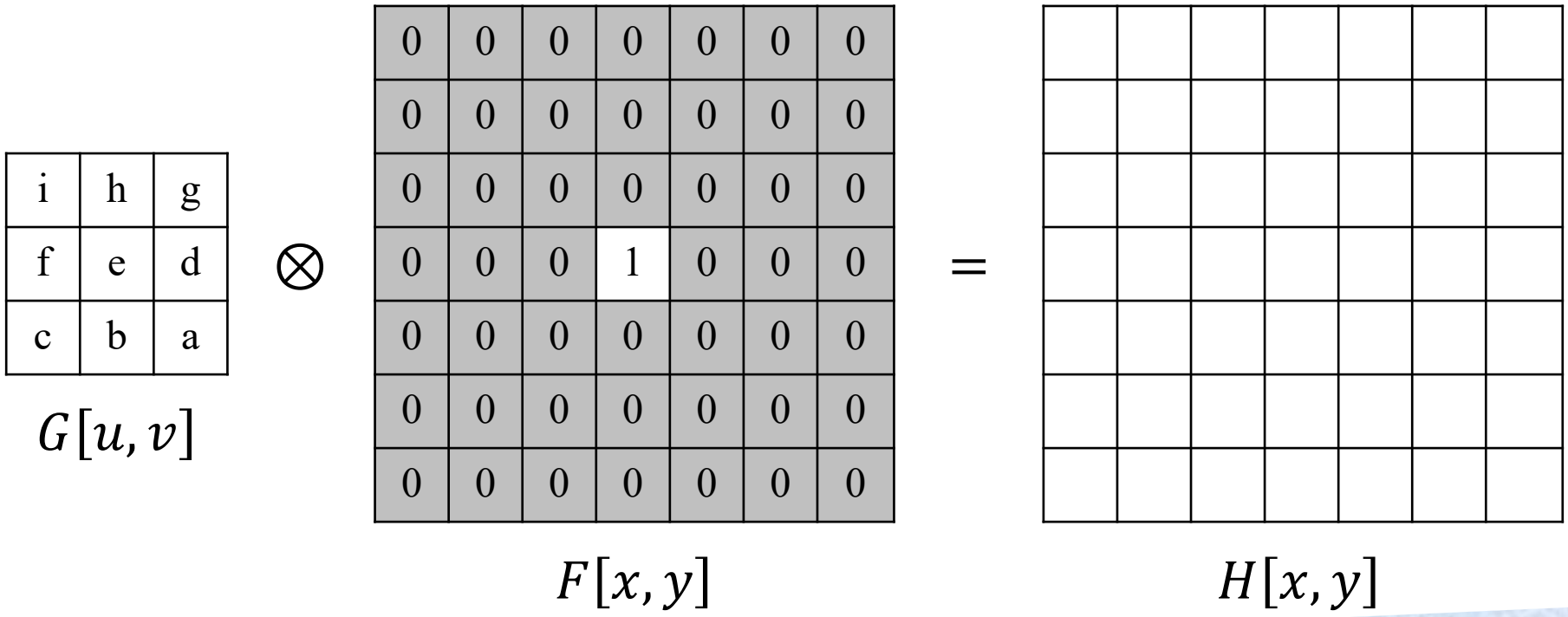
对冲击响应滤波



如何避免输出反转呢？

反转滤波器

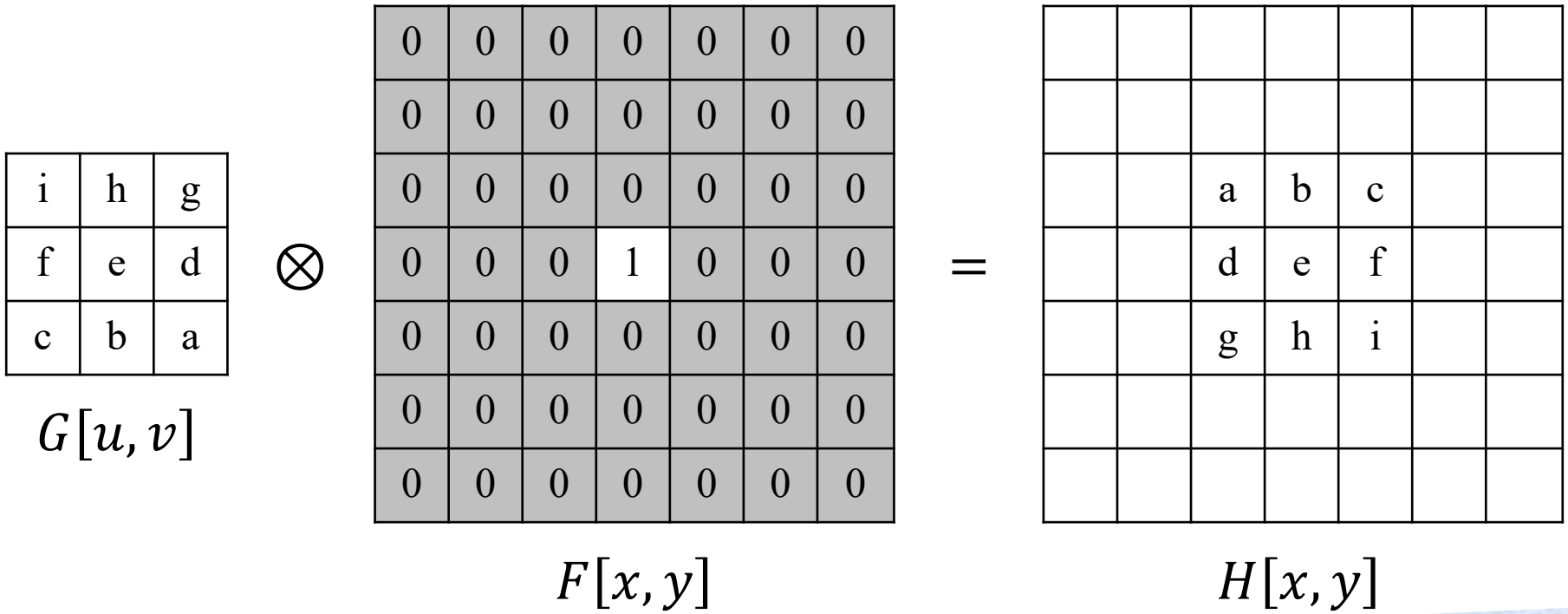
对冲击响应滤波



如何避免输出反转呢？

反转滤波器

对冲击响应滤波



如何避免输出反转呢？

反转滤波器

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

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$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

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$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

该式叫作卷积，记作$H=F*G$

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]G[u,v]$$

该式叫作互相关，记作$H=F\otimes G$

令滑动平均的窗口大小为$(2K+1)\times(2K+1)$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x+u,y+v]G[u,v]$$

该式叫作互相关，记作$H=F\otimes G$

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

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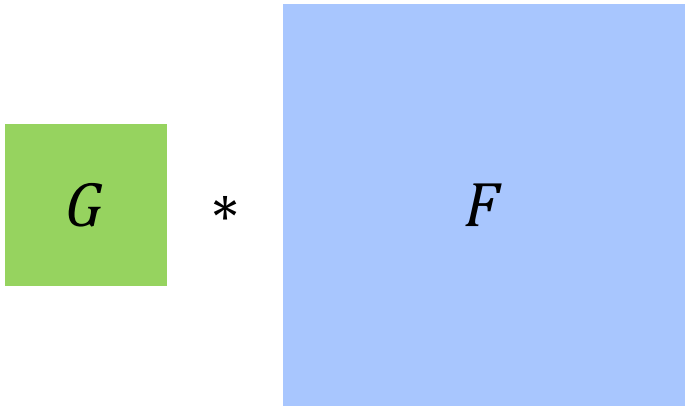
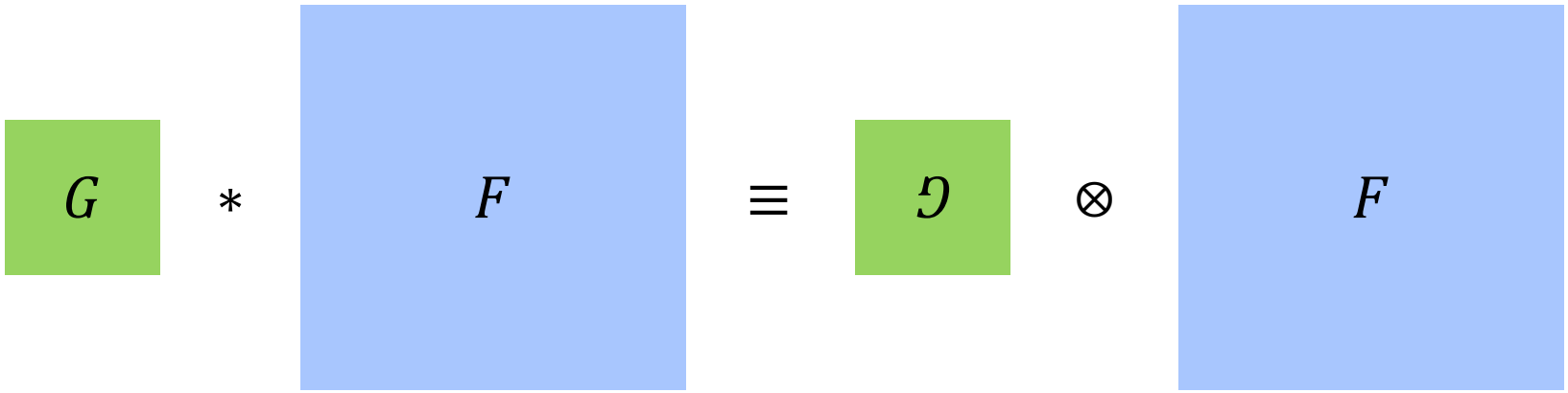


$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

该式叫作卷积，记作$H=F*G$



卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

$$F[x,y]*\delta[x,y]=F[x,y]$$

卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

线性运算

$$G*(\alpha F_1[x,y]+\beta F_2[x,y])=\alpha H_1[x,y]+\beta H_2[x,y]$$

卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

平移不变性

$$G*F[x-\alpha,y-\beta]=H[x-\alpha,y-\beta]$$

卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

分配律

$$G*(E[x,y]+F[x,y])=(G*E[x,y])+(G*F[x,y])$$

卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

结合律

$$(E[x,y]*F[x,y])*G[x,y]=E[x,y]*(F[x,y]*G[x,y])$$

卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

结合律

$$(E[x,y]*F[x,y])*G[x,y]=E[x,y]*(F[x,y]*G[x,y])$$

依次应用若干个滤波器
相当于应用一个滤波器

卷积性质

$$H[x,y]=\sum_{u=-K}^{K}\sum_{v=-K}^{K}F[x-u,y-v]G[u,v]$$

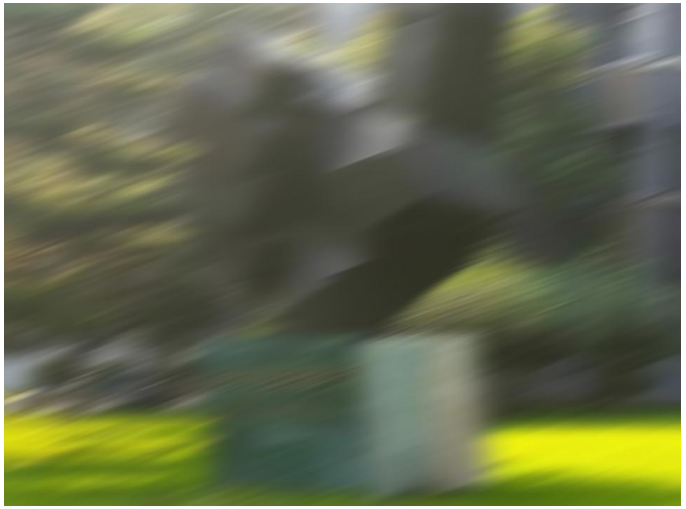
交换律

$$F[x,y]*G[x,y]=G[x,y]*F[x,y]$$

证明略…


OFF
ON
RAW
JPEG
MENU
INFO.
AF-ON
START
STOP
SET
LOCK
Canon

运动模糊



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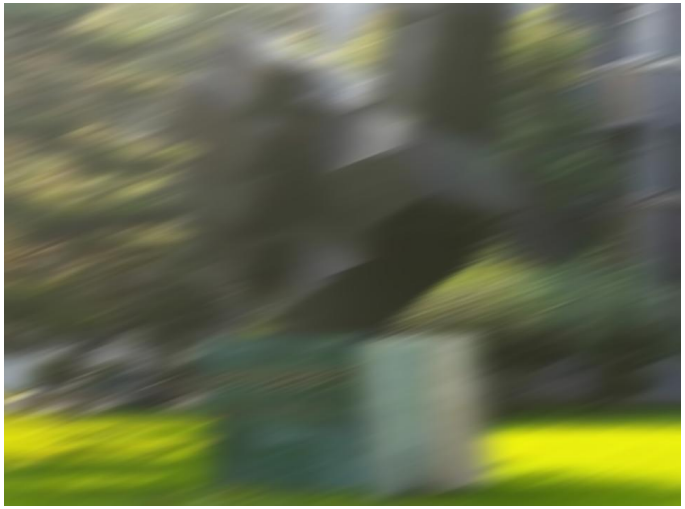
模糊图像

运动模糊



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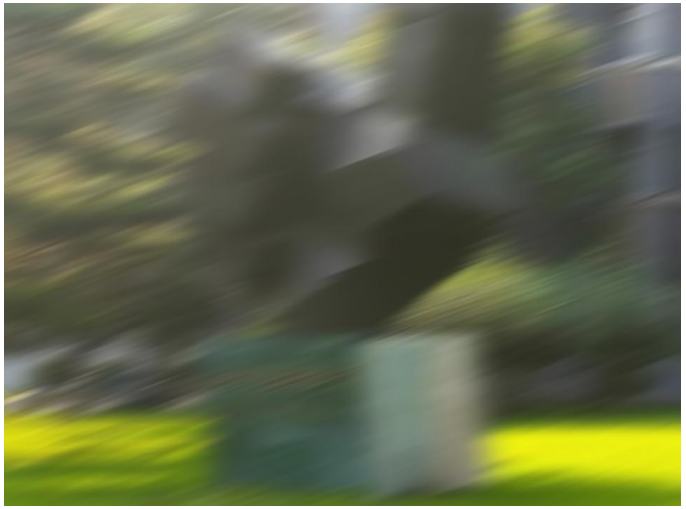
模糊图像

完美图像

运动模糊



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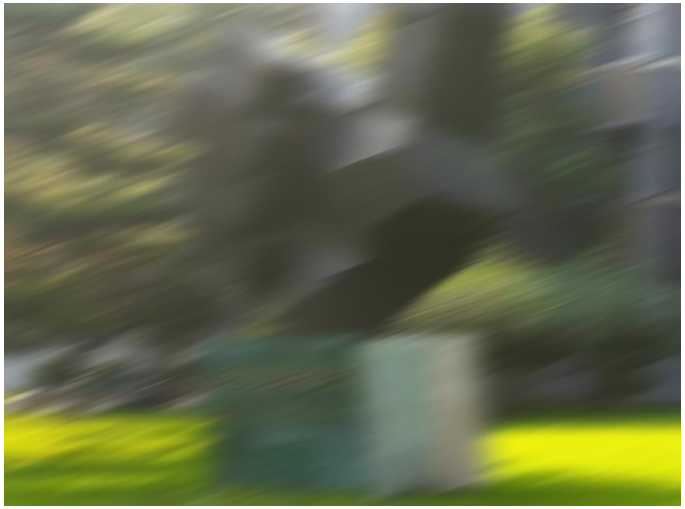
模糊图像

完美图像

运动模糊



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核函数

模糊图像

完美图像

$$H[x,y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} F[x-u, y-v] G[u,v]$$

$$H[x,y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} F[x-u, y-v] G[u,v]$$

卷积每像素需要多少个操作?

$$H[x,y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} F[x-u, y-v] G[u,v]$$

卷积每像素需要多少个操作?

$(2K+1)^2$

假设滤波器可以改写成

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$$F[x, y] = U[x]V[y]$$

假设滤波器可以改写成

$$F[x, y] = U[x]V[y]$$

该滤波器称作可分离的

假设滤波器可以改写成

$$F[x, y] = U[x]V[y]$$

该滤波器称作可分离的

可以让卷积更快!

假设滤波器可以改写成

$$F[x, y] = U[x]V[y]$$

假设滤波器可以改写成

$$F[x, y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u, v]F[x - u, y - v]$$

假设滤波器可以改写成

$$F[x, y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u, v]F[x - u, y - v]$$

代入滤波器定义

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

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$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

眼熟吗？

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

1D水平卷积

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

眼熟吗？

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$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

1D垂直卷积

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

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代入滤波器定义

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$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

先水平滤波再垂直滤波

假设滤波器可以改写成

$$F[x,y] = U[x]V[y]$$

$$H * F = \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]F[x-u,y-v]$$

代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

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卷积每像素需要多少个操作?

假设滤波器可以改写成

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代入滤波器定义

$$= \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]V[y-v]$$

提出因子

$$= \sum_{v=-\infty}^{\infty} V[y-v] \sum_{u=-\infty}^{\infty} H[u,v]U[x-u]$$

卷积每像素需要多少个操作?

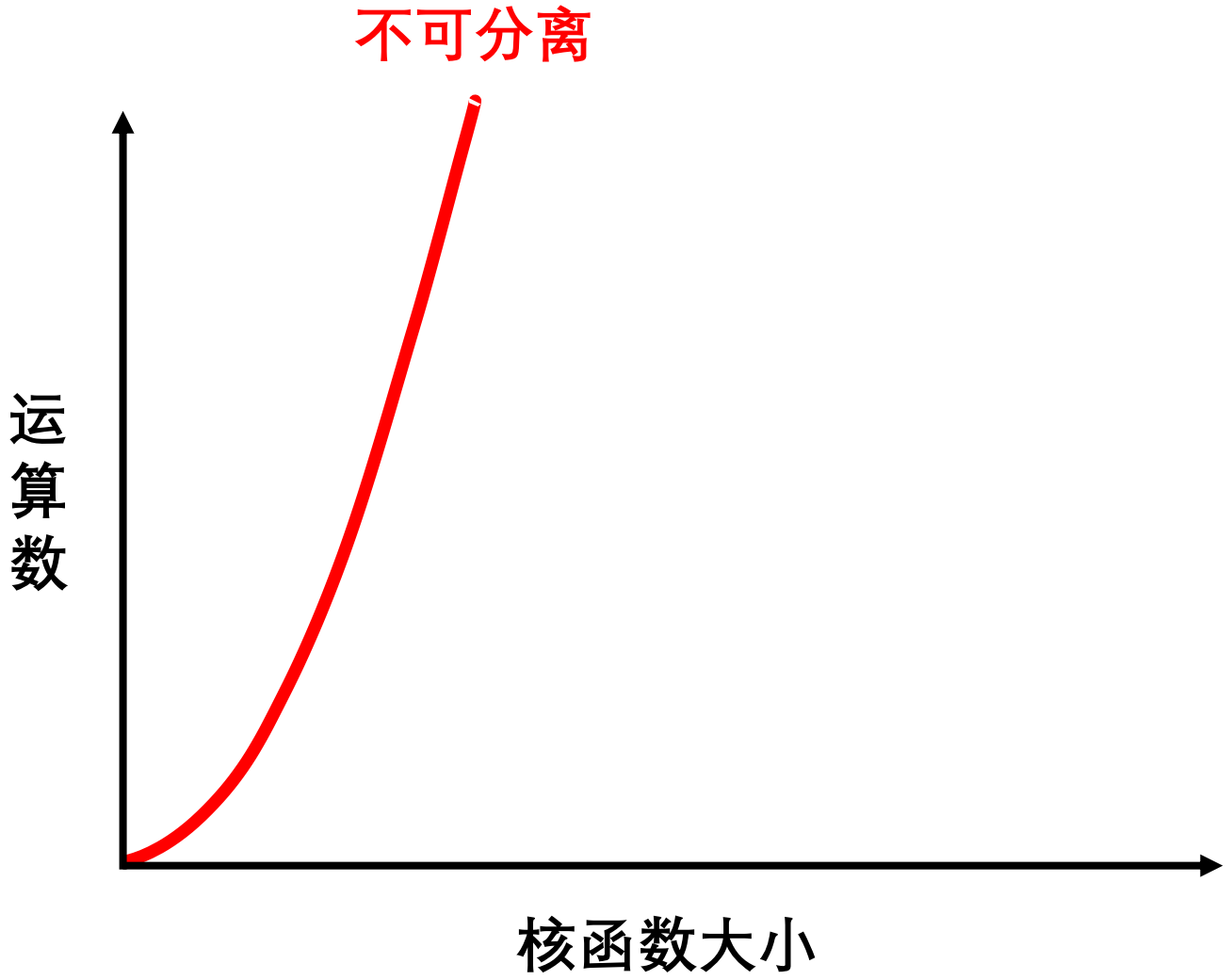
$2(2K+1)$

$$H[x,y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} F[x-u, y-v] G[u,v]$$

卷积每像素需要多少个操作?

$(2K+1)^2$


运算数
核函数大小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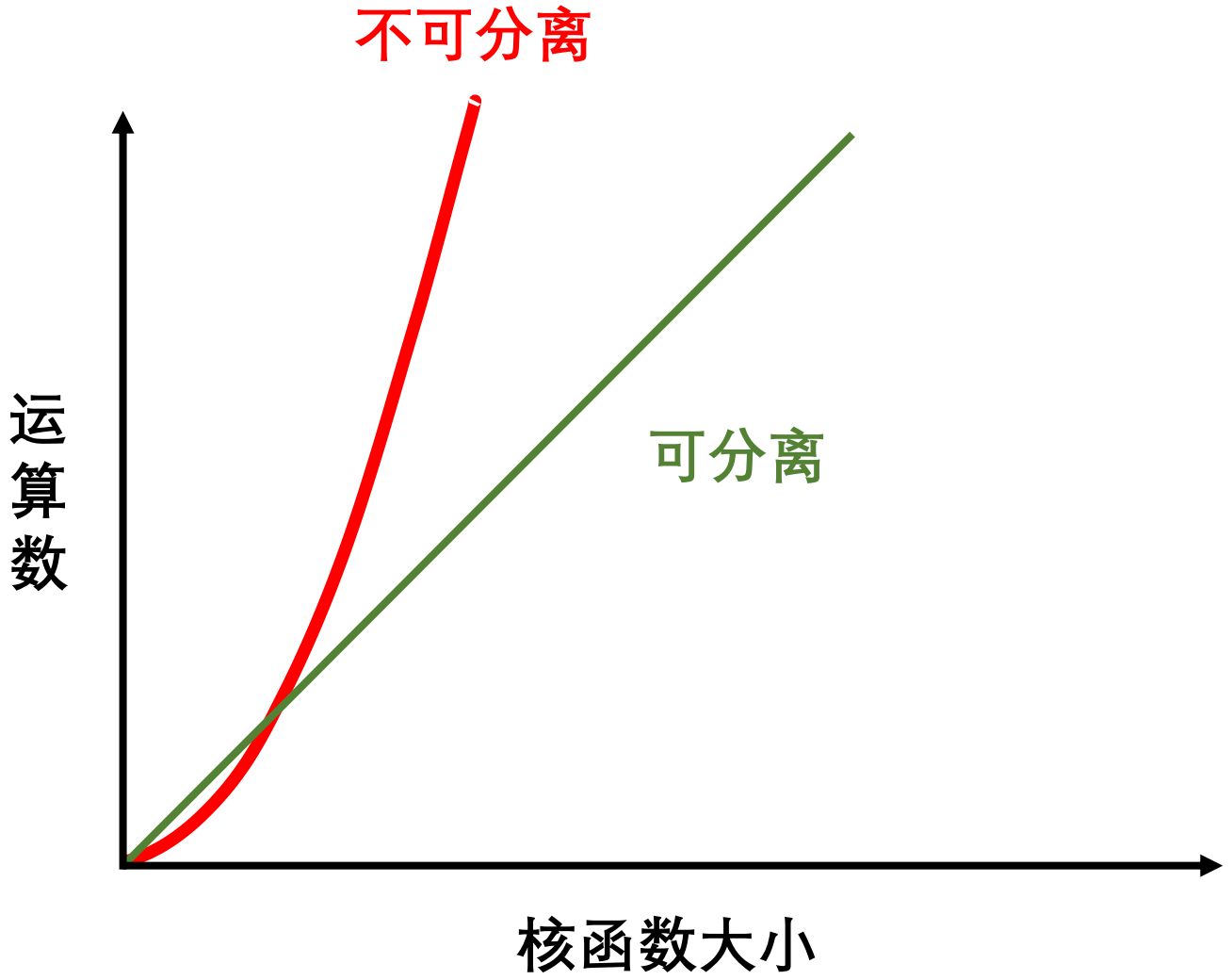
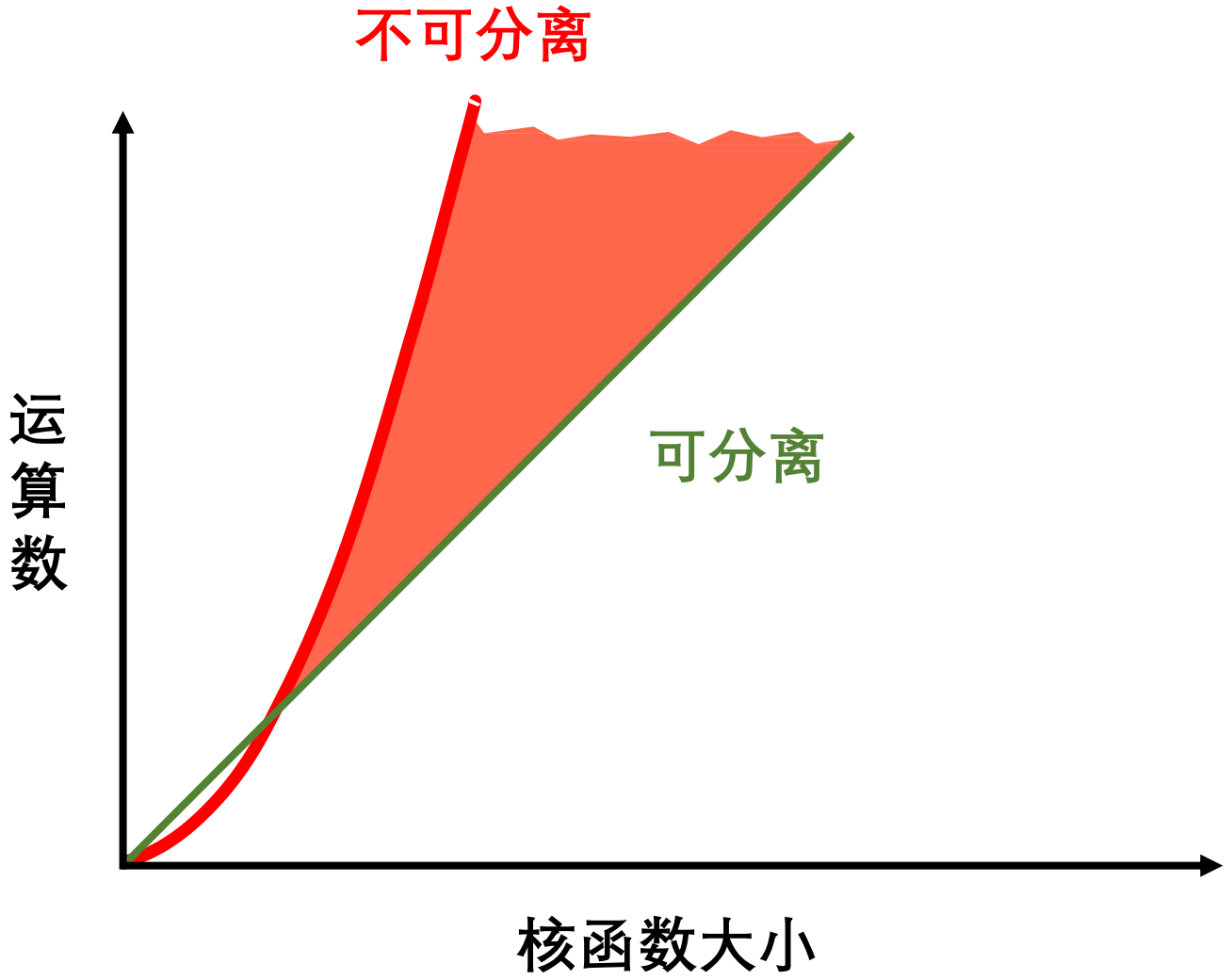
运算数
核函数大小


不可分离
可分离
运算数
核函数大小


不可分离
可分离
运算数
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证明2D高斯滤波器是可分离的

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证明2D高斯滤波器是可分离的

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$$= \left[\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right]\left[\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{y^2}{2\sigma^2}\right)\right]$$

证明2D高斯滤波器是可分离的

$$
\begin{aligned}
G(x,y) &= \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \\
&= \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)\exp\left(-\frac{y^2}{2\sigma^2}\right) \\
&= \left[\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right]\left[\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{y^2}{2\sigma^2}\right)\right] \\
&= G_1(x)G_1(y)
\end{aligned}
$$

回顾

互相关 $H = G \otimes F$

$$H[x,y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} G[u,v]F[x+u, y+v]$$

卷积 $H = G * F$

$$H[x,y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} G[u,v]F[x-u, y-v]$$

回顾

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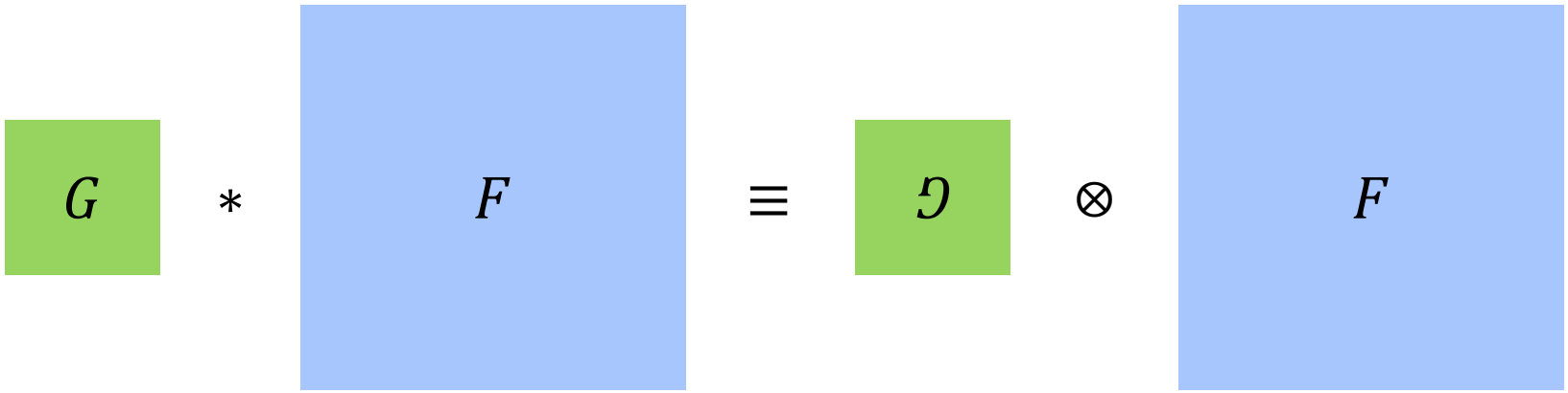

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回顾

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对于对称滤波器，输出有什么不同？

卷积 $H = G * F$

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回顾

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没区别

卷积 $H = G * F$

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实践

用例

图像锐化

输入

图像锐化

输入

图像锐化

输入

模糊的

图像锐化

输入

模糊的

图像锐化

输入

模糊的

图像锐化

输入

模糊的

“锐利的东西”

图像锐化

=

+

输入 **模糊的** **“锐利的东西”**

图像锐化

模糊的

+

“锐利的东西”

图像锐化

模糊的

+

“锐利的东西”

=

图像锐化

模糊的

+

“锐利的东西”

=

锐化的

图像锐化

输入

锐化的

预测滤波器输出

输入

*

0	0	0
0	1	0
0	0	0

=

?

输入

*

0	0	0
0	1	0
0	0	0

=

没变化

输入

*

0	0	0
0	0	1
0	0	0

=

?

输入

*

0	0	0
0	0	1
0	0	0

=

向右平移1像素

输入

$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

?

输入

$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

模糊了

Back

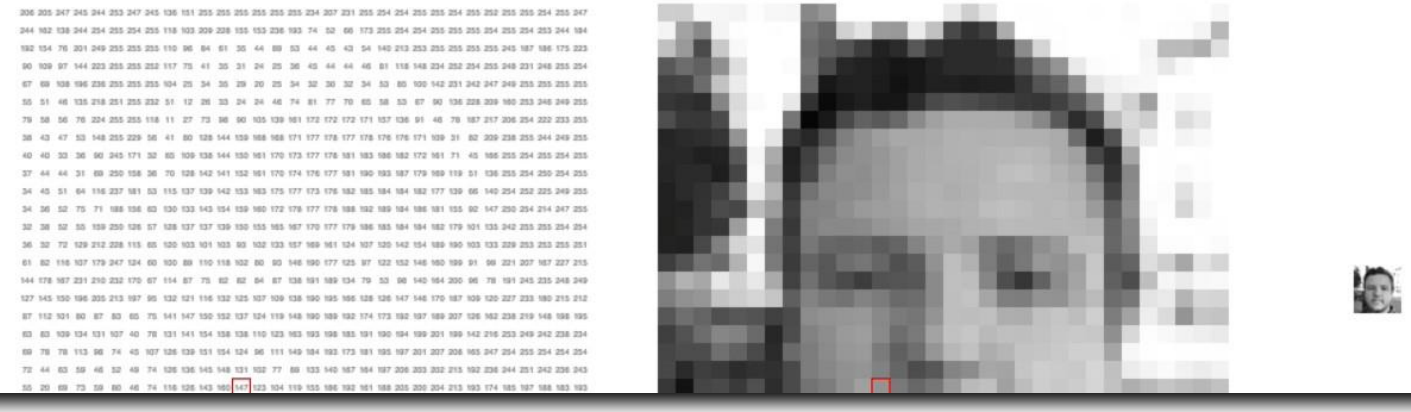
Image kernels

Explained Visually

By Victor Powell

An image kernel is a small matrix used to apply effects like the ones you might find in Photoshop or Gimp, such as blurring, sharpening, outlining or embossing. They're also used in machine learning for 'feature extraction', a technique for determining the most important portions of an image. In this context the process is referred to more generally as "convolution" (see: convolutional neural networks.)

To see how they work, let's start by inspecting a black and white image. The matrix on the left contains numbers, between 0 and 255, which each correspond to the brightness of one pixel in a picture of a face. The large, granulated picture has been blown up to make it easier to see; the last image is the "real" size.



tinyurl.com/image-kernels

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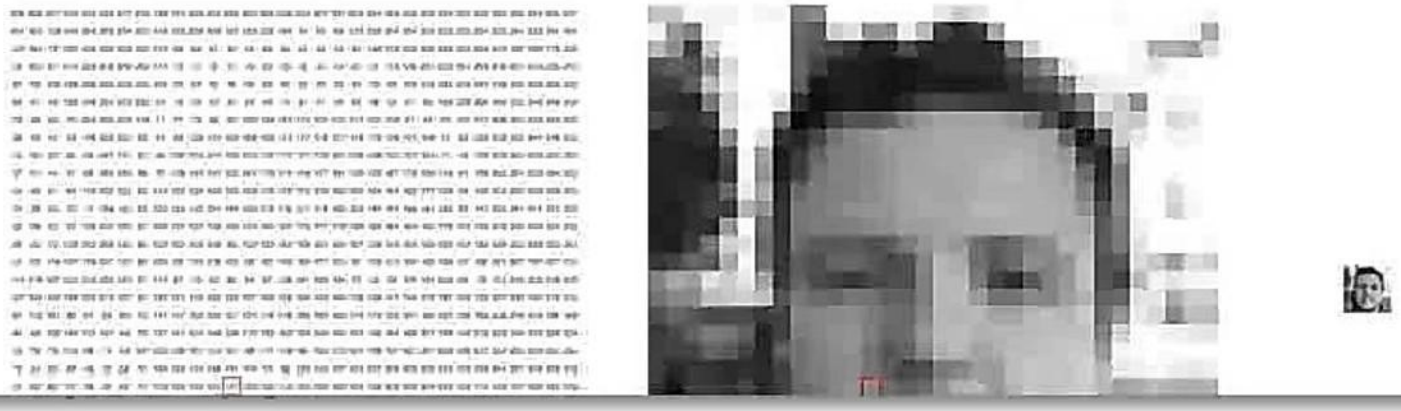

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非线性滤波


椒盐噪声


椒盐噪声

10	15	20
23	90	27
33	31	30

10	15	20
23	90	27
33	31	30

将像素值排序

10 15 20 23 27 30 31 33 90

10	15	20
23	90	27
33	31	30

将像素值排序

10 15 20 23 27 30 31 33 90

用中值替换像素

10	15	20
23	90	27
33	31	30

将像素值排序

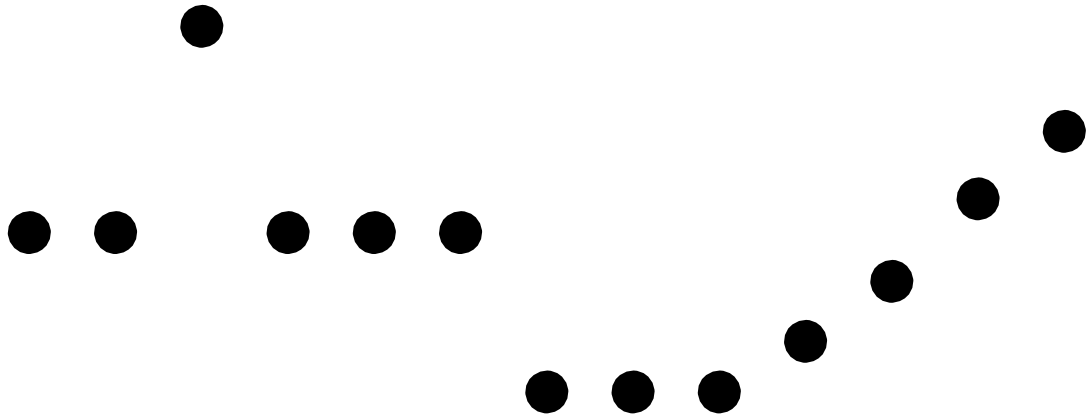
10 15 20 23 27 30 31 33 90

用中值替换像素

中值滤波

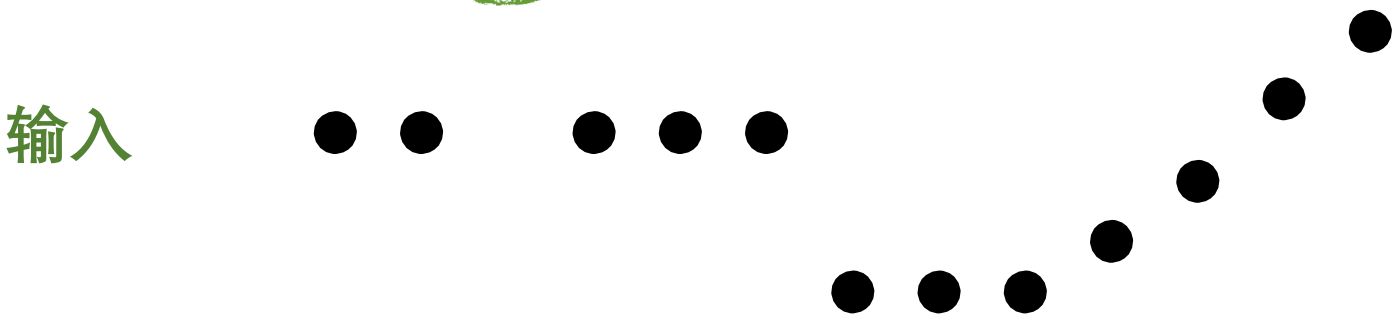
中值滤波

输入

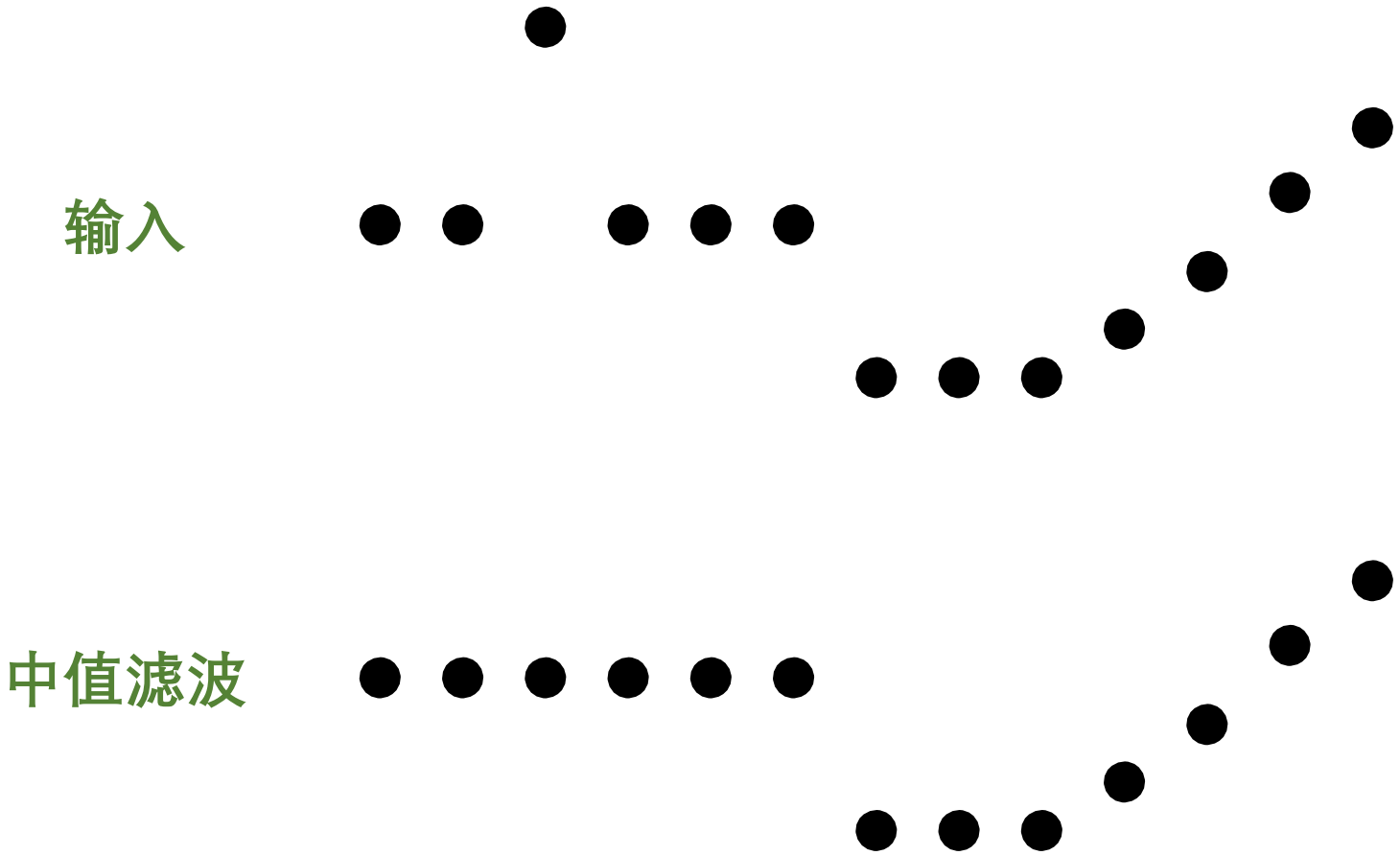


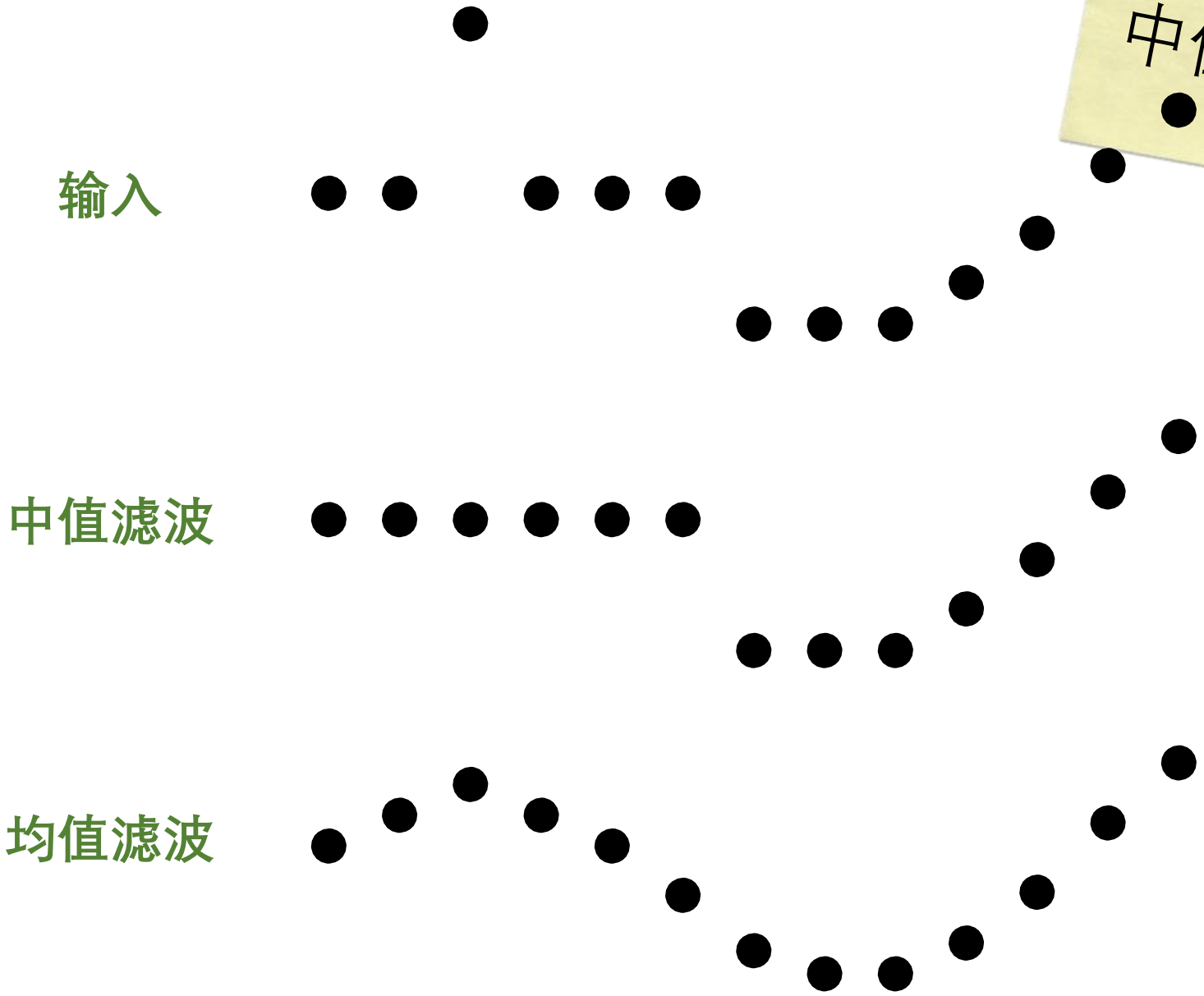
中值滤波

局外点

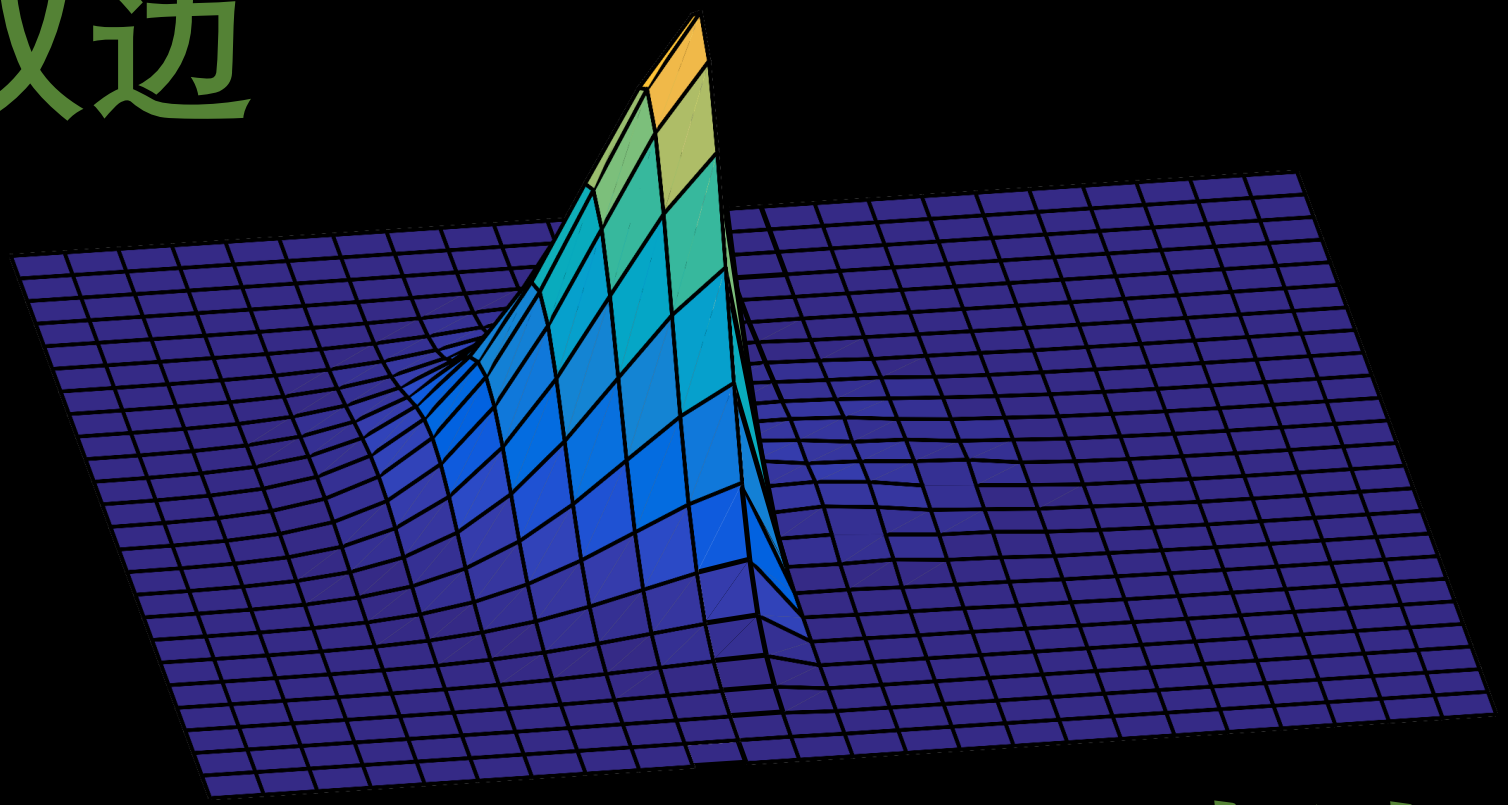


中值滤波




中值滤波
输入
中值滤波
均值滤波

双边



滤波器

Bilateral Filtering for Gray and Color Images

C. Tomasi *

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Stanford University
Stanford, CA 94305
tomasi@cs.stanford.edu

R. Manduchi

Interactive Media Group
Apple Computer, Inc.
Cupertino, CA 95014
manduchi@apple.com

Abstract

Bilateral filtering smooths images while preserving edges, by means of a nonlinear combination of nearby image values. The method is noniterative, local, and sim-

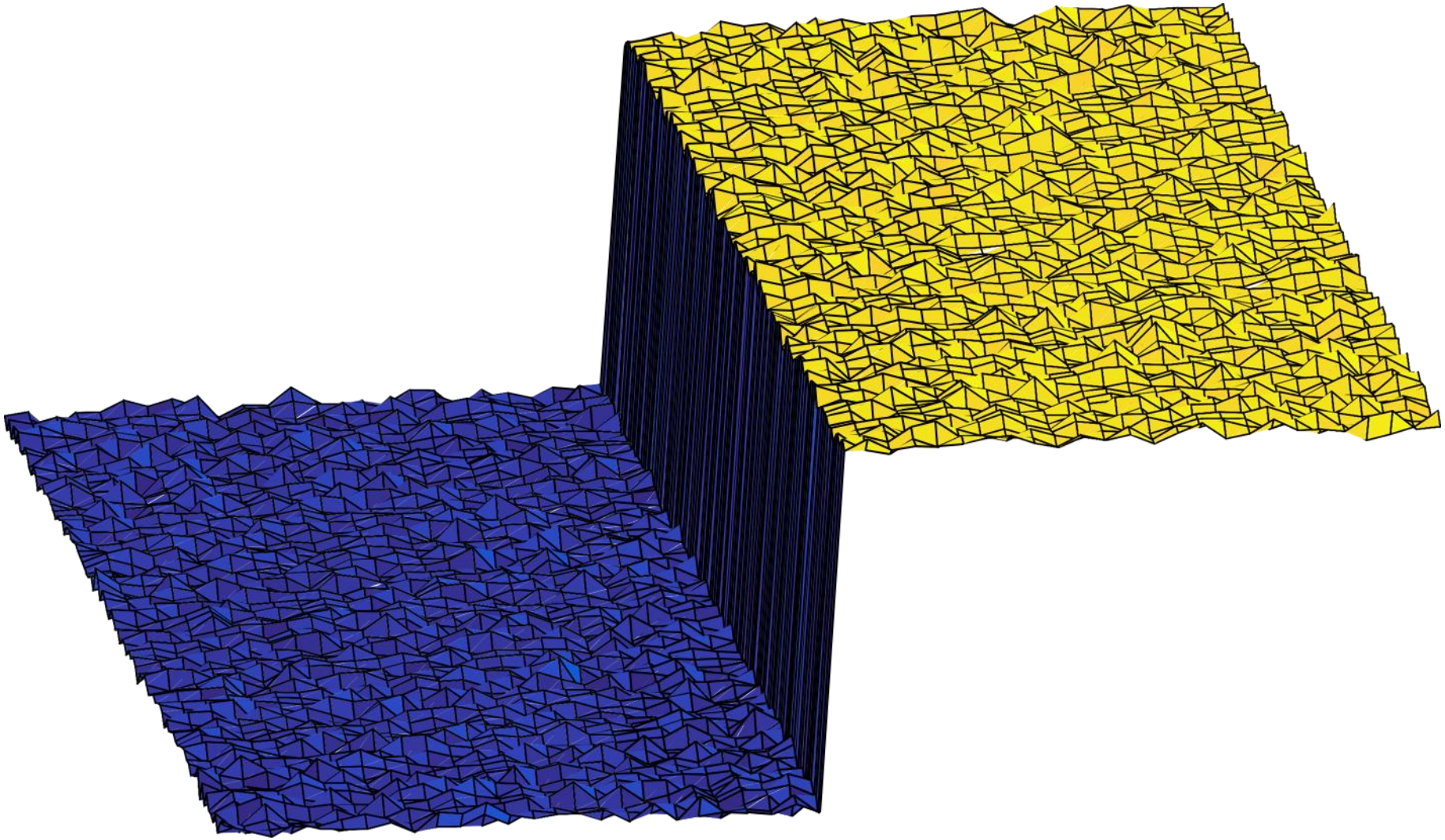
levels or colors based on both their

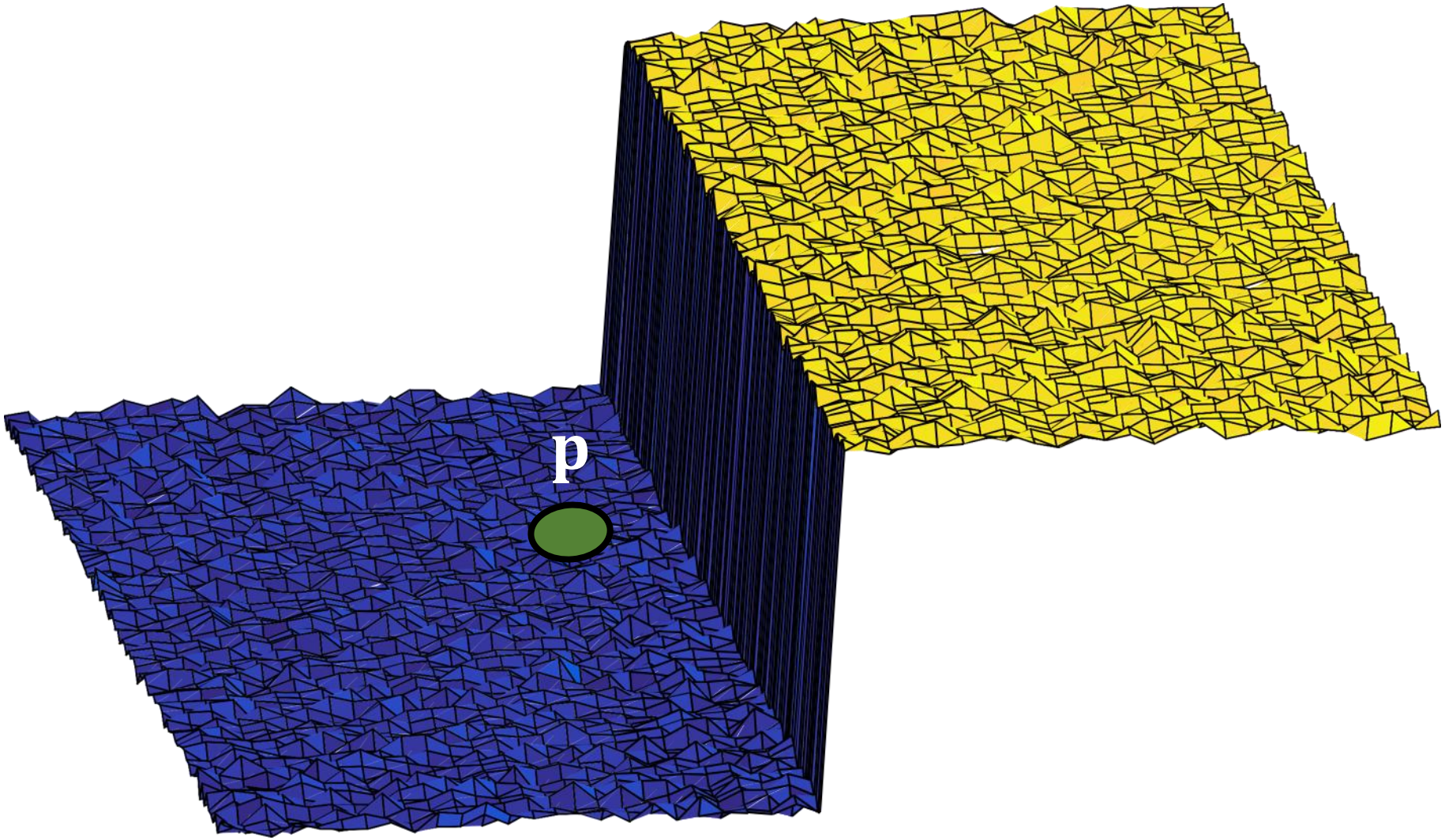
we prevent averaging across edges, while still averaging within smooth regions? Anisotropic diffusion [12, 14] is a popular answer: local image variation is measured at every point, and pixel values are averaged from neighborhoods whose size and shape depend on local variation. Diffusion

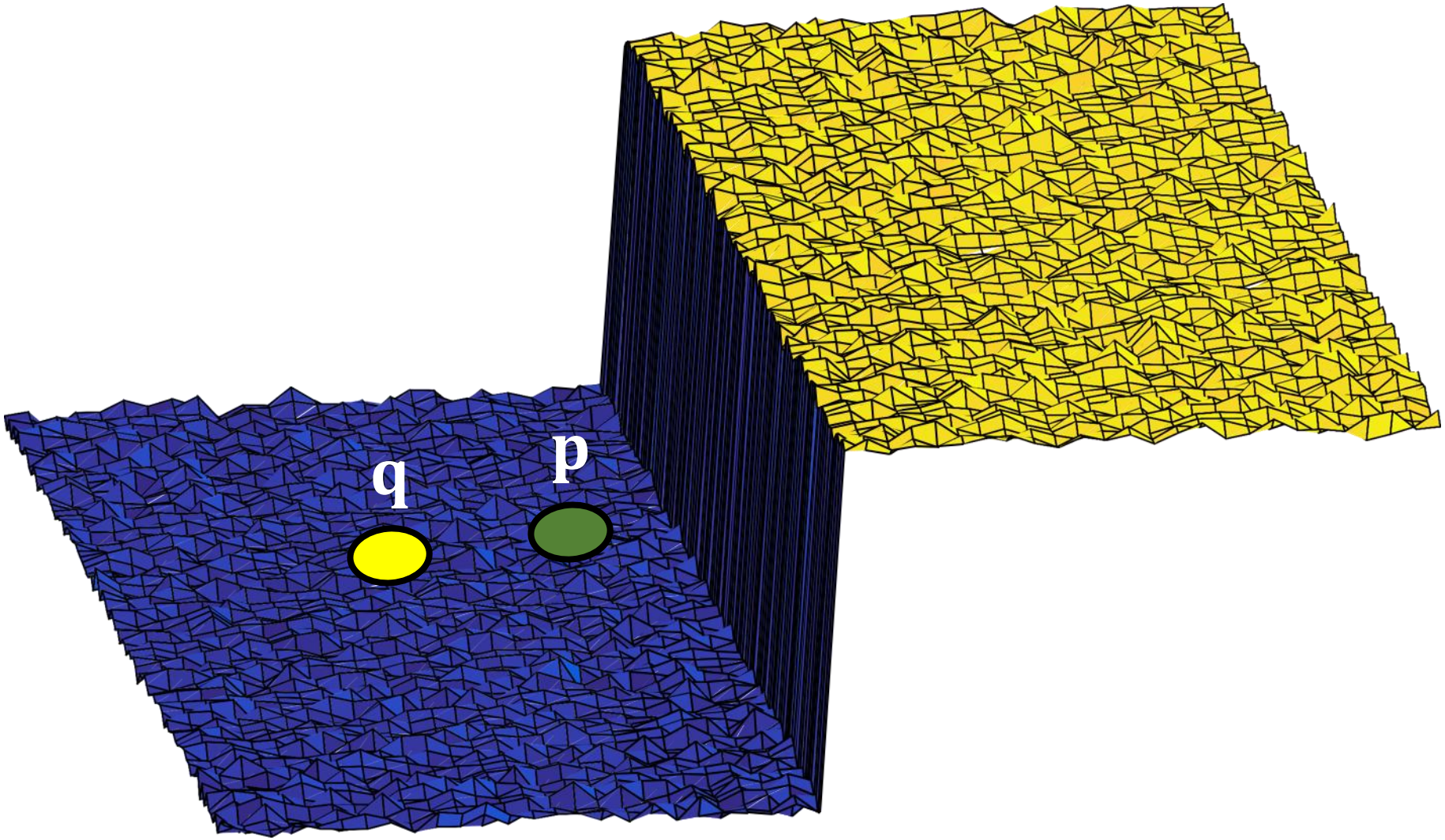
IEEE International Conference on Computer Vision (ICCV) 1998


高斯平滑

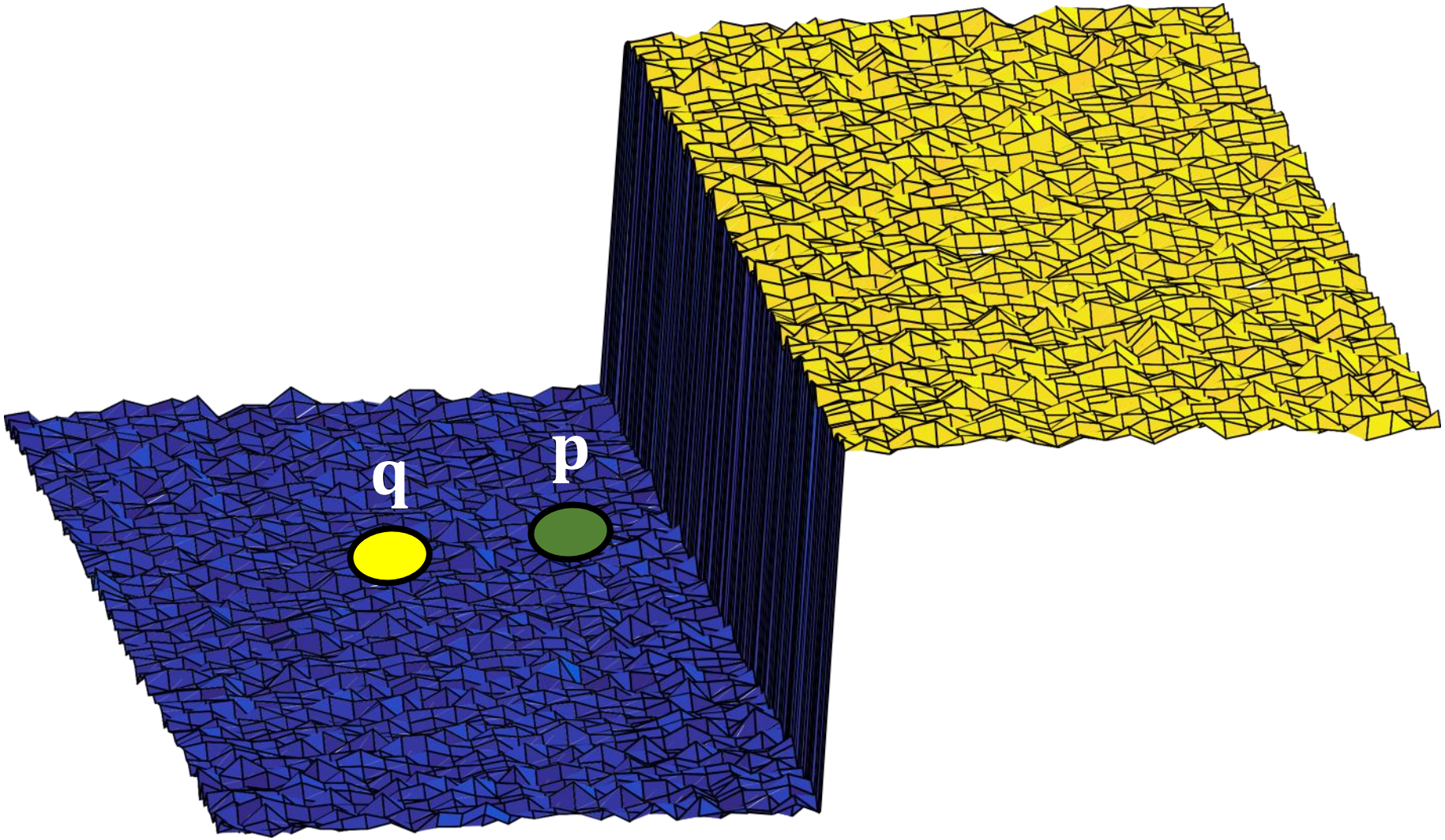

高斯平滑




p


q
p

$$I'(\mathbf{p}) = \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) I(\mathbf{q})$$



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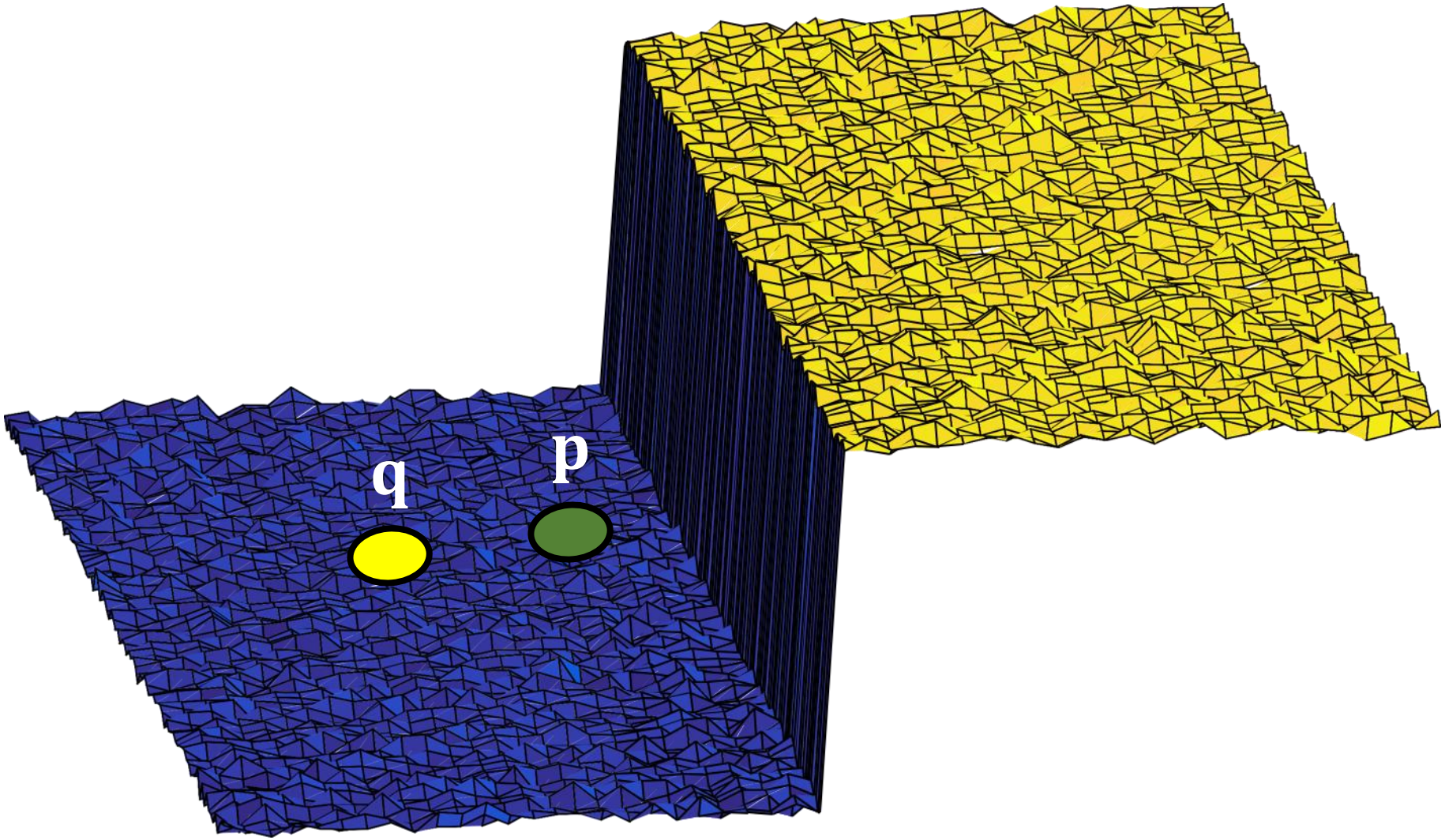
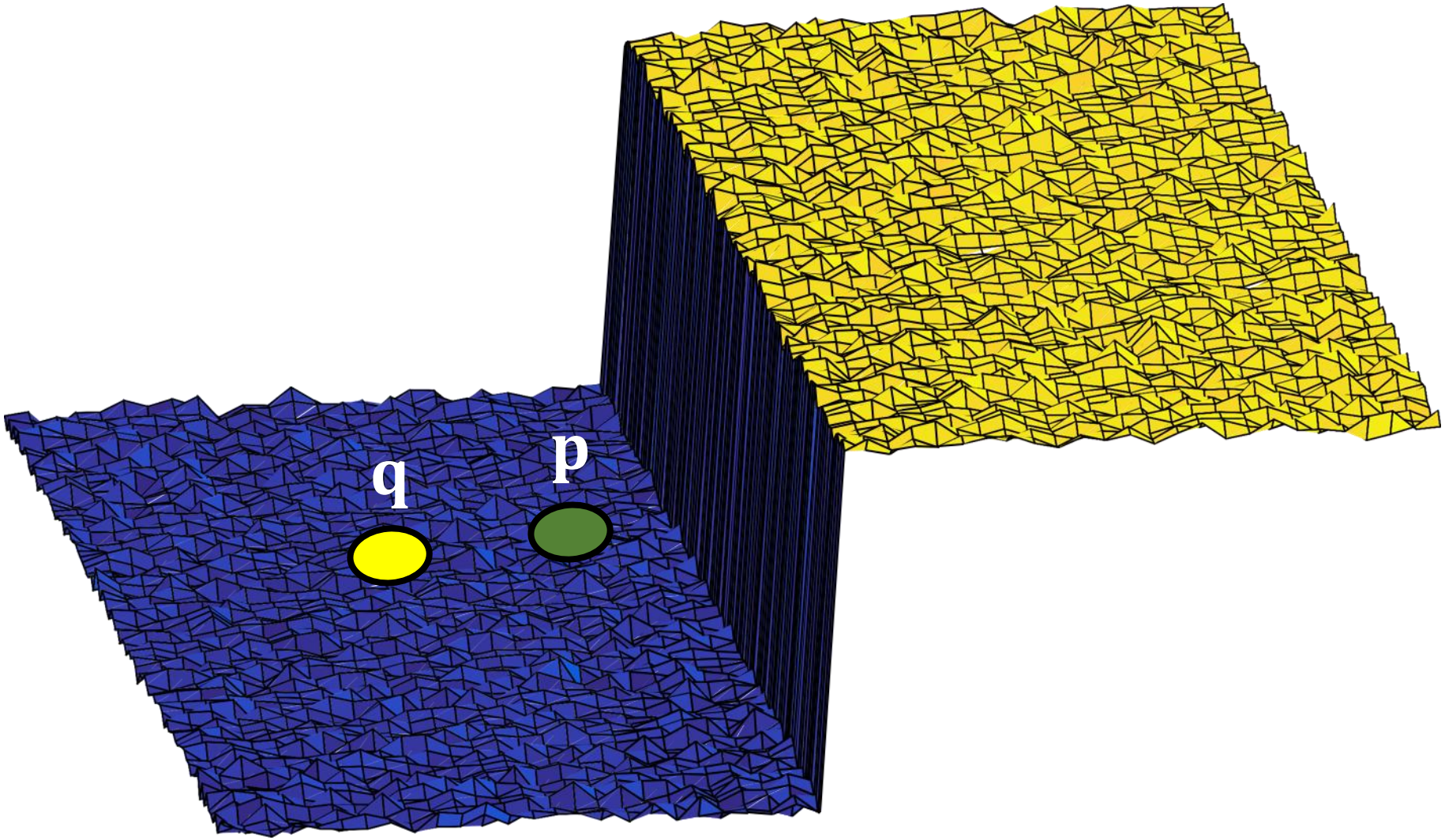


空间域权重

$$I'(\mathbf{p}) = \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) I(\mathbf{q})$$



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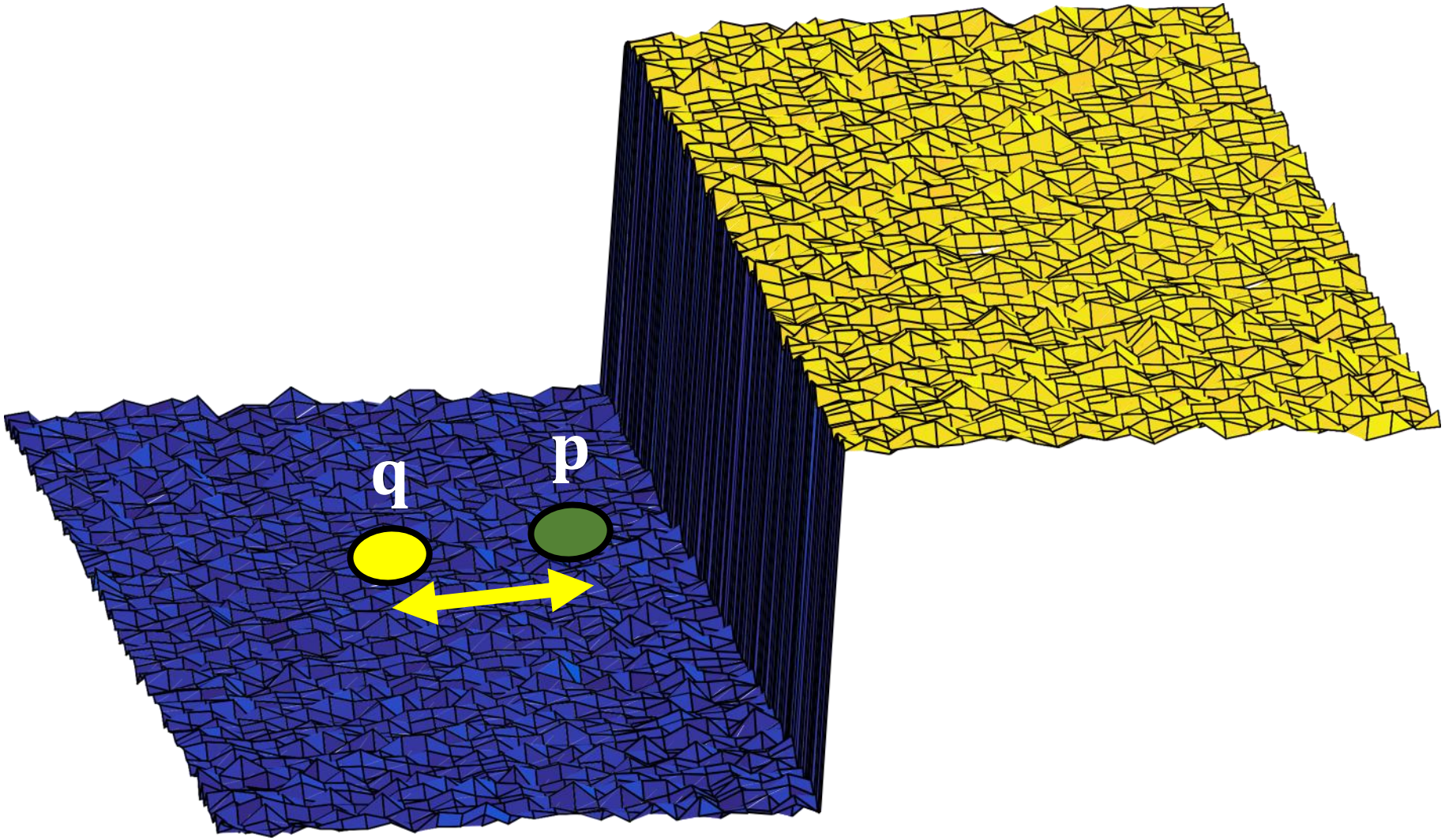

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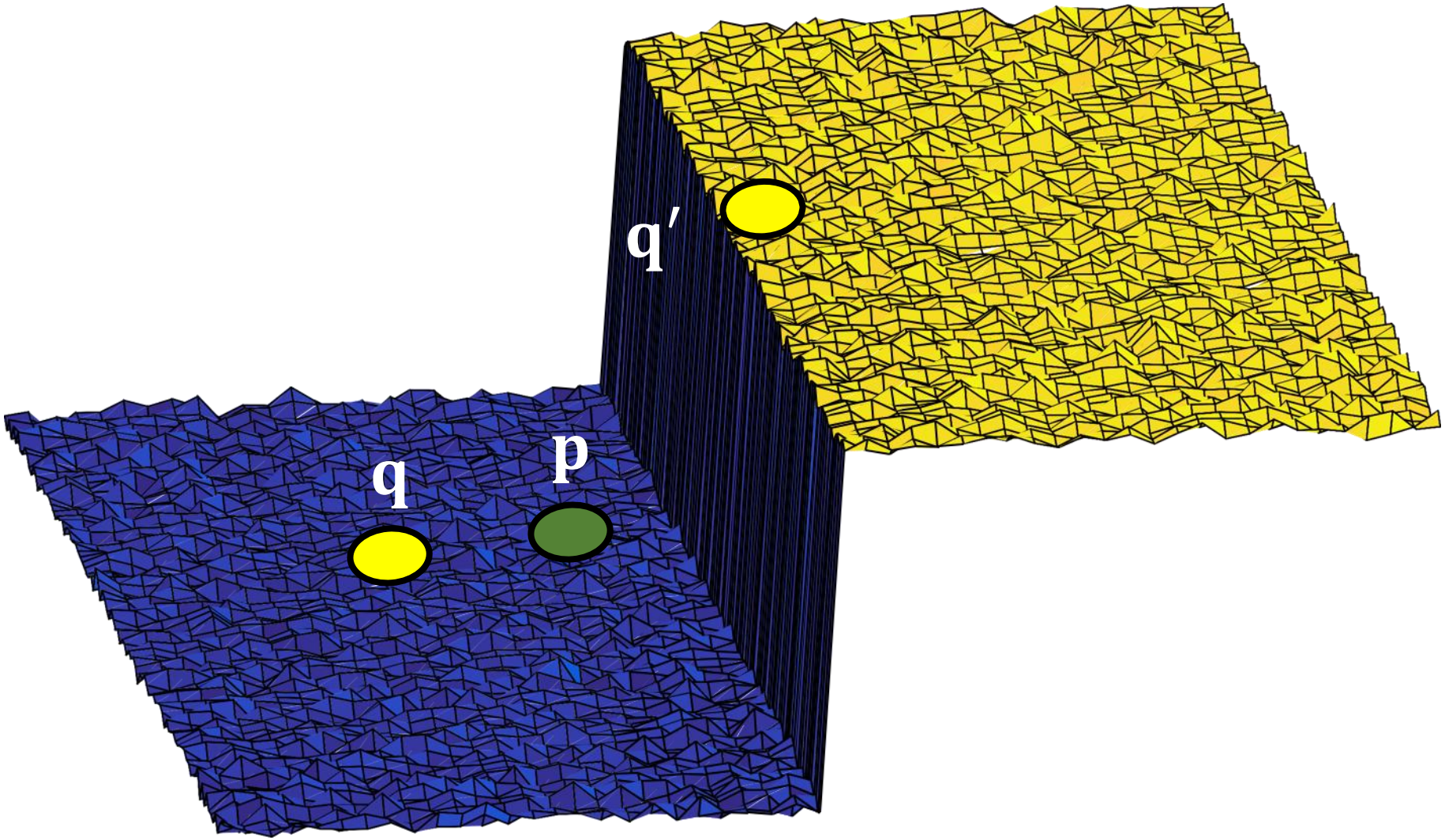

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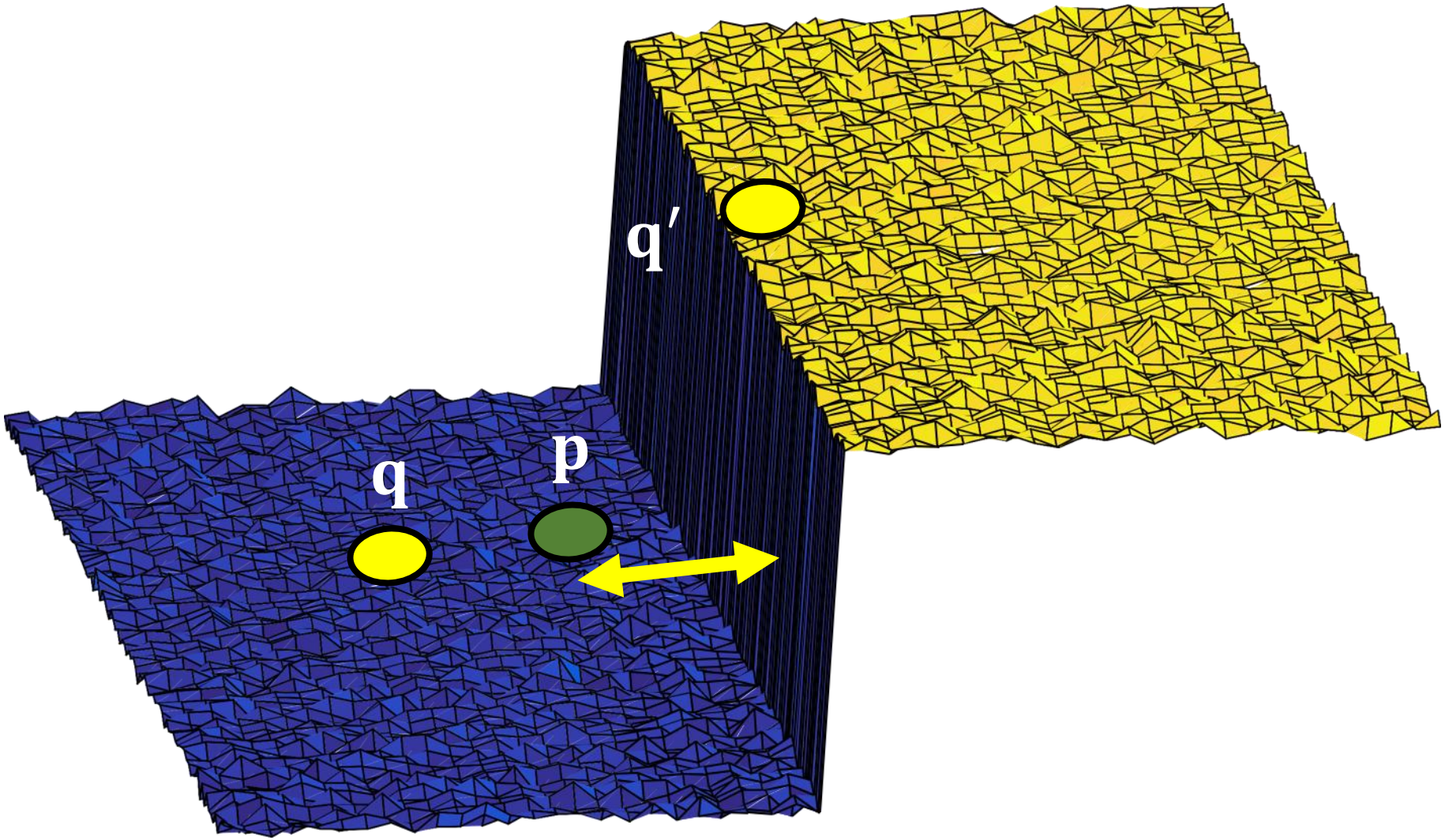

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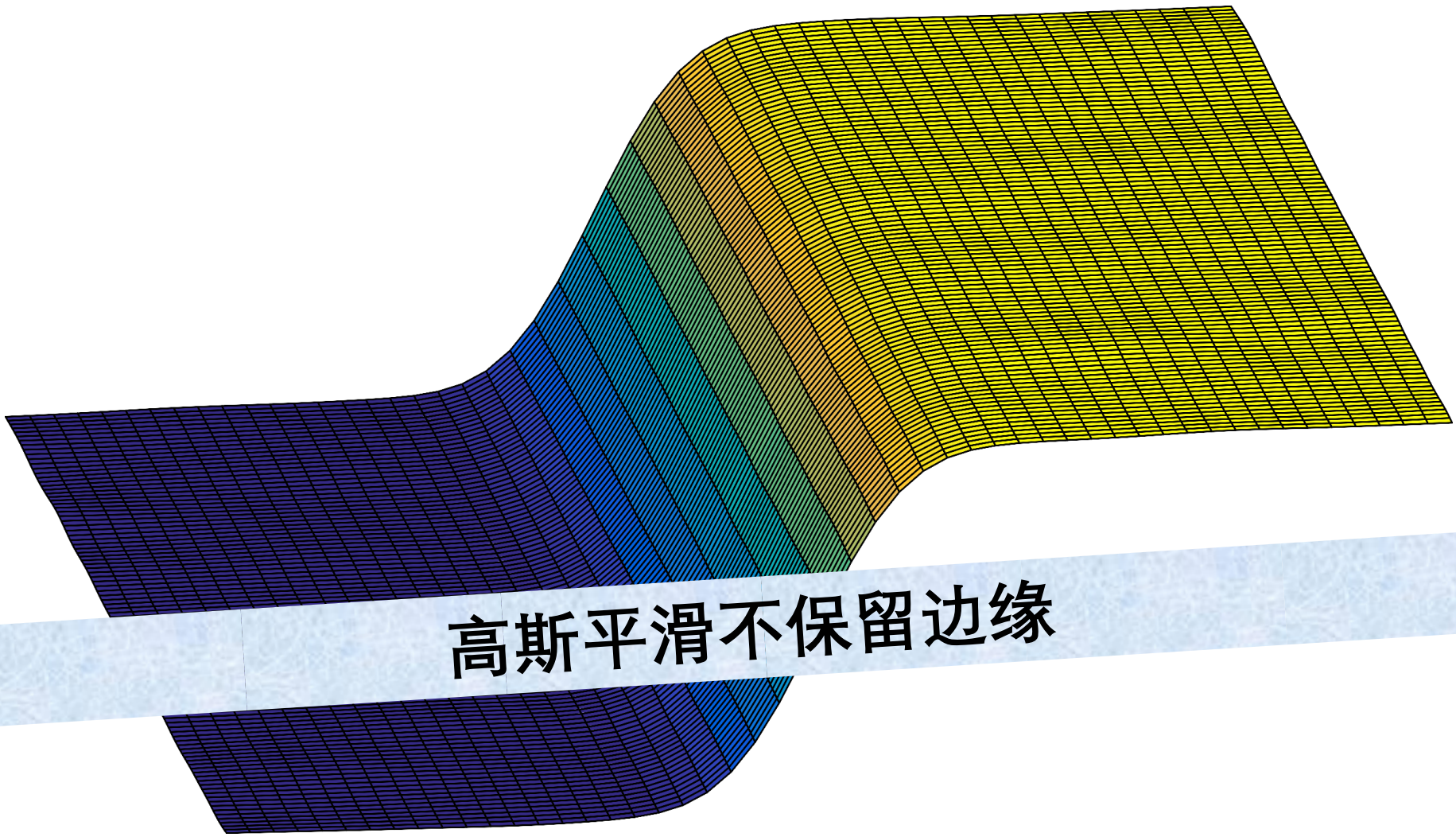
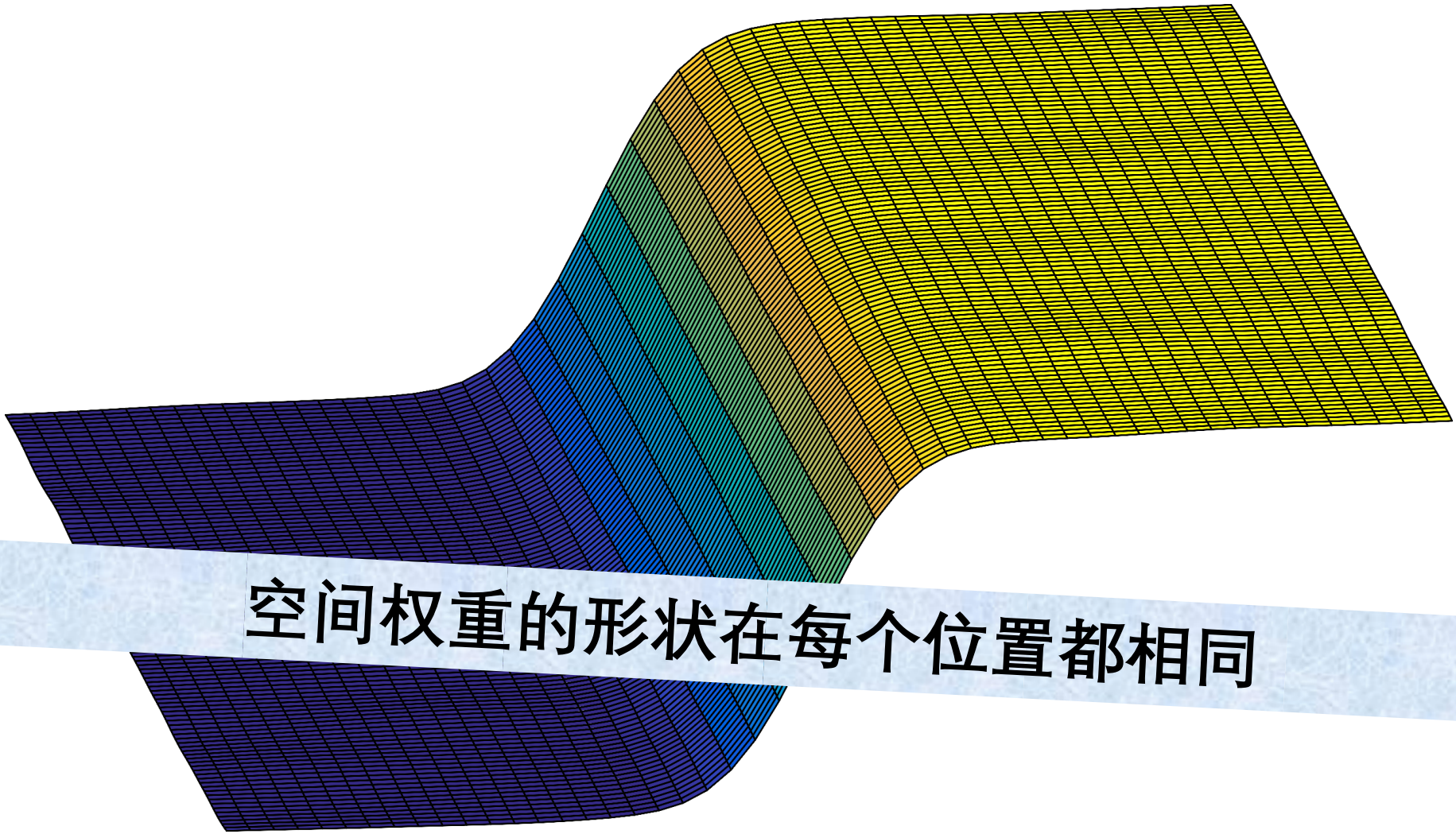


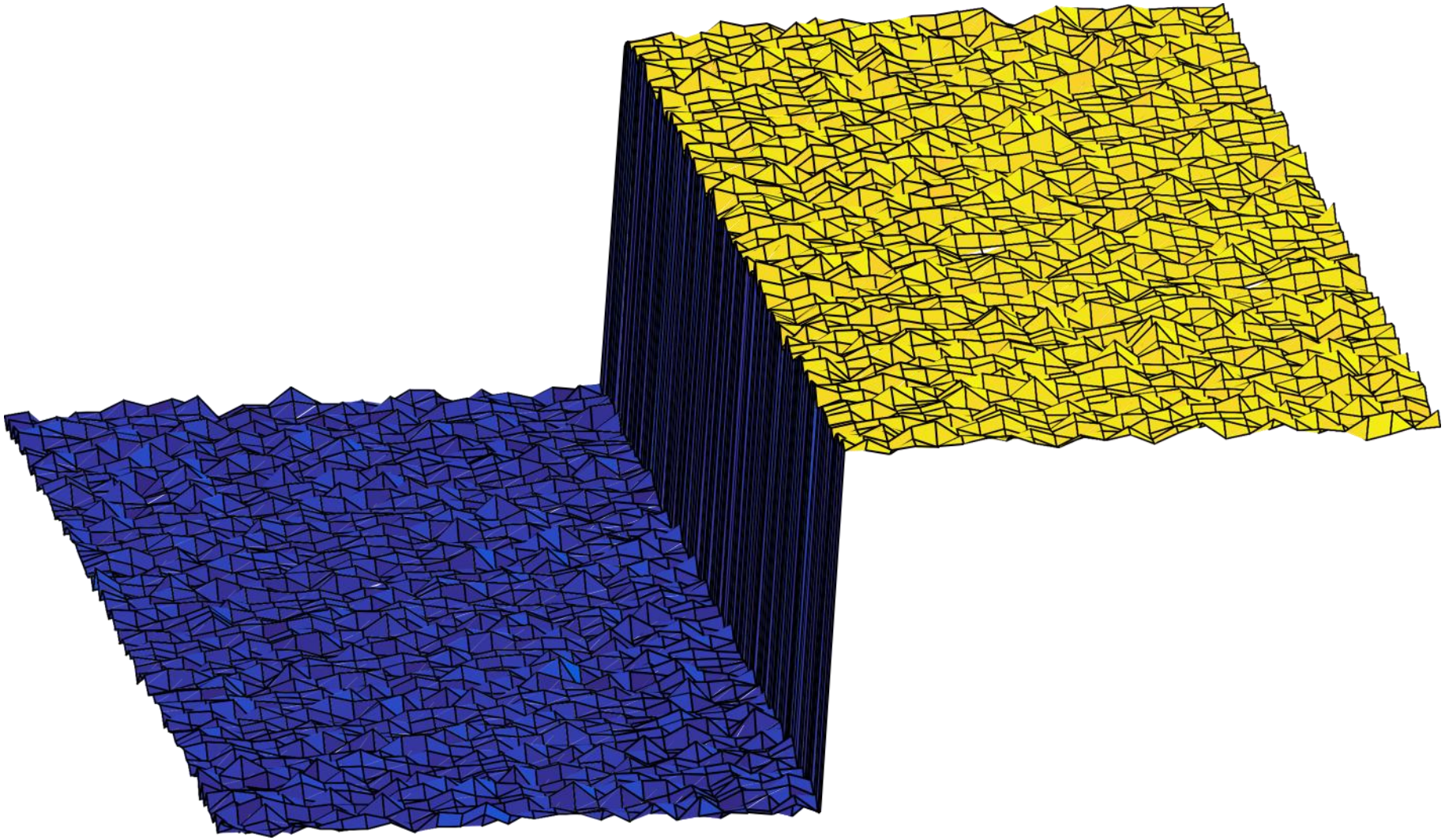
高斯平滑不保留边缘

$$I'(\mathbf{p}) = \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) I(\mathbf{q})$$

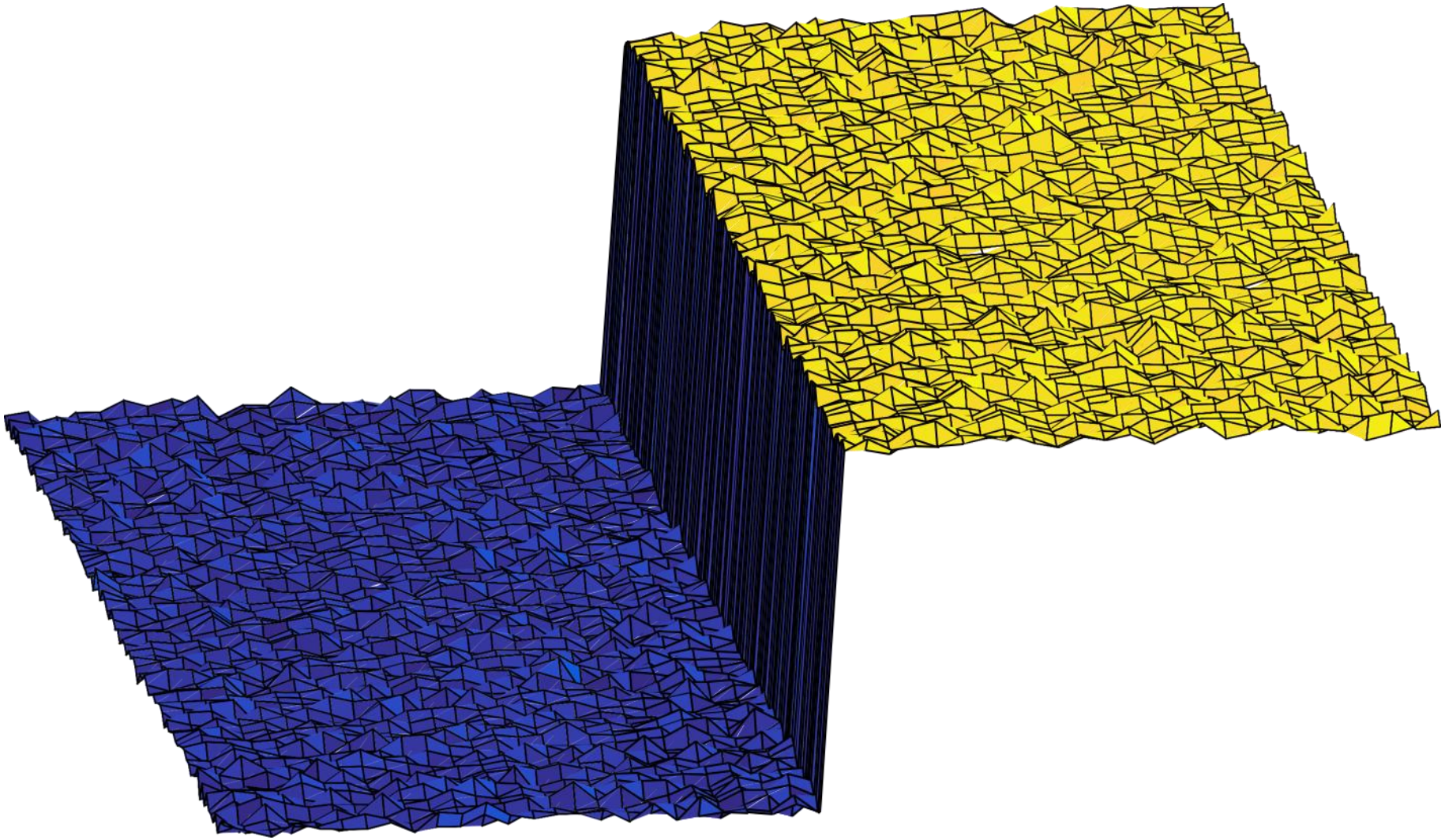


空间权重的形状在每个位置都相同

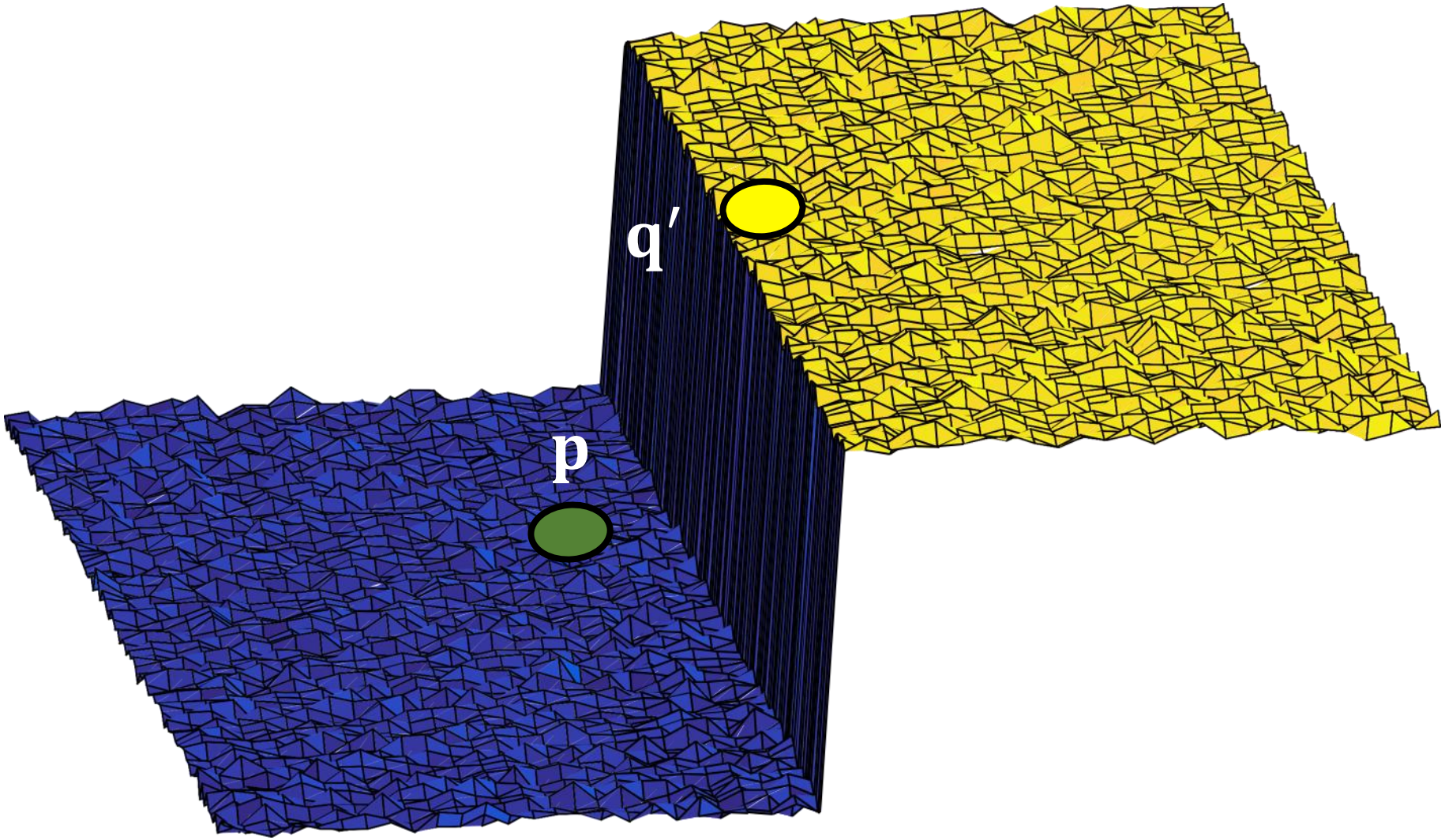
$$I'(\mathbf{p}) = \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) I(\mathbf{q})$$



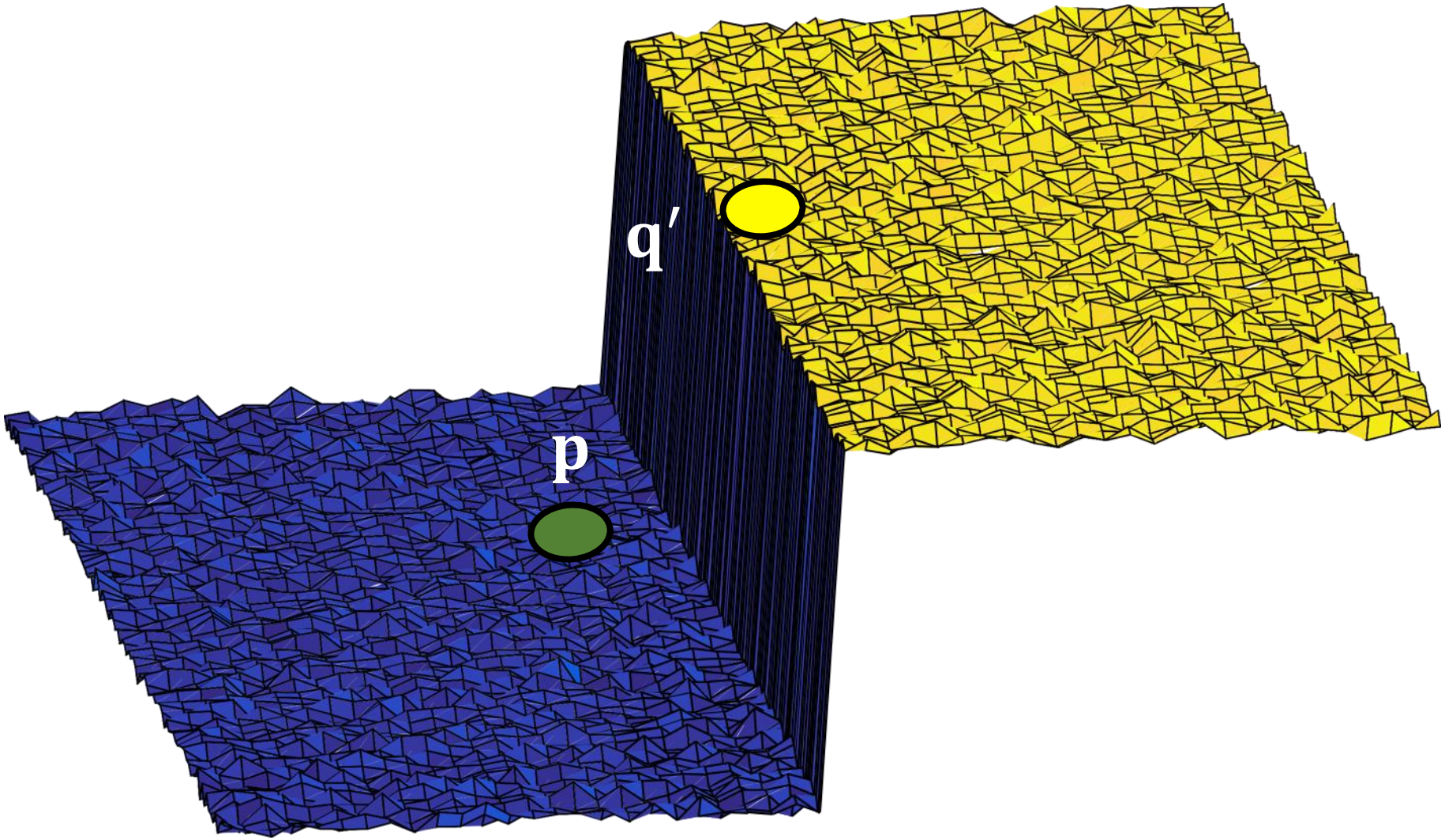
$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$



$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$

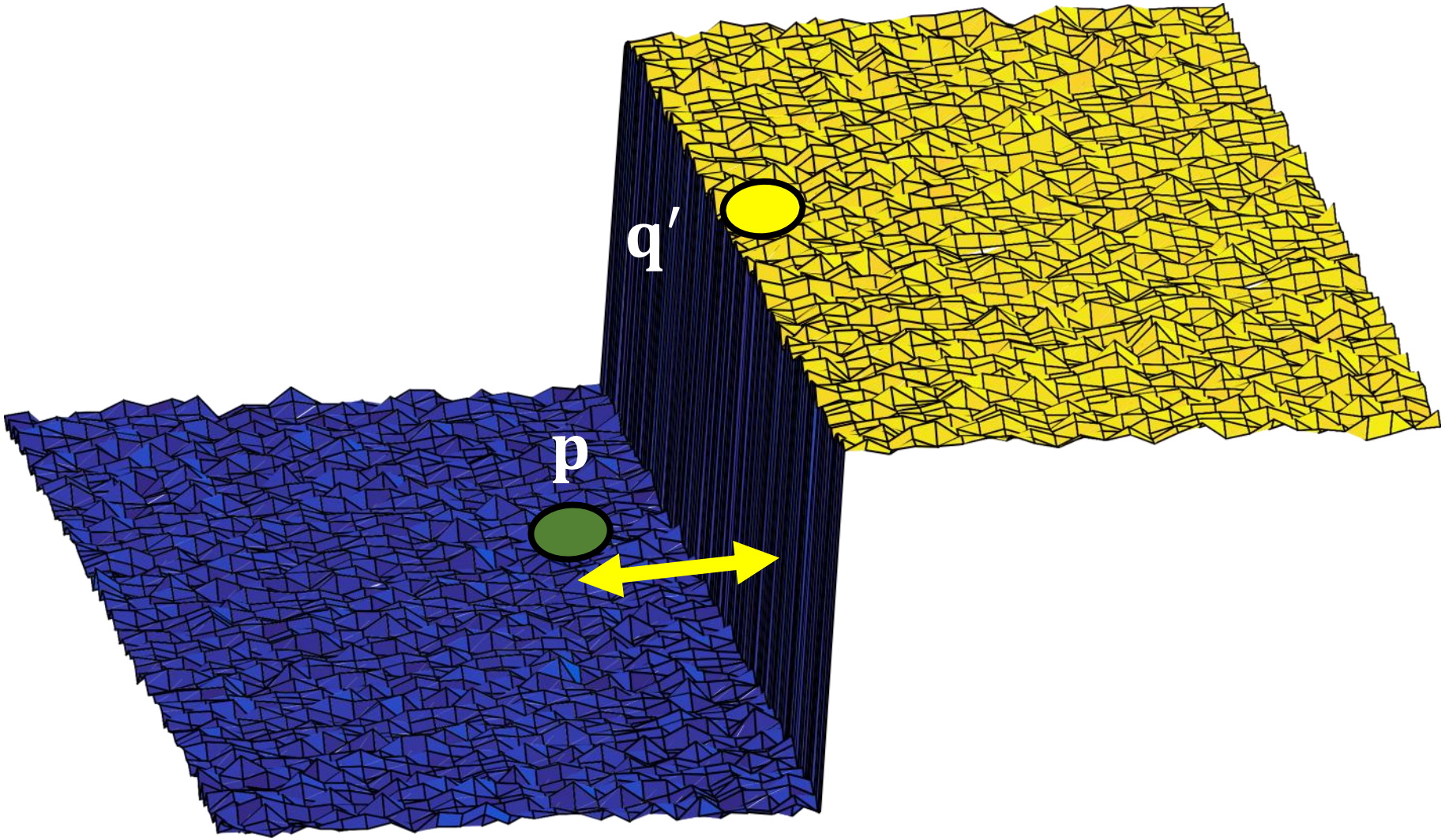


$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$



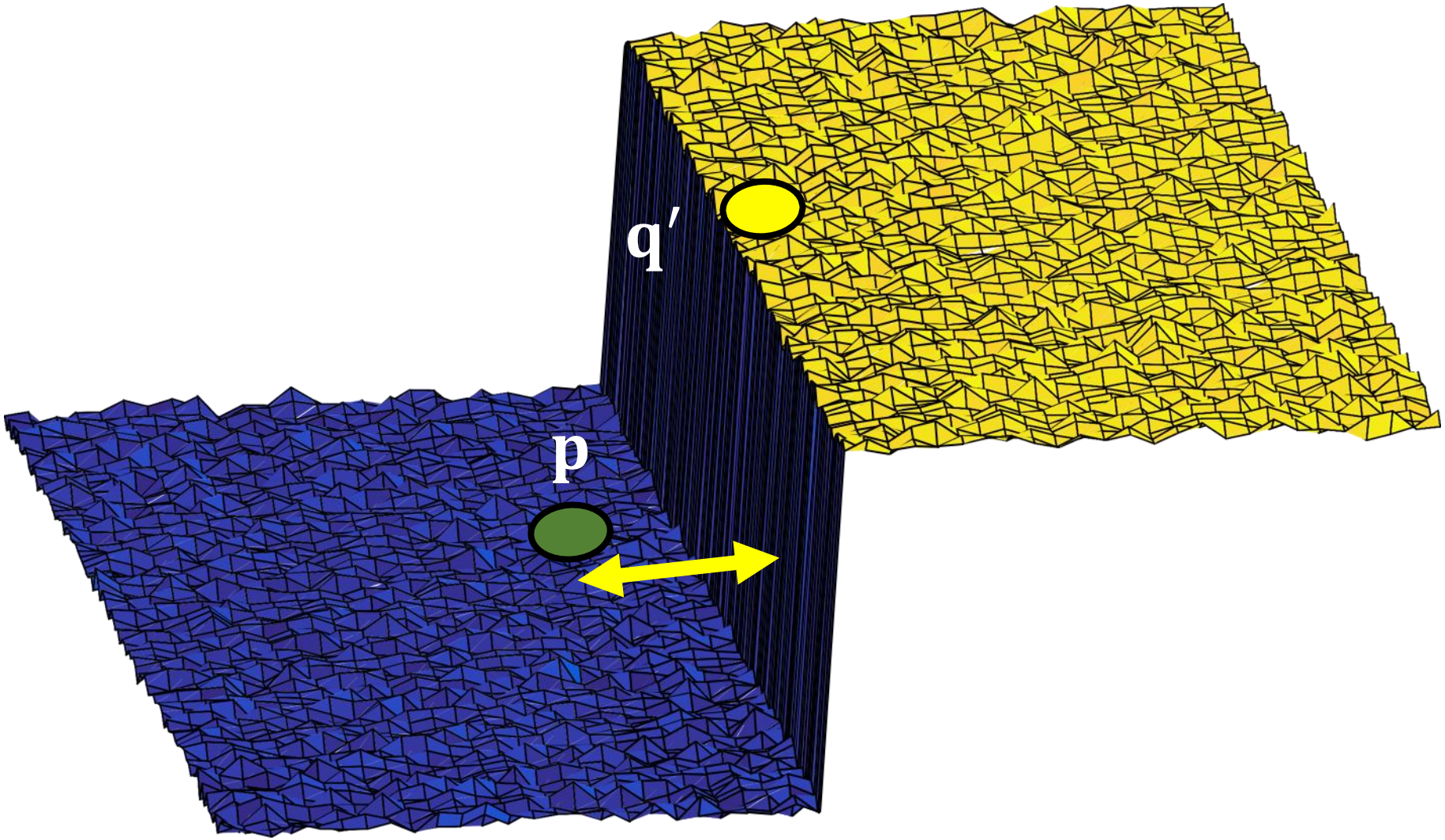
空间域权重

$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$



空间域权重

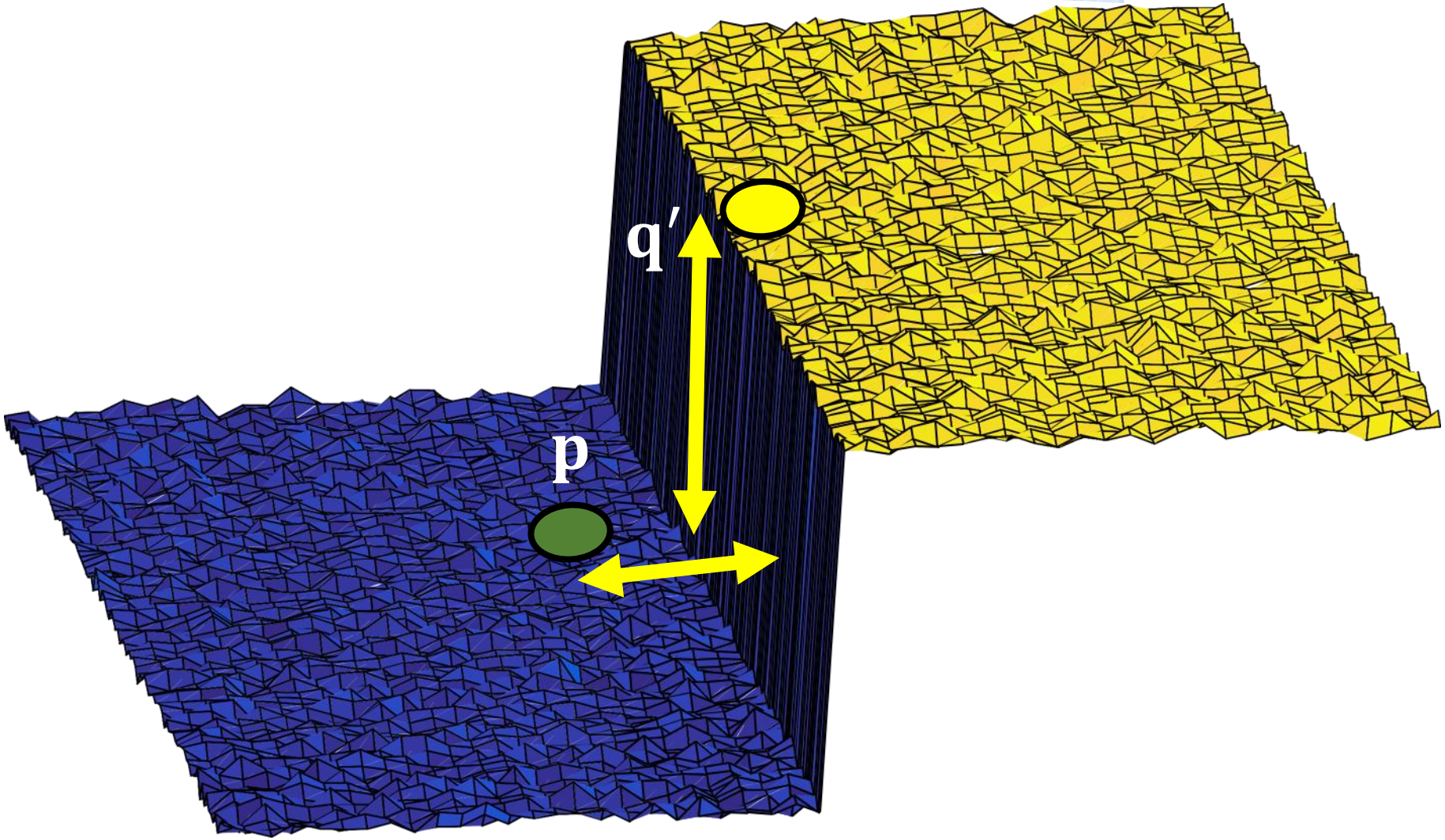
$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$



空间域权重

$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$

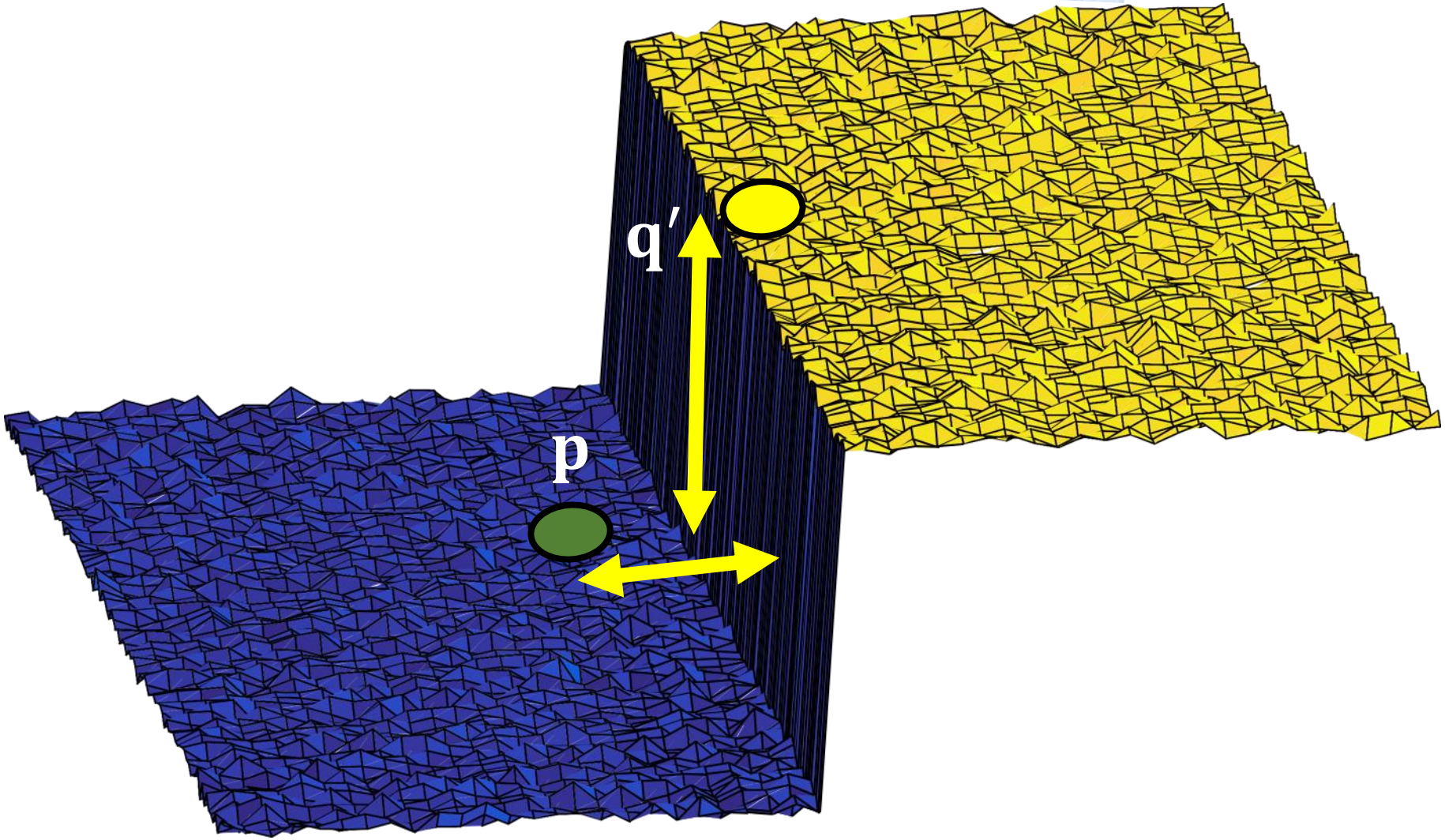
值域权重



空间域权重

$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q}\in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p})-I(\mathbf{q})\|) I(\mathbf{q})$$

值域权重

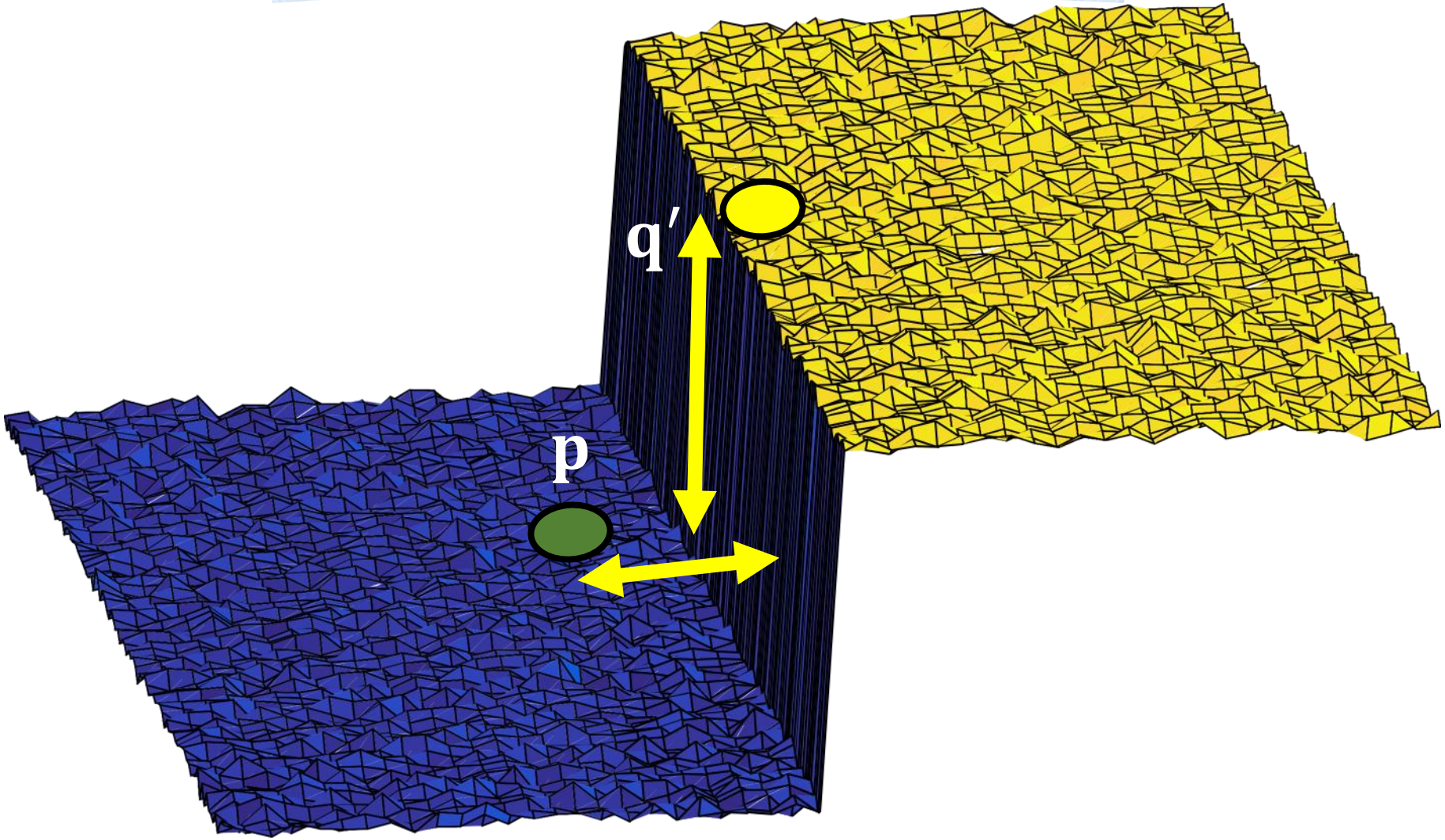


空间域权重

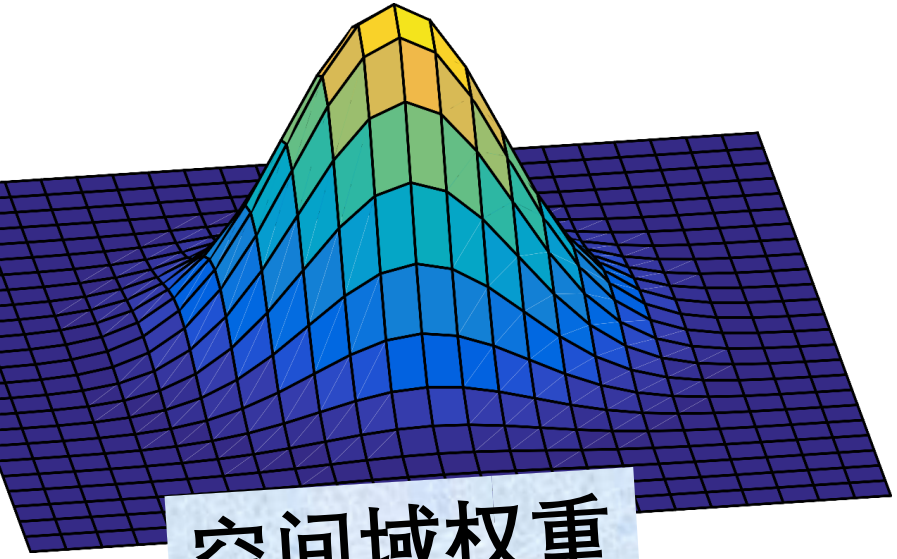
$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q}\in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p})-I(\mathbf{q})\|) I(\mathbf{q})$$

归一化

值域权重

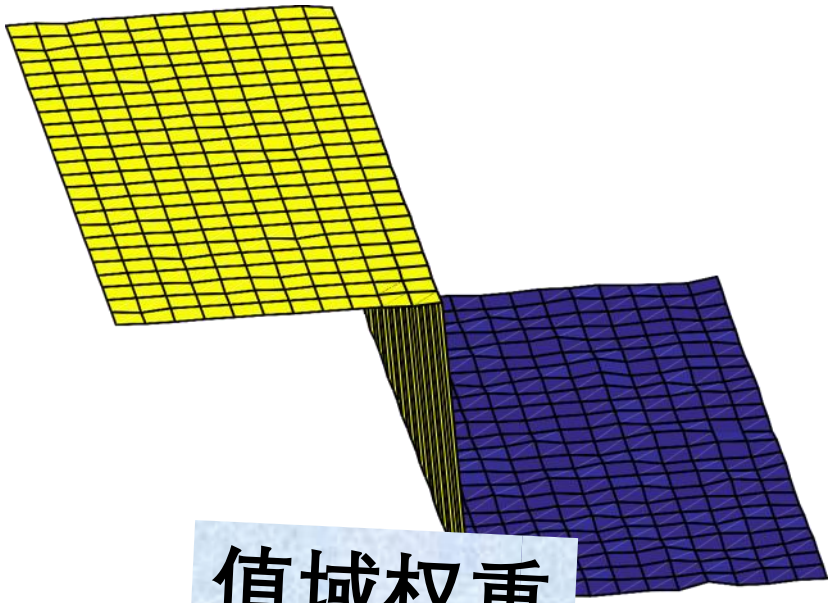


$G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)$



空间域权重

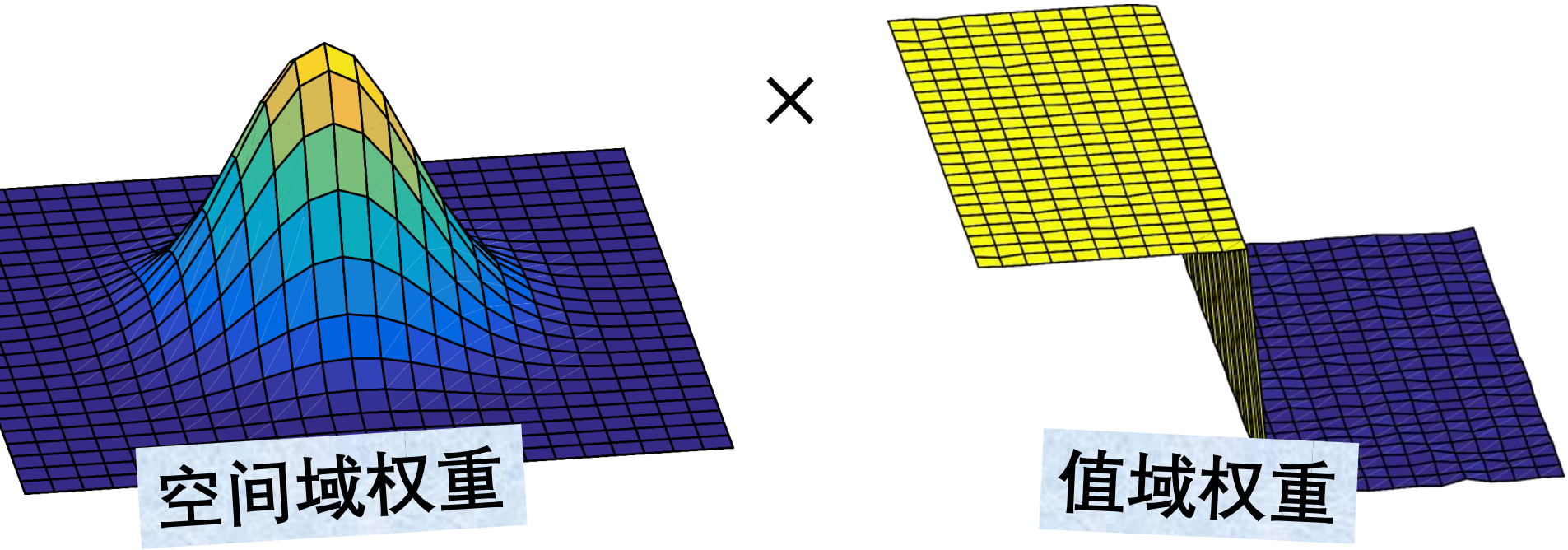
$G_{\sigma_r}(\|I(\mathbf{p})-I(\mathbf{q})\|)$



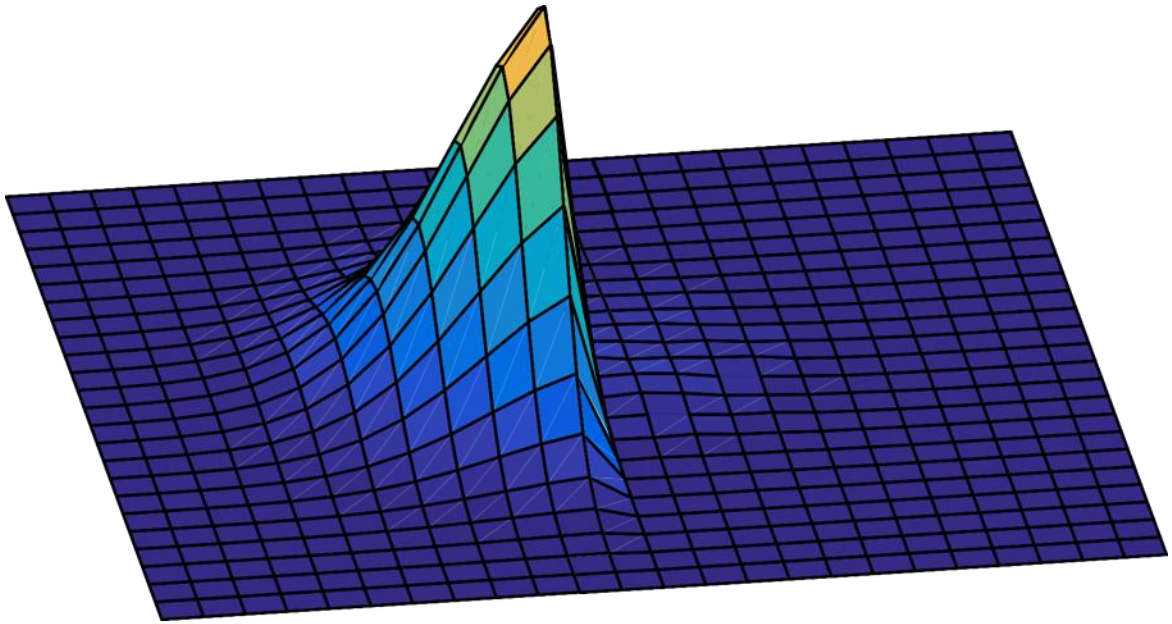
值域权重

$G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)$

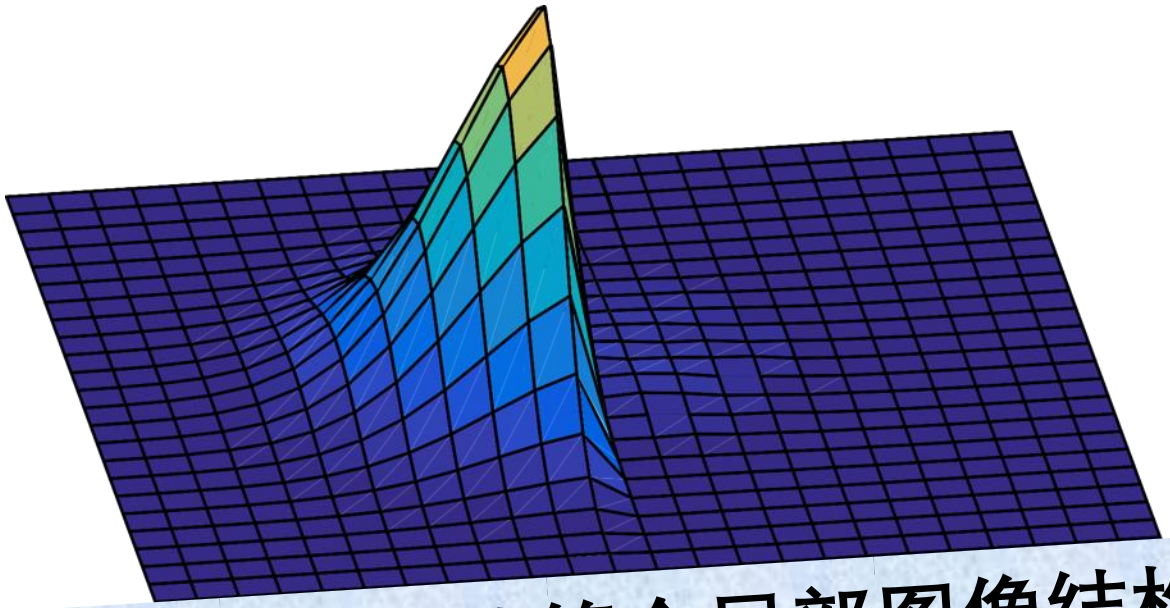
$G_{\sigma_r}(\|I(\mathbf{p})-I(\mathbf{q})\|)$



$$w(\mathbf{p})G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)G_{\sigma_r}(\|I(\mathbf{p})-I(\mathbf{q})\|)$$

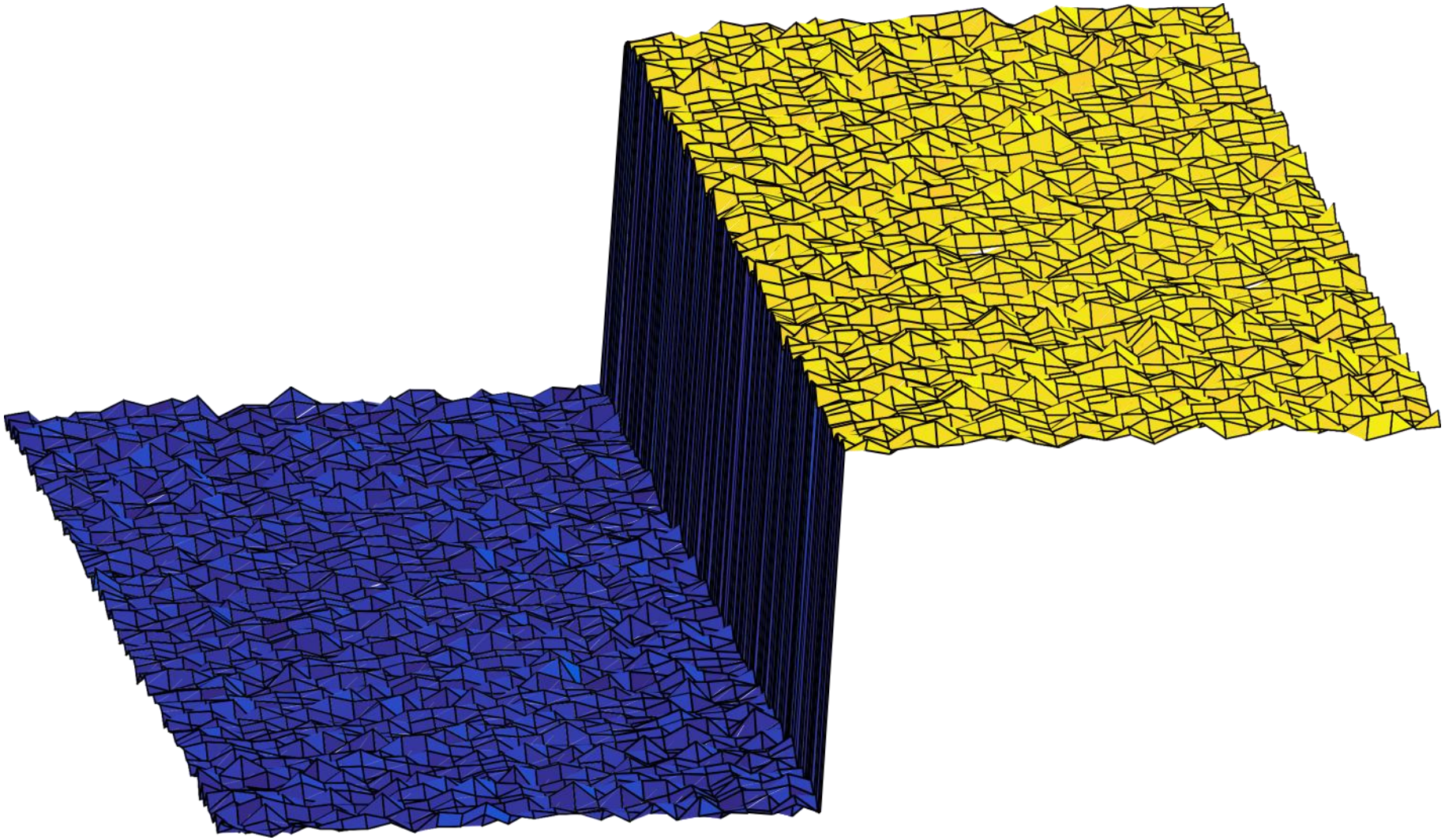


$$w(\mathbf{p})G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)G_{\sigma_r}(\|I(\mathbf{p})-I(\mathbf{q})\|)$$



权重滤波器的形状符合局部图像结构

$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$



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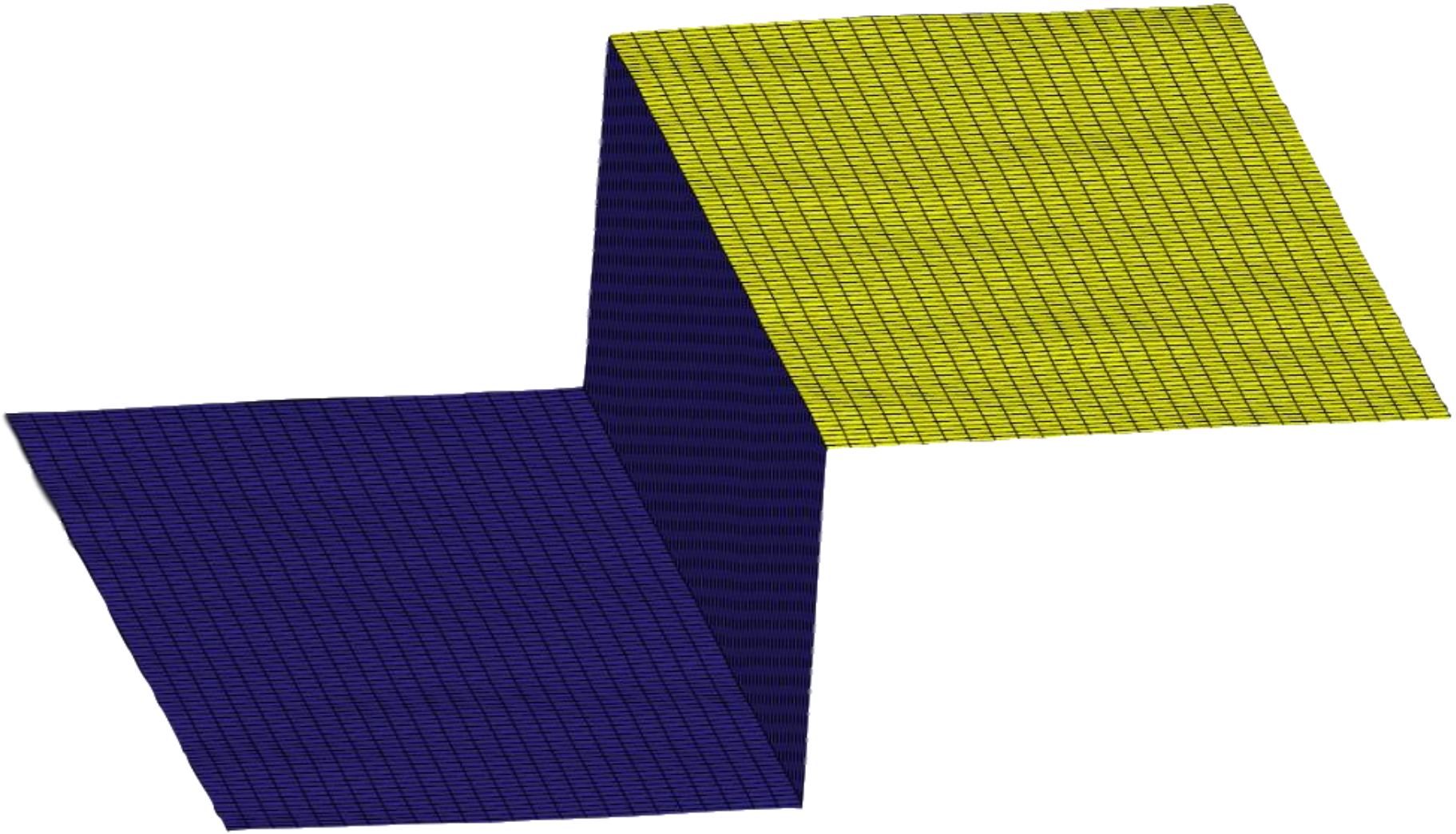
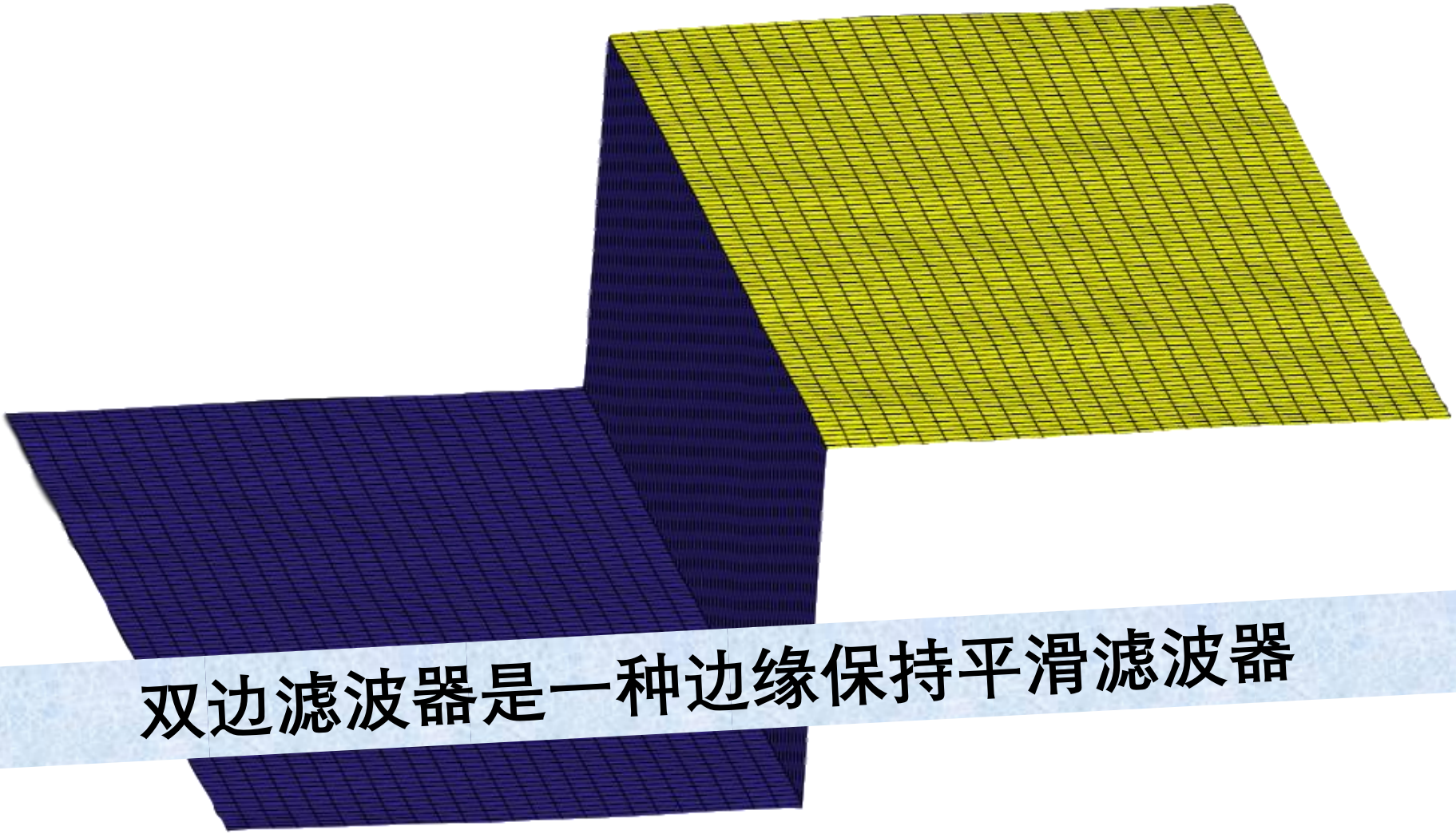


$$I'(\mathbf{p}) = w(\mathbf{p}) \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I(\mathbf{p}) - I(\mathbf{q})\|) I(\mathbf{q})$$



双边滤波器是一种边缘保持平滑滤波器

双边滤波

双边滤波