计算机视觉

3D结构与运动

中国传媒大学
COMMUNICATION UNIVERSITY OF CHINA

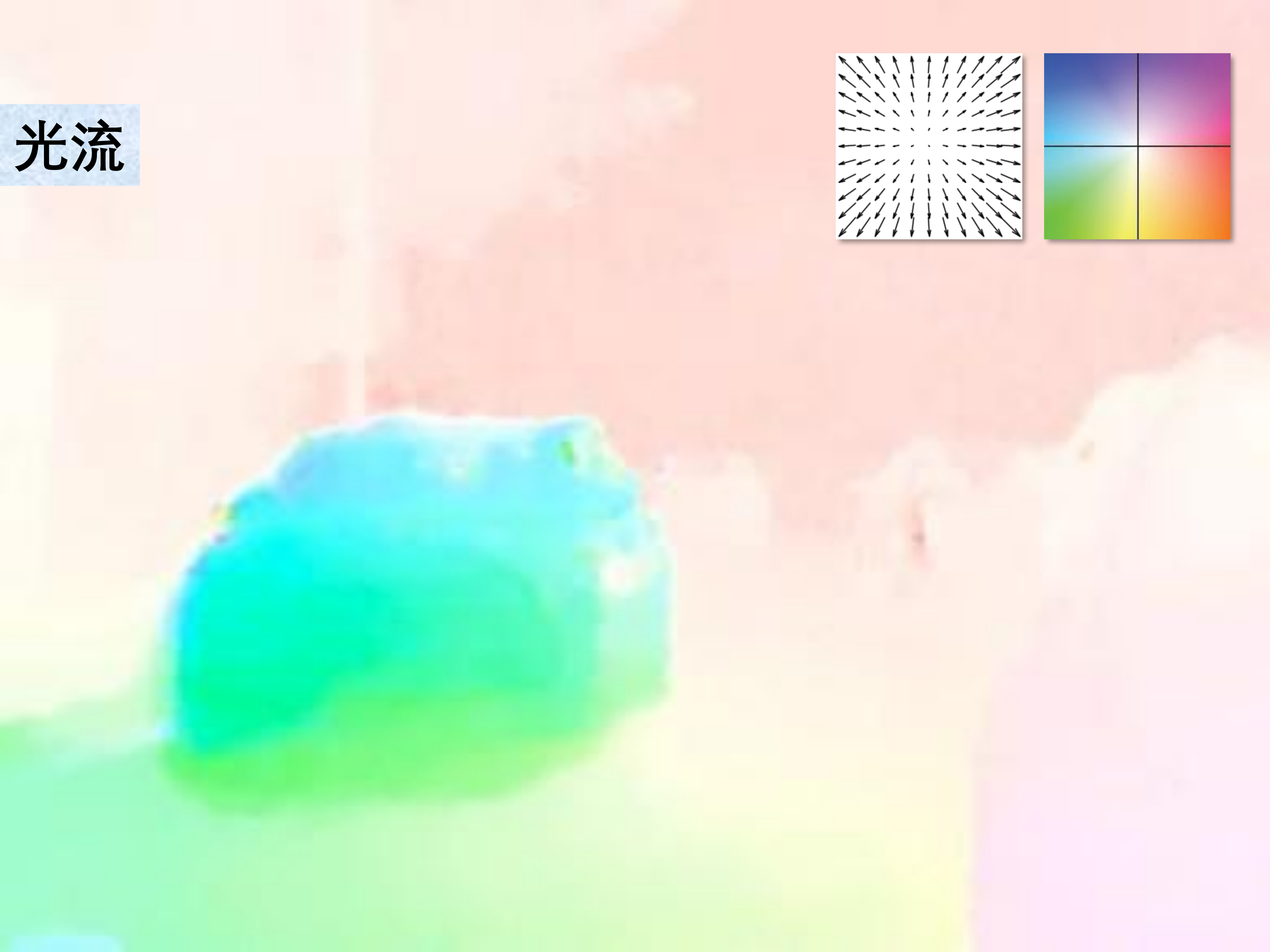
上节课

从图像中恢复运动

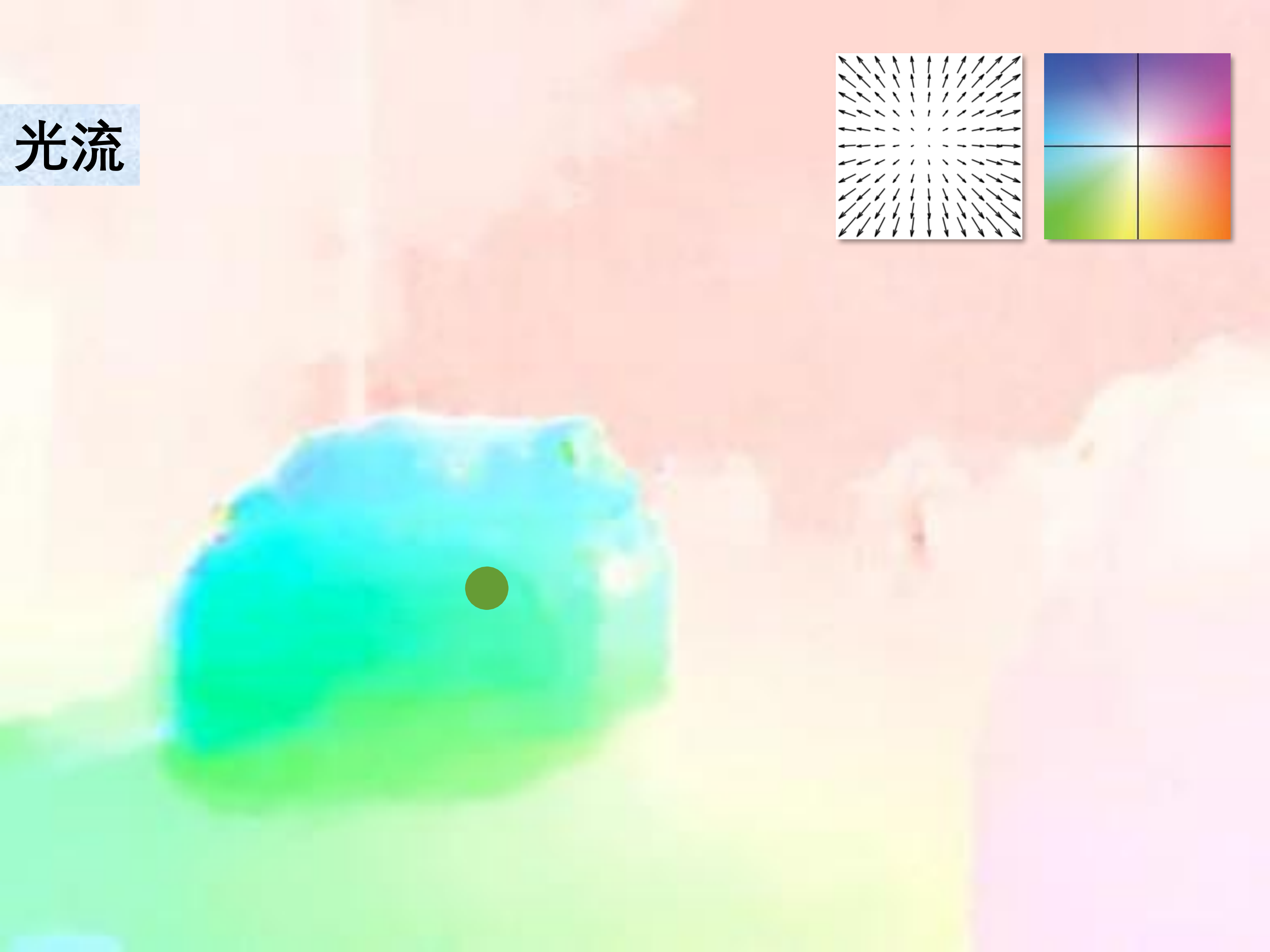
回顾


输入序列
30

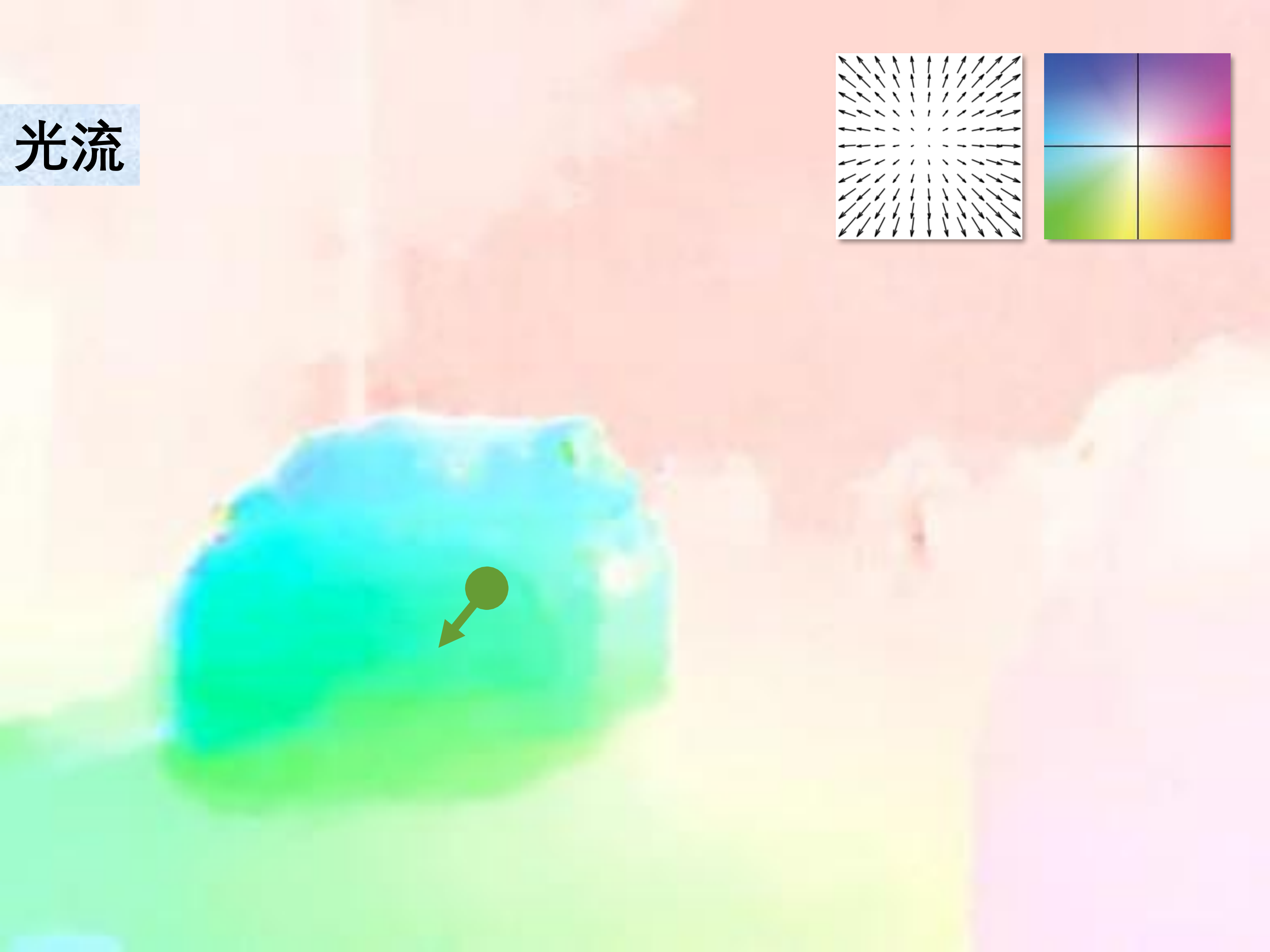
光流



光流



光流



运动场
vs.
光流

几何结构

运动场

vs.

光流

几何结构

运动场

vs.

光流

光度概念

运动场

$\neq$

光流

运动场

$\neq$

光流

光流近似运动场

亮度恒常假设

$$I(x, y, t) = I(x + u\Delta t, y + v\Delta t, t + \Delta t)$$

亮度恒常约束： $I_x u + I_y v + I_t = 0$

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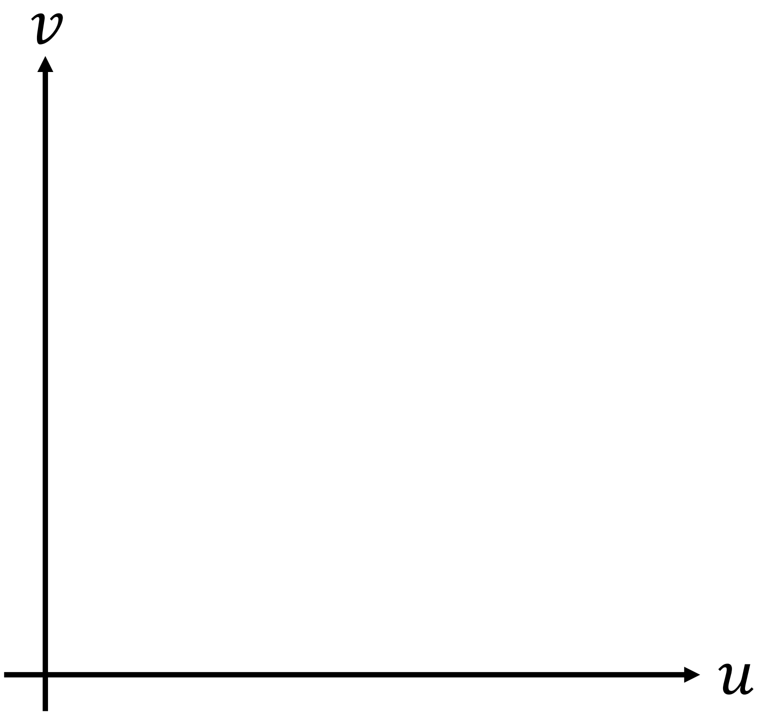
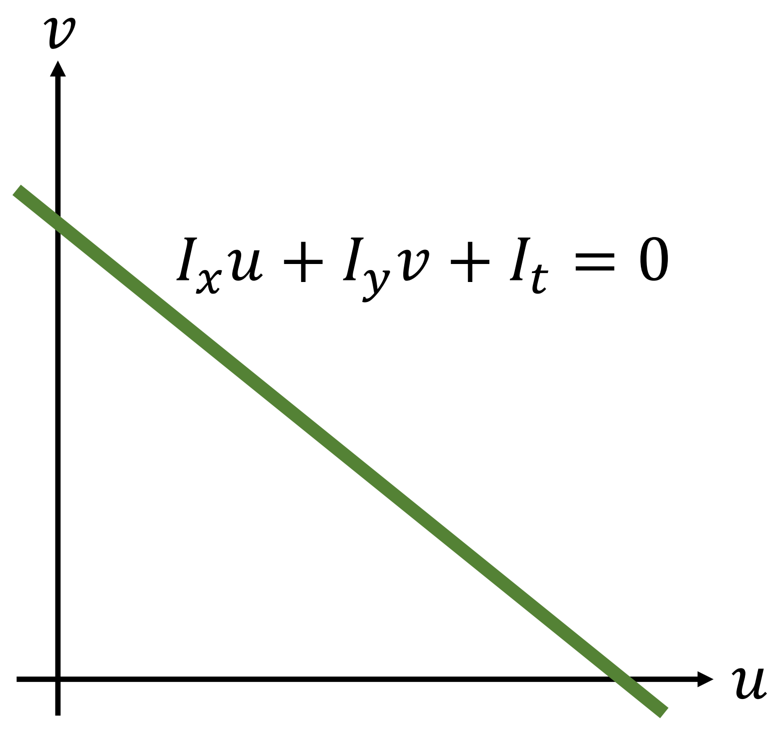


亮度恒常约束： $I_x u + I_y v + I_t = 0$



解决方案：引入其他约束

An Iterative Image Registration Technique
with an Application to Stereo Vision

Bruce D. Lucas
Takeo Kanade

Computer Science Department
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

局部方法

Determining Optical Flow

Berthold K.P. Horn and Brian G. Schunck
Artificial Intelligence Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

全局方法

由运动恢复结构

无穷小方法

问题陈述

给定一个由相机运动产生的光流场

问题陈述

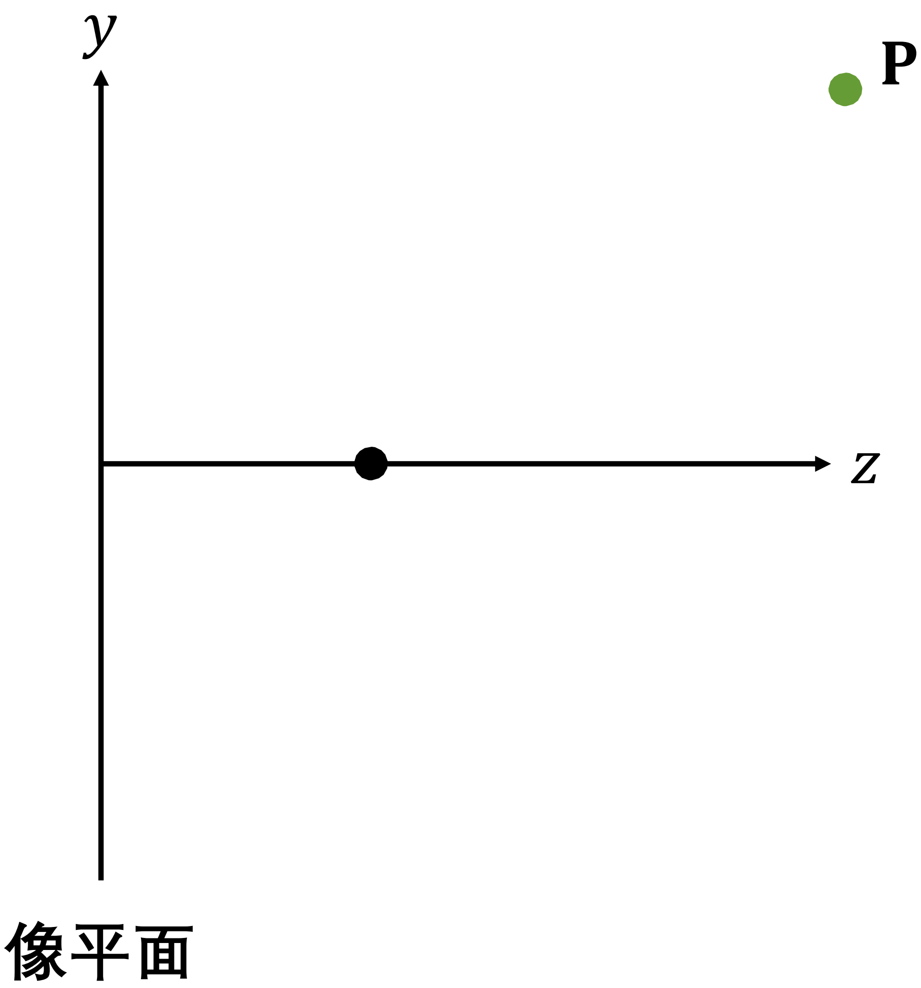
给定一个由相机运动产生的光流场

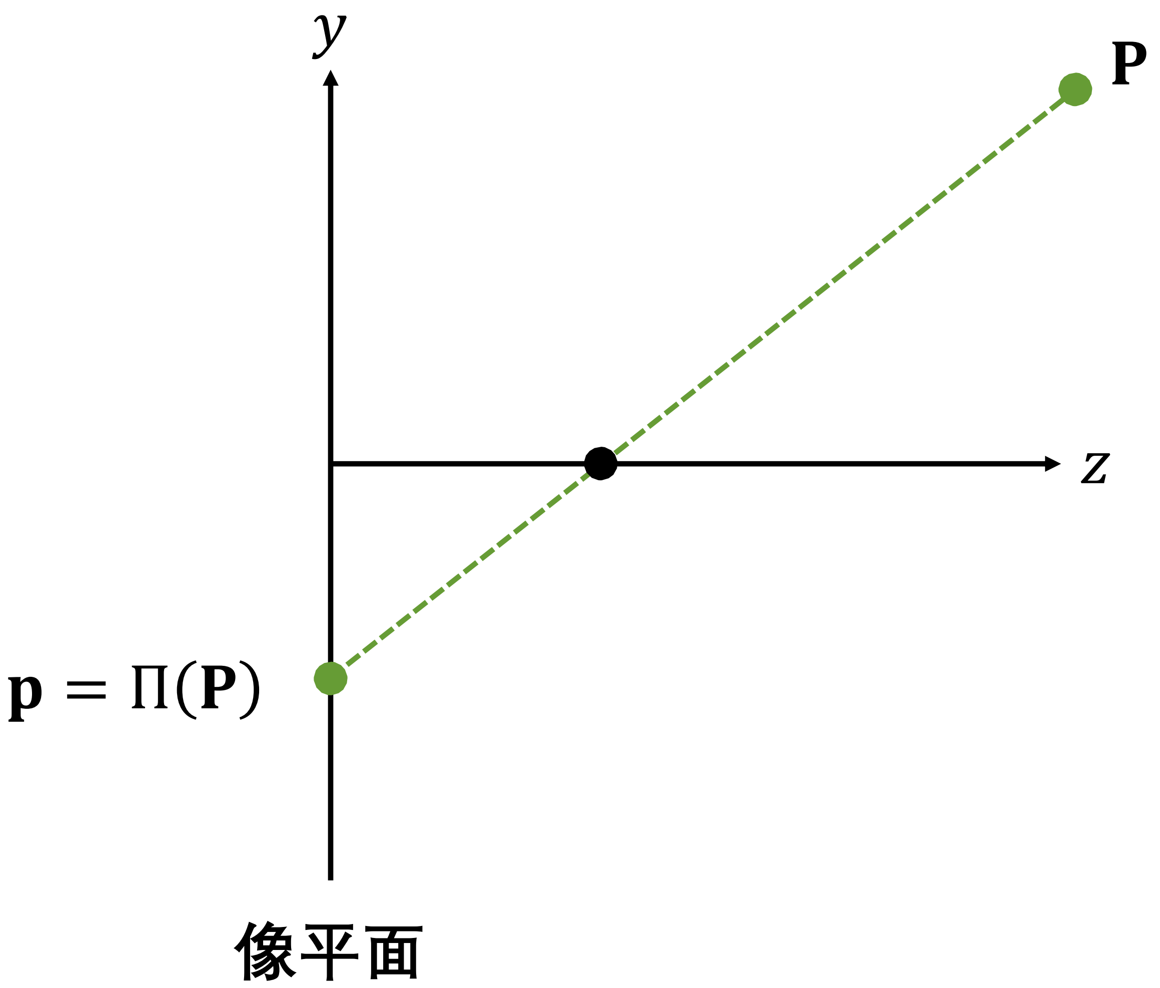
恢复相机的3D旋转和平移速度

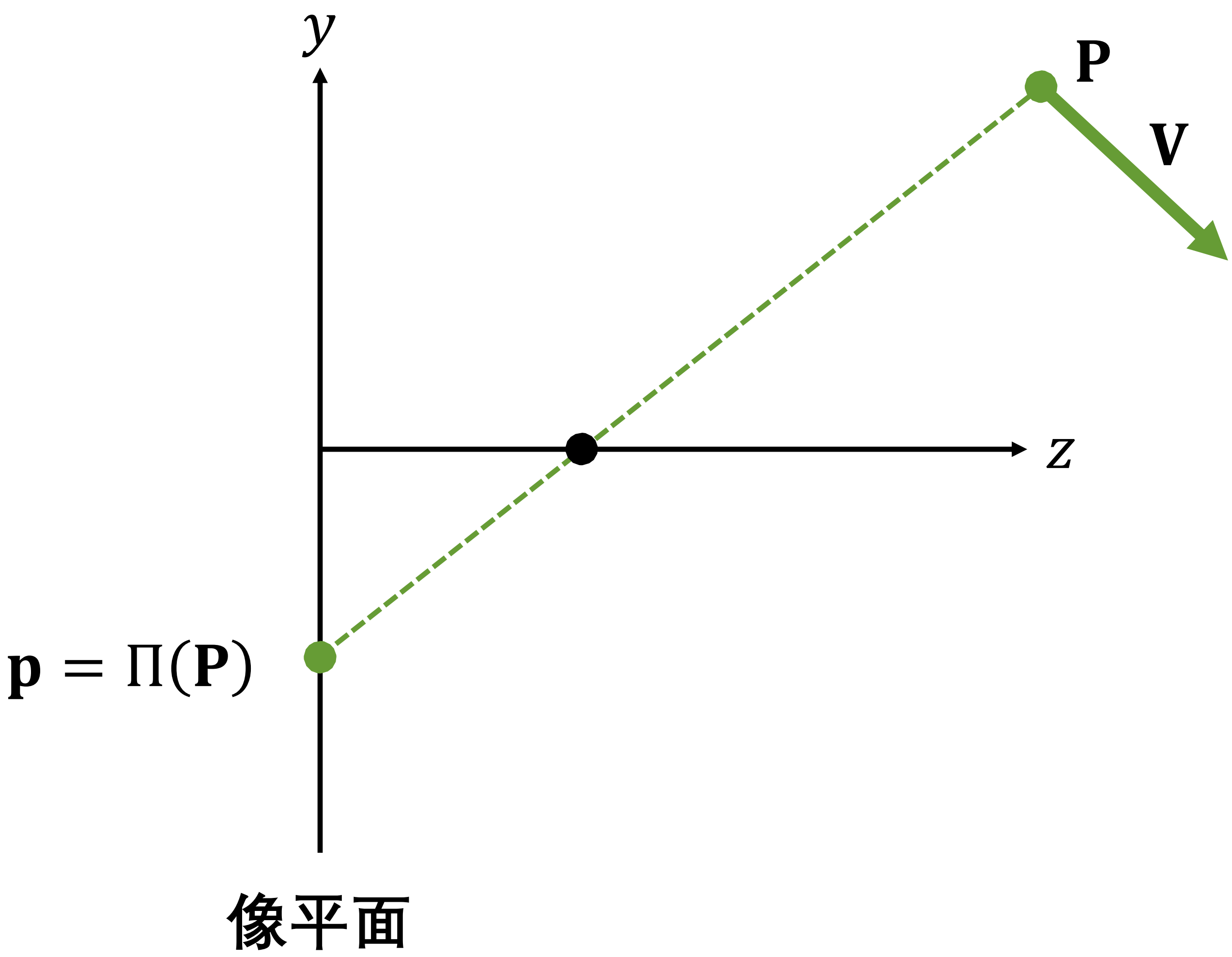
问题陈述

给定一个由相机运动产生的光流场

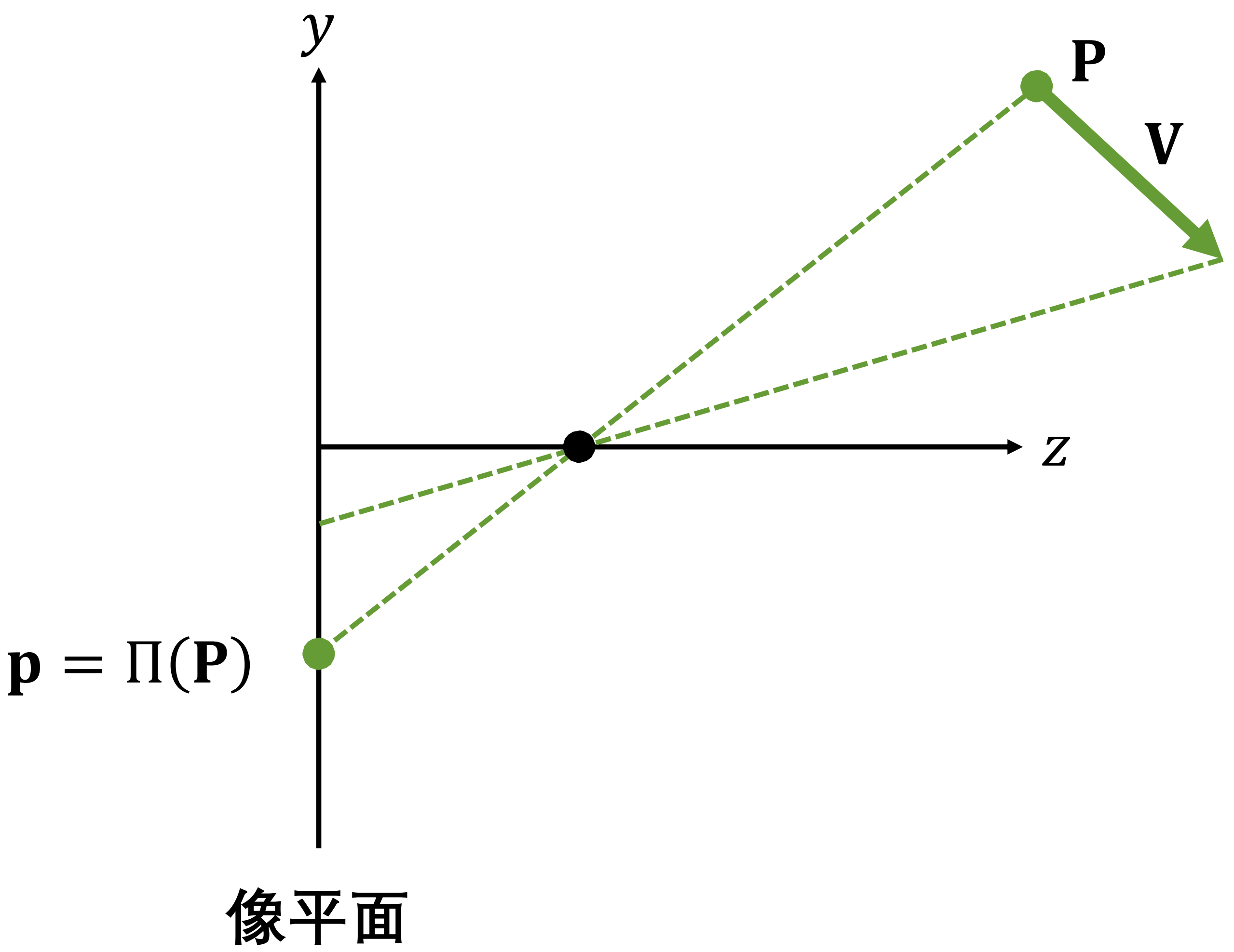
恢复相机的3D旋转和平移速度
恢复3D场景结构


y
P
z
像平面

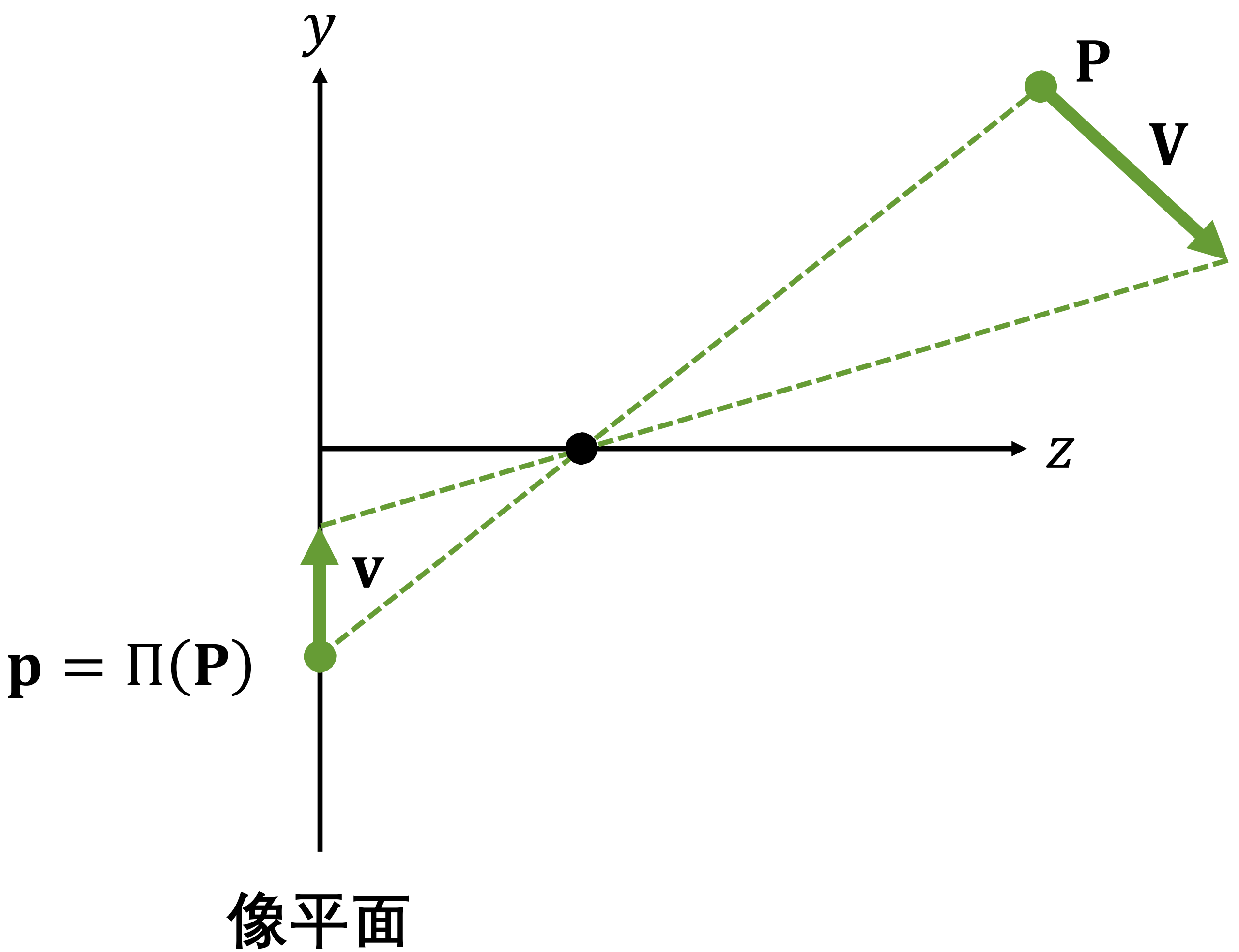

y
$\mathbf{P}$
z
$\mathbf{p} = \Pi(\mathbf{P})$
像平面

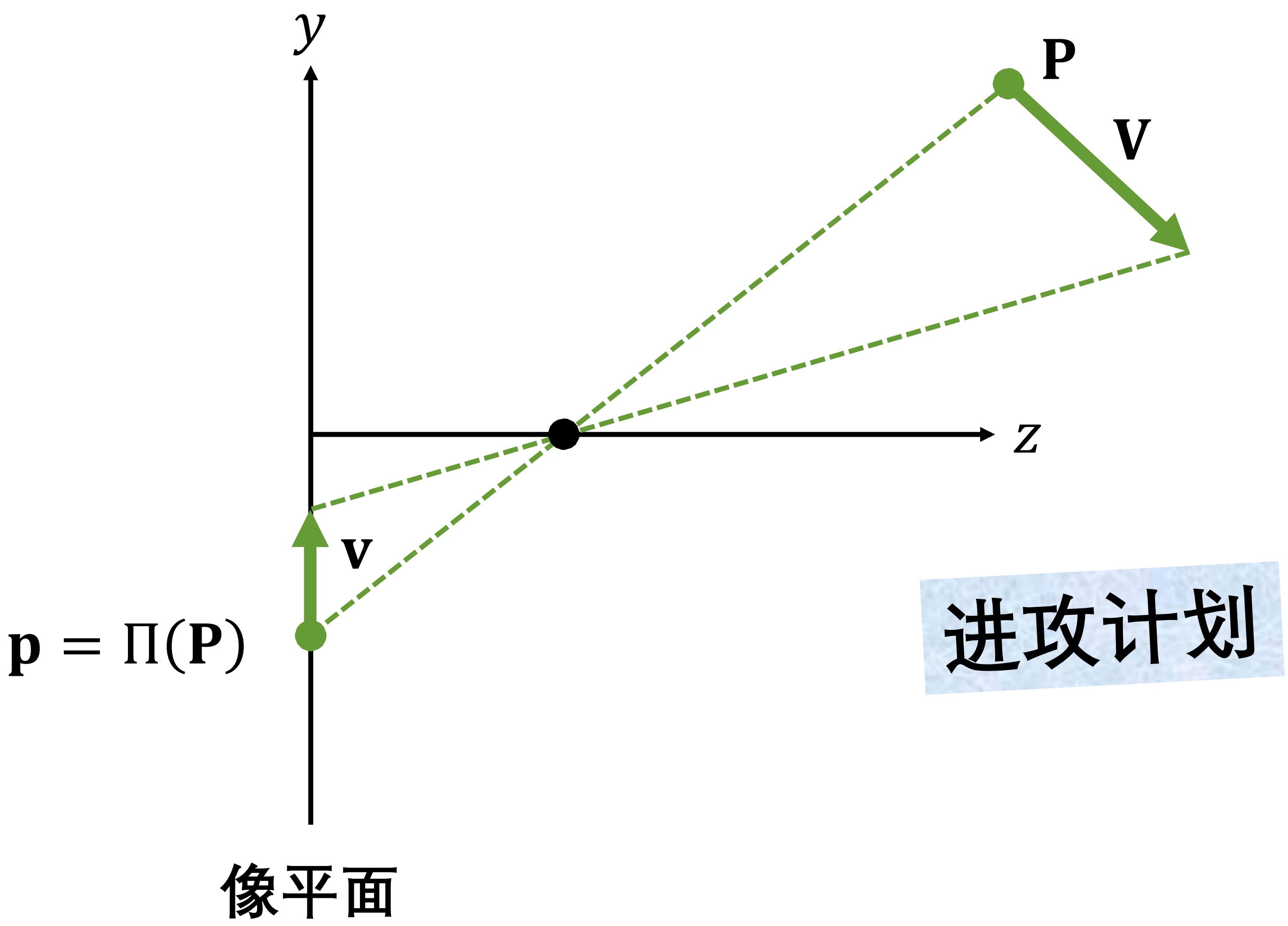


y
$\mathbf{P}$
$\mathbf{V}$
z
$\mathbf{p} = \Pi(\mathbf{P})$
像平面

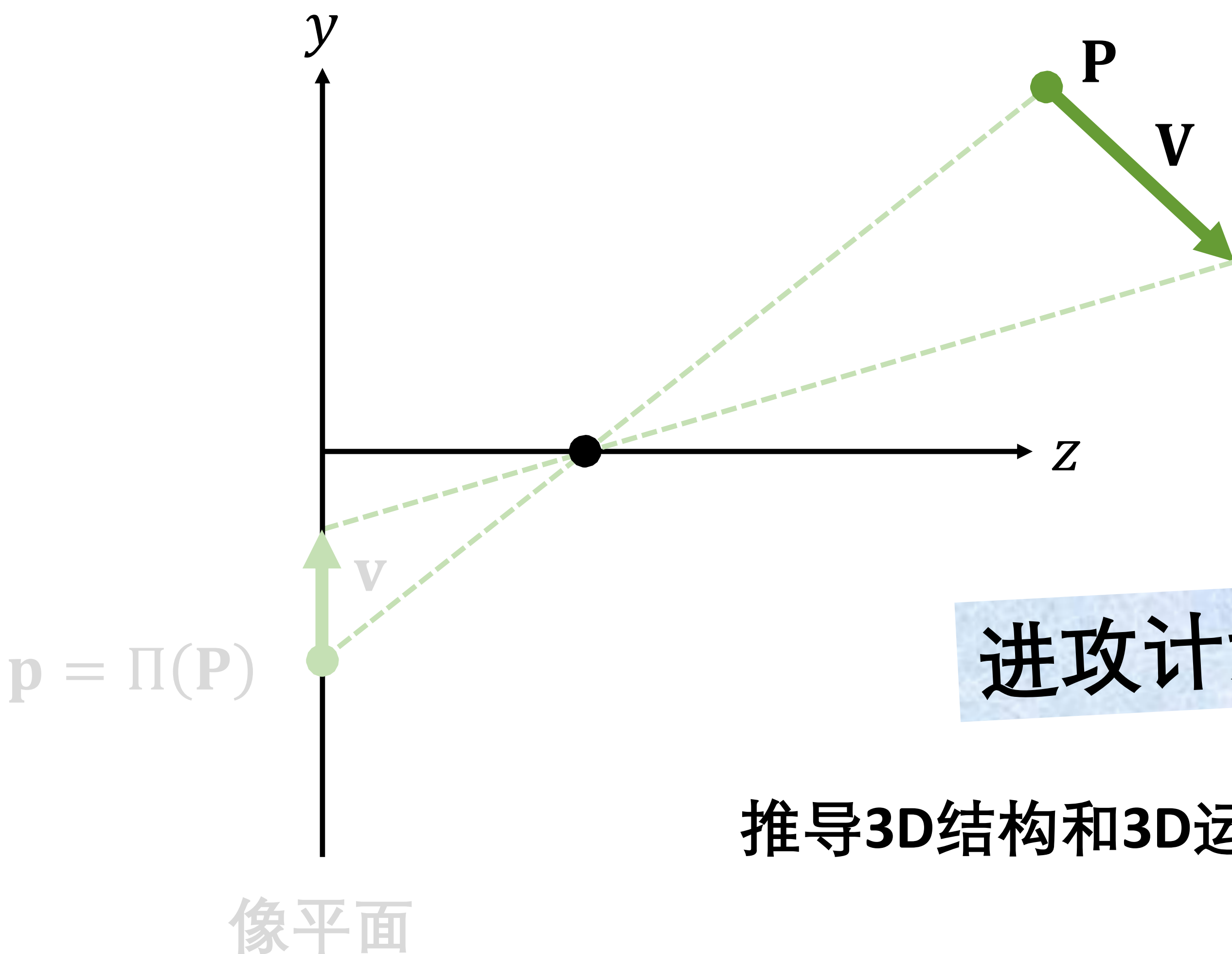


y
P
V
z
p = Π(P)
像平面


y
$\mathbf{P}$
$\mathbf{V}$
z
$\mathbf{v}$
$\mathbf{p} = \Pi(\mathbf{P})$
像平面

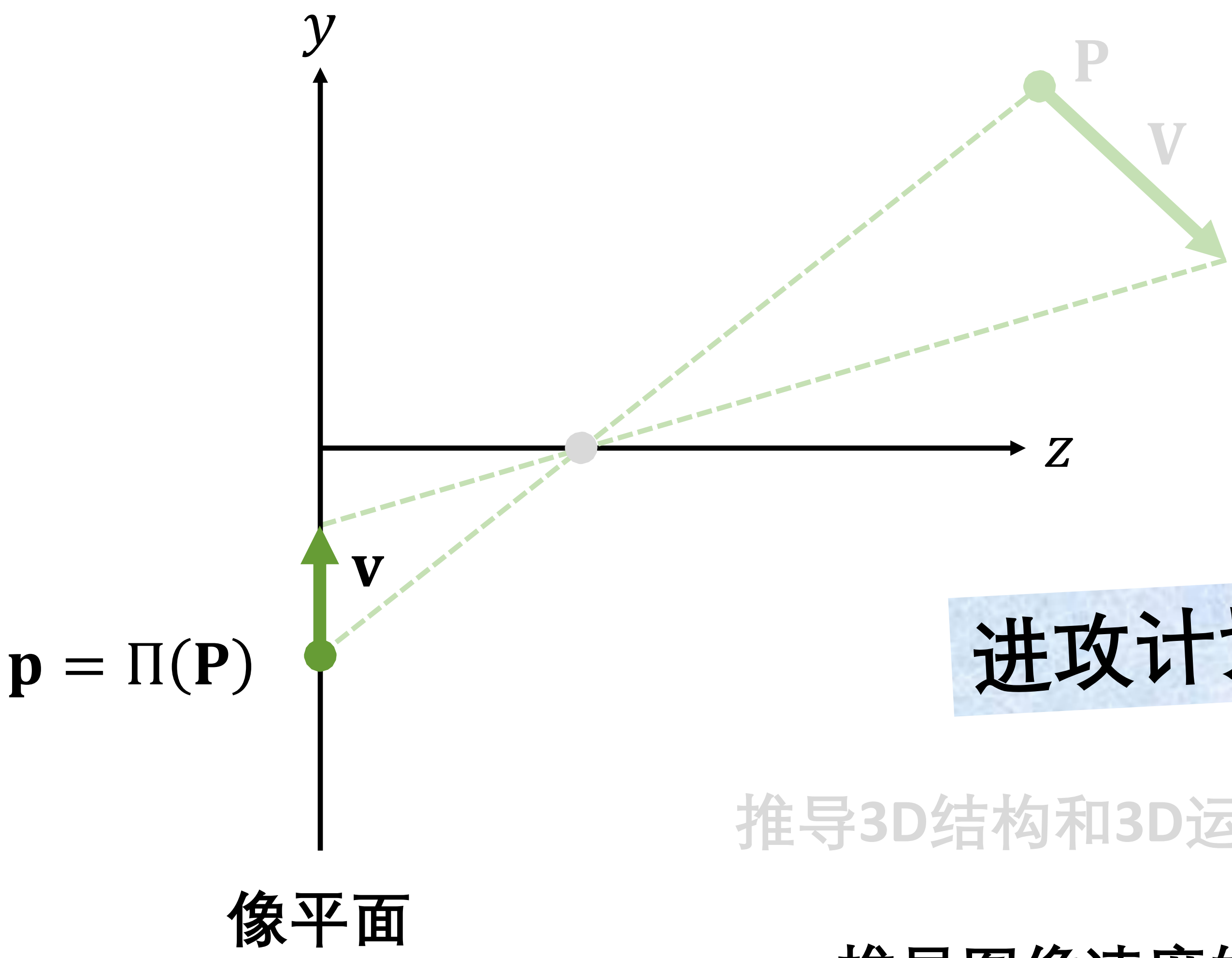


y
P
V
z
v
$\mathbf{p} = \Pi(\mathbf{P})$
进攻计划
像平面



进攻计划

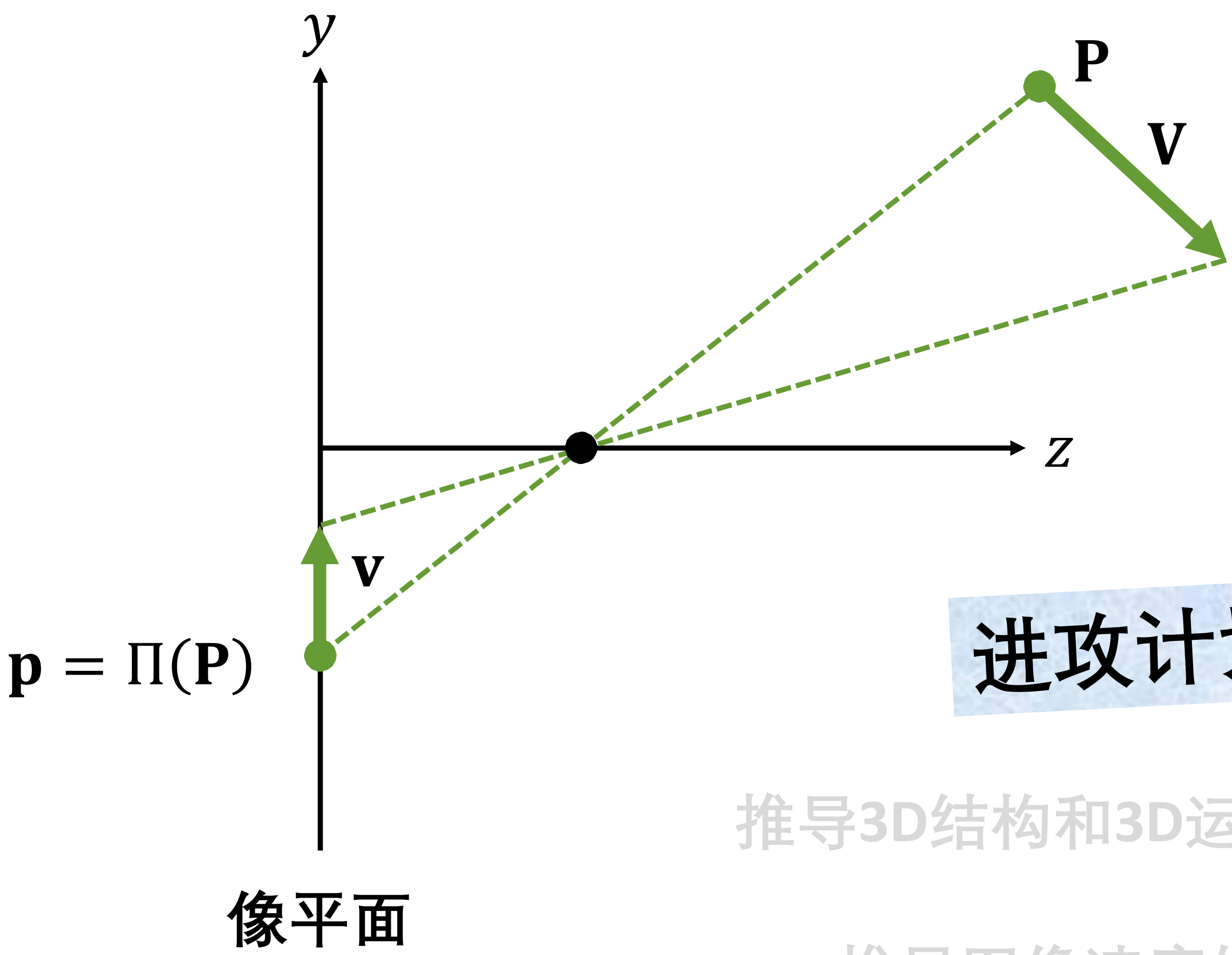
推导3D结构和3D运动的表达式



进攻计划

推导3D结构和3D运动的表达式

推导图像速度的表达式

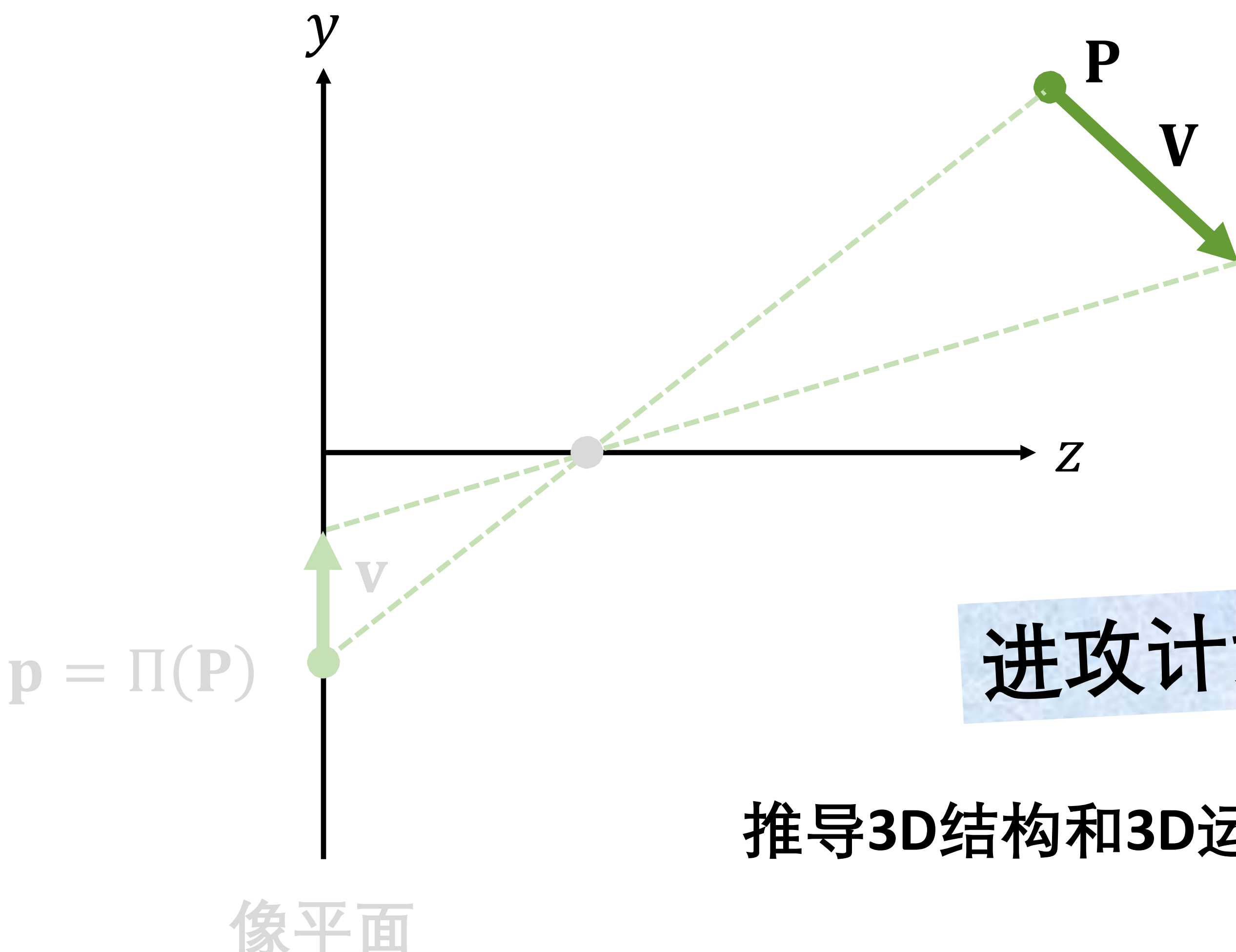


进攻计划

推导3D结构和3D运动的表达式

推导图像速度的表达式

关联3D和2D参数



进攻计划

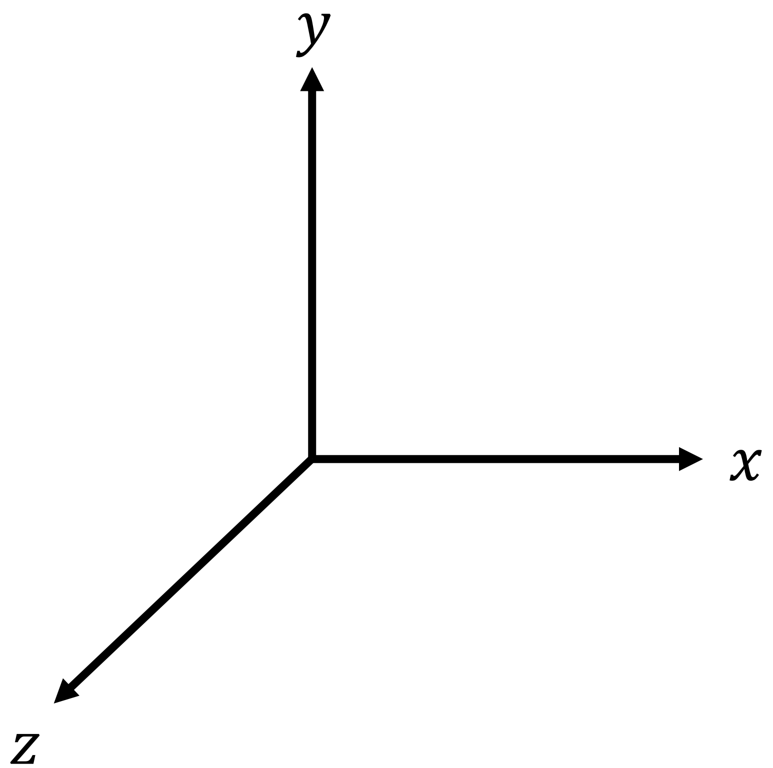
推导3D结构和3D运动的表达式

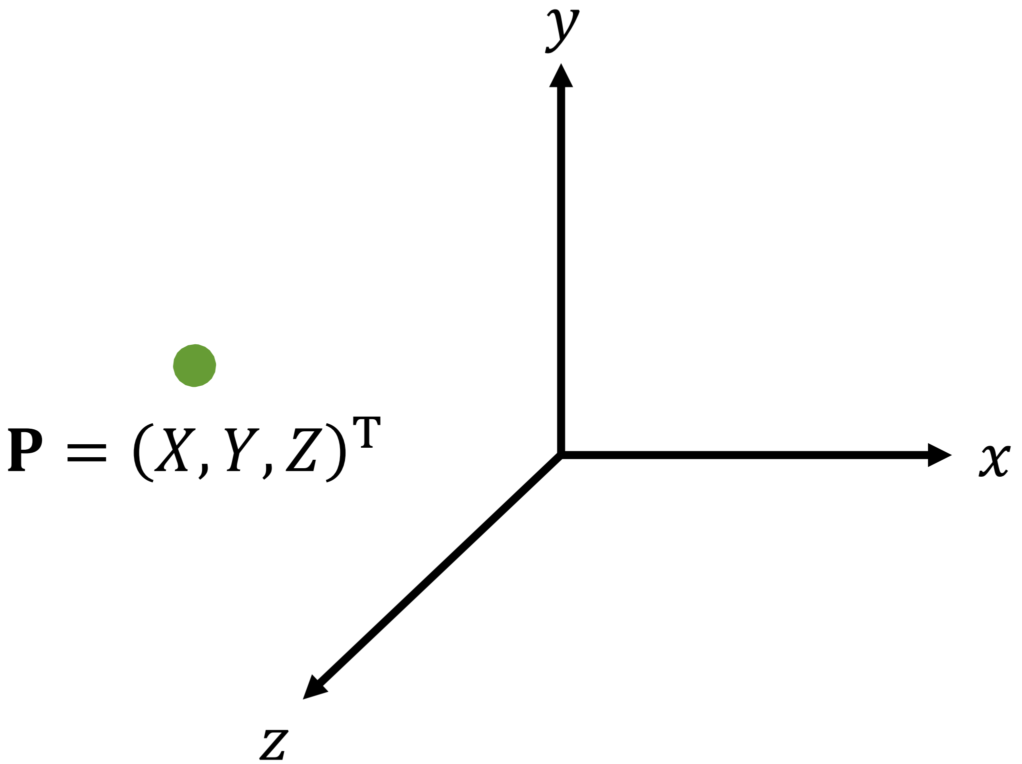
推导图像速度的表达式

关联3D和2D参数

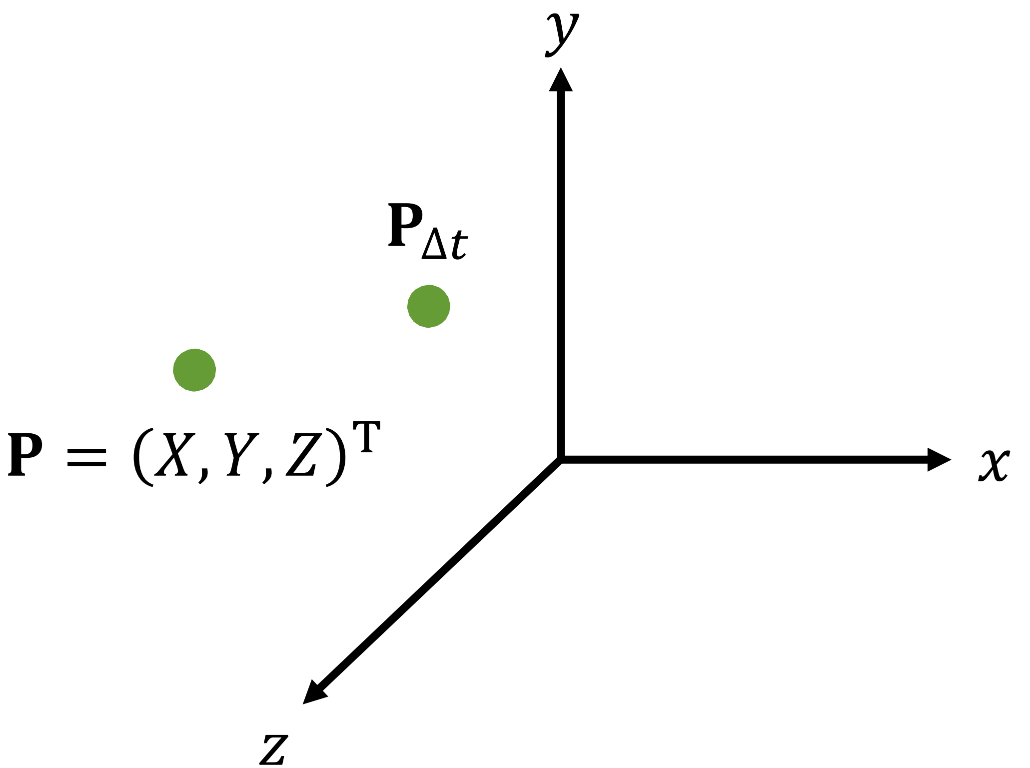
假设场景是刚性的

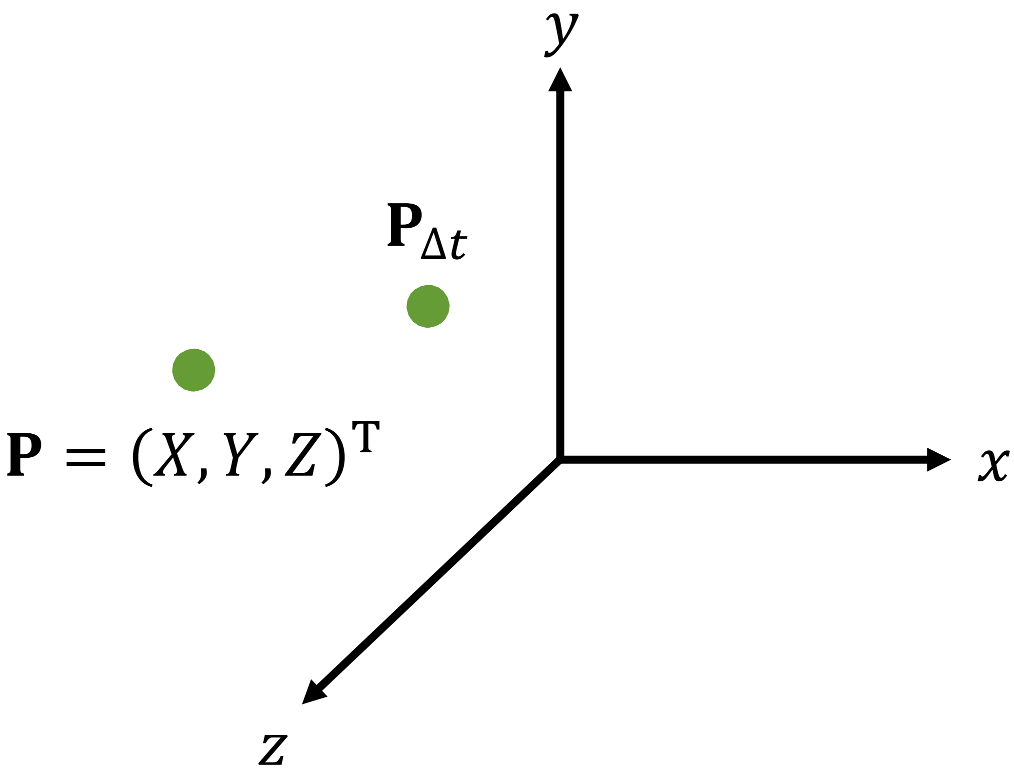
3D结构与运动




$\mathbf{P} = (X, Y, Z)^{\mathrm{T}}$
y
x
z

3D结构与运动



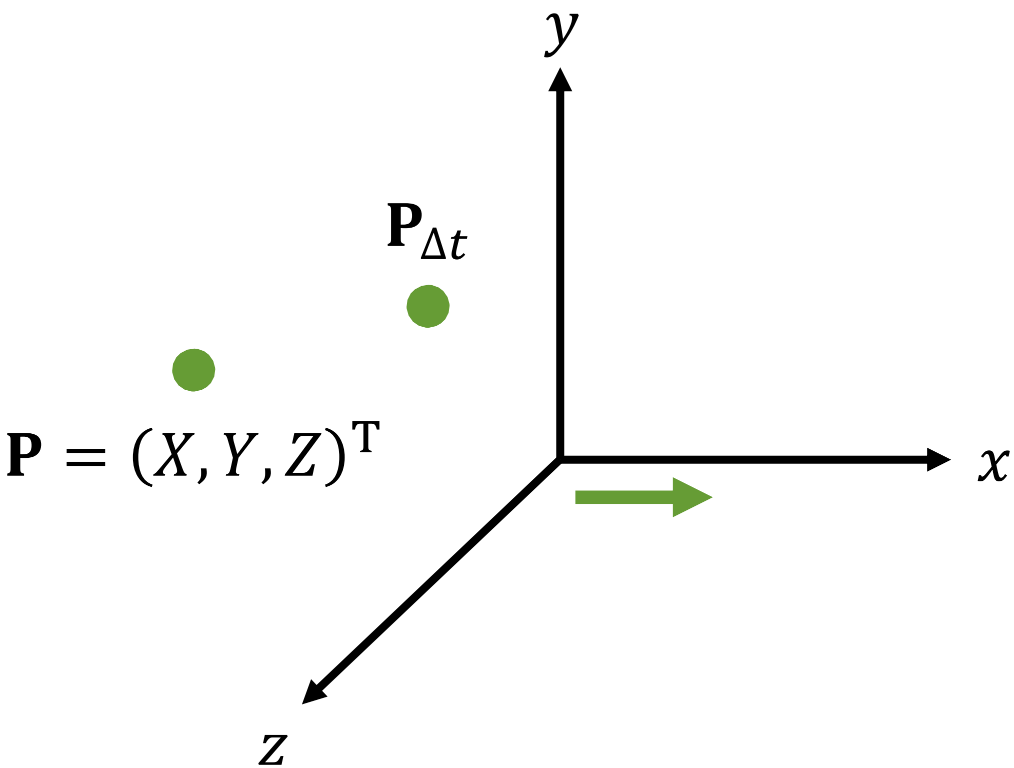


点的运动可以分解为两部分

平移

$$\mathbf{T} = \left(t_x, t_y, t_z\right)^{\mathrm{T}}$$

点的运动可以分解为两部分



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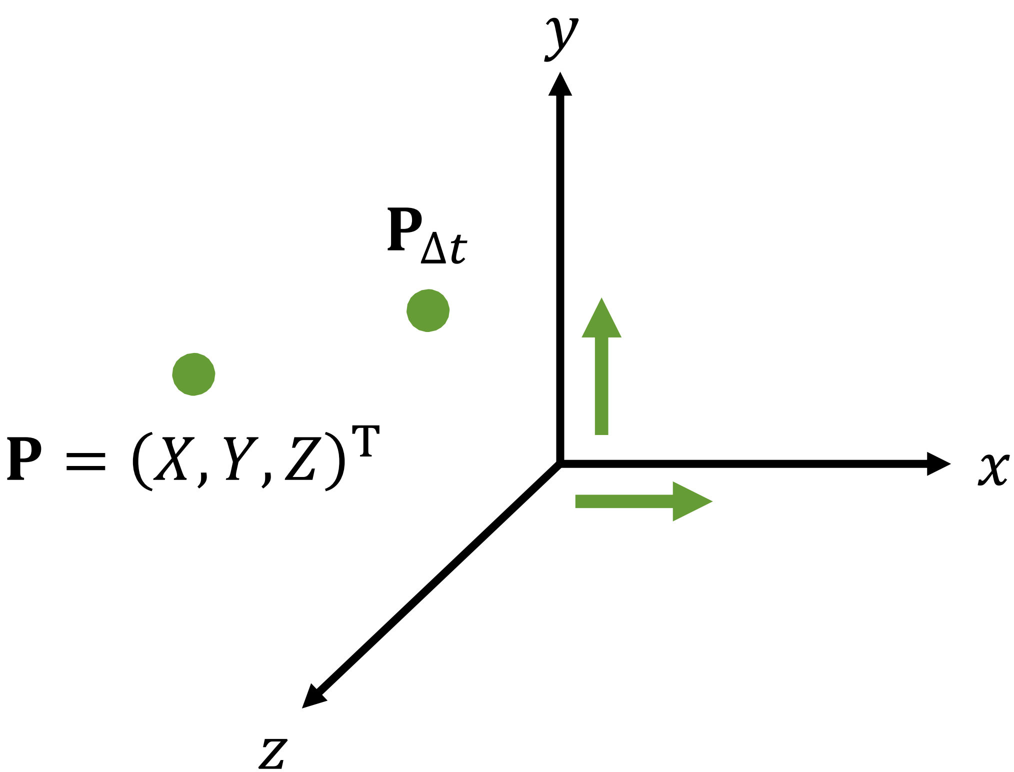

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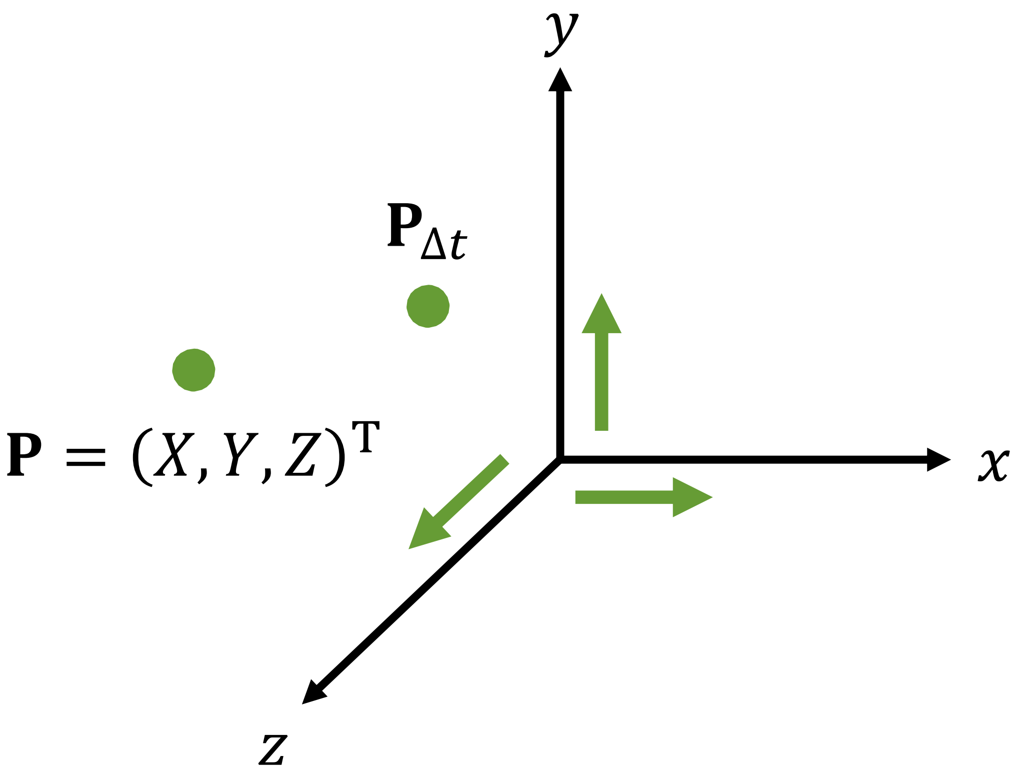

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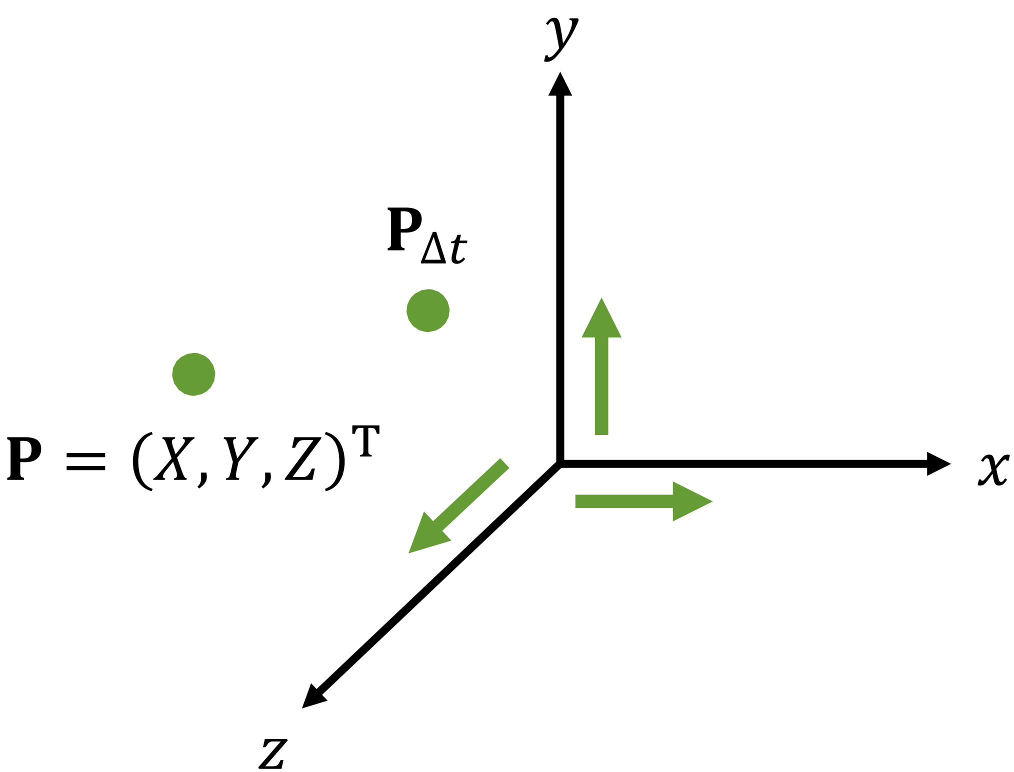

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平移

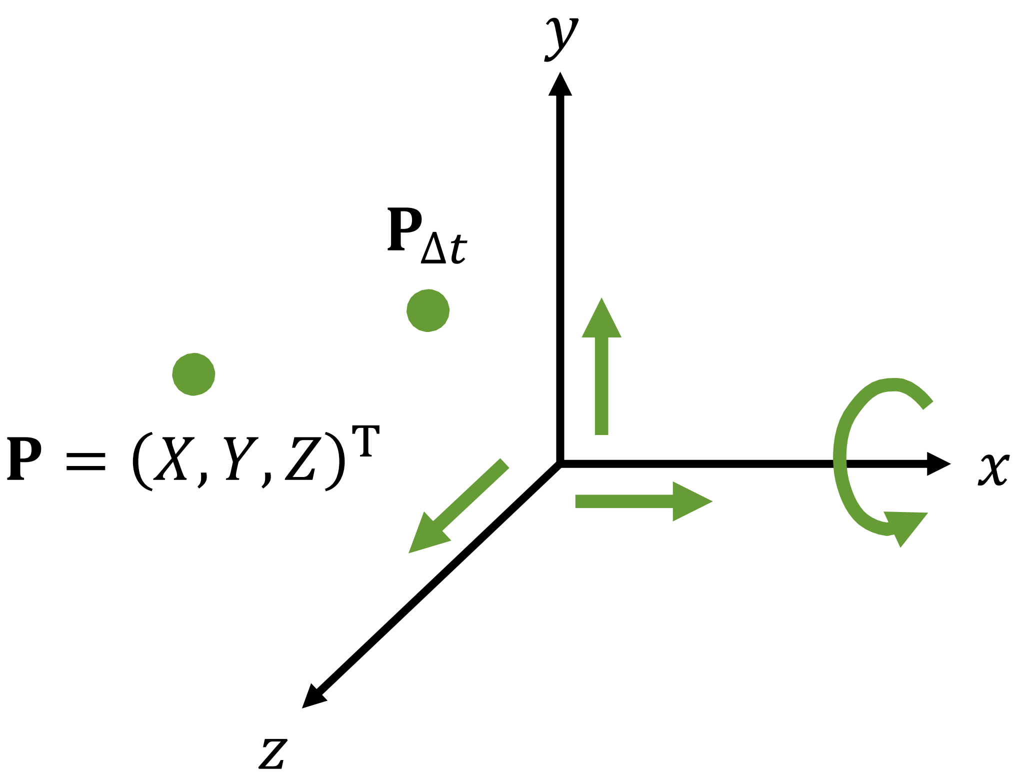
$$\mathbf{T} = \left(t_x, t_y, t_z\right)^{\mathrm{T}}$$

旋转

$$\mathbf{R}(\mathbf{\Omega})$$

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}}$$

点的运动可以分解为两部分



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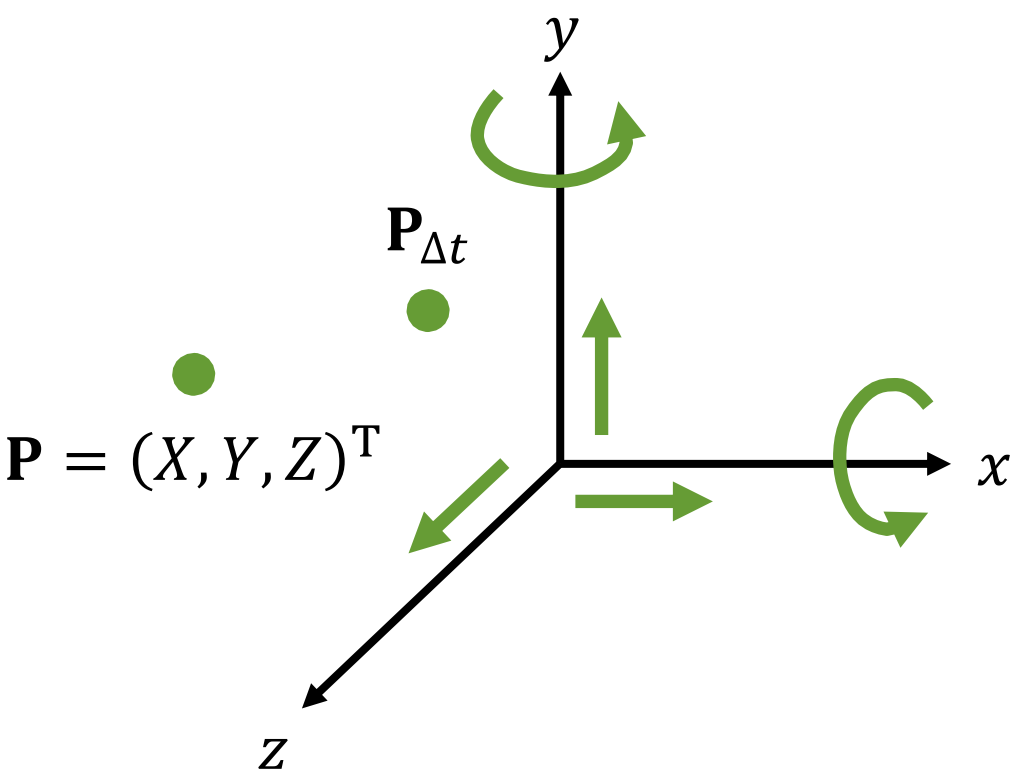

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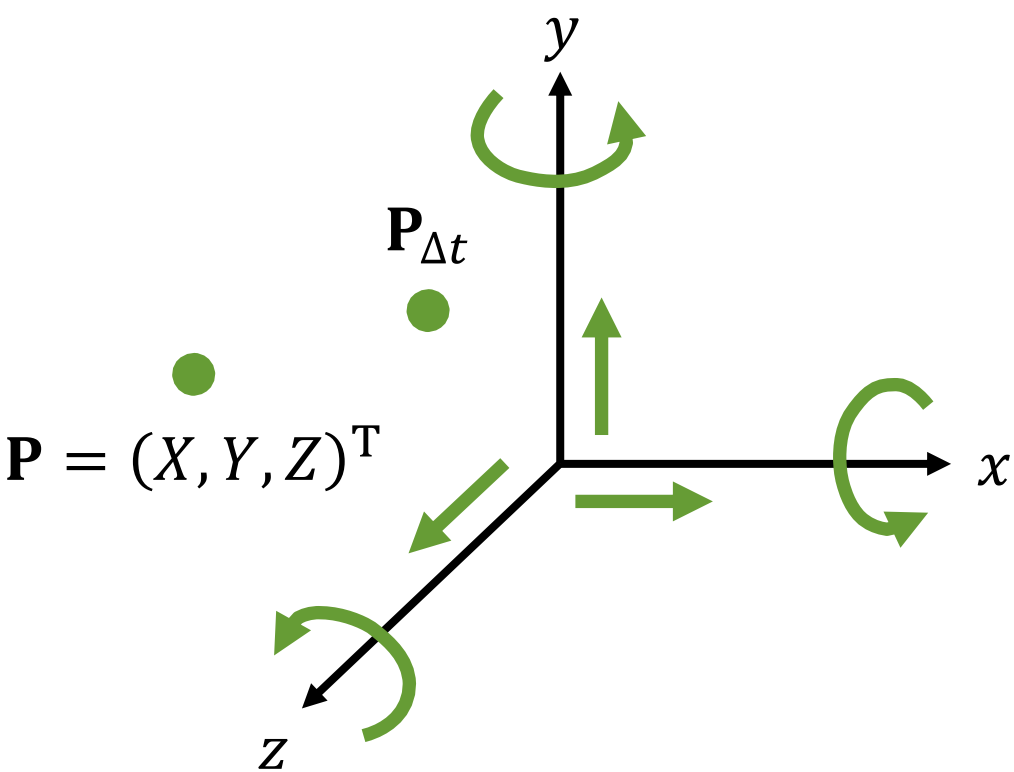

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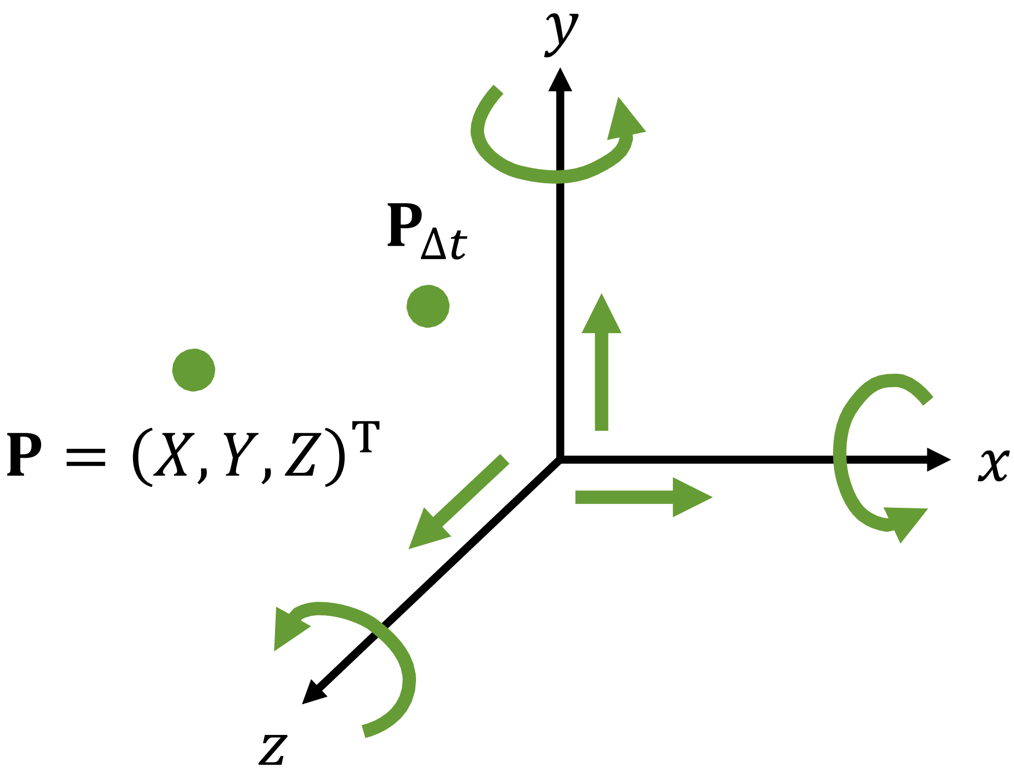

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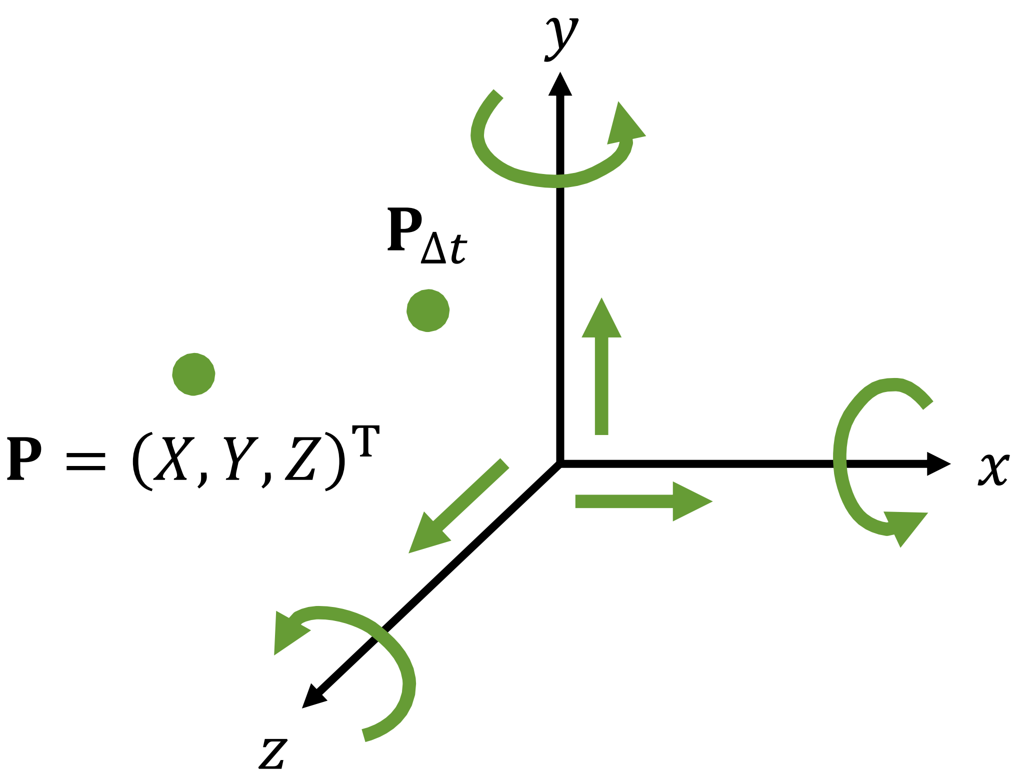
$$\mathbf{P}_{\Delta t} = \mathbf{R}(\boldsymbol{\Omega})\mathbf{P} + \mathbf{T}$$

点的运动可以分解为两部分



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任意旋转可以表示为

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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_x & -\sin\omega_x \\ 0 & \sin\omega_x & \cos\omega_x \end{pmatrix} \begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix} \begin{pmatrix} \cos\omega_z & -\sin\omega_z & 0 \\ \sin\omega_z & \cos\omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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因为我们要处理的是速度，所以可以用无穷小近似

插曲

$$\cos(0 + \Delta\theta)$$

$$\cos(0 + \Delta\theta) = 1 - \frac{\Delta\theta^2}{2!} + \frac{\Delta\theta^4}{4!} - \frac{\Delta\theta^6}{6!} + \cdots$$

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这是如何得来的?

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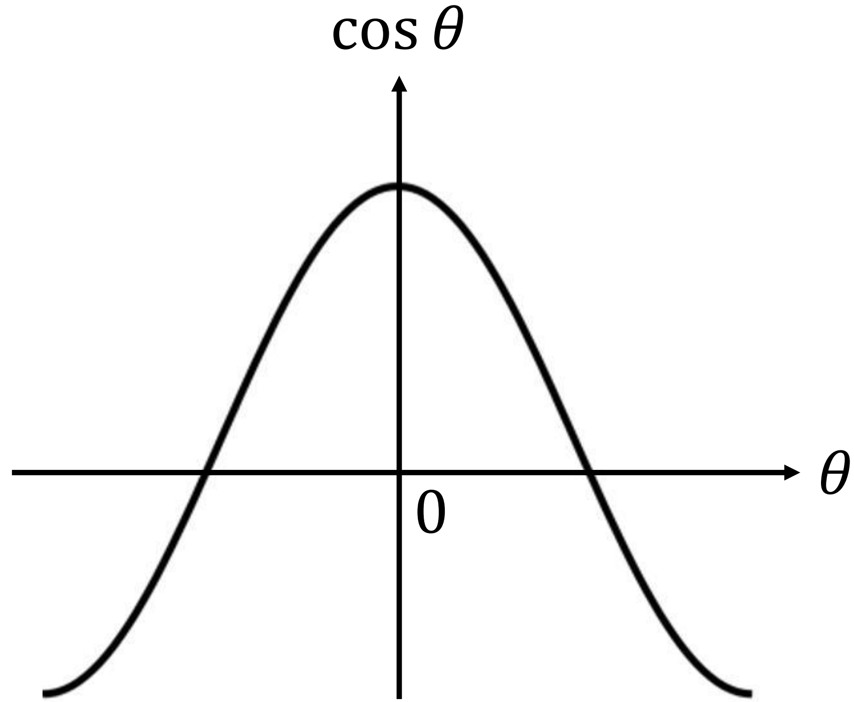
这是如何得来的?

泰勒级数

$$\cos(0+\Delta\theta) = 1 - \frac{\Delta\theta^2}{2!} + \frac{\Delta\theta^4}{4!} - \frac{\Delta\theta^6}{6!} + \cdots$$

假设一个小 (角度) 位移

$$
\begin{aligned}
\cos(0 + \Delta\theta) &= 1 - \frac{\Delta\theta^2}{2!} + \frac{\Delta\theta^4}{4!} - \frac{\Delta\theta^6}{6!} + \cdots \\
&\approx 1
\end{aligned}
$$



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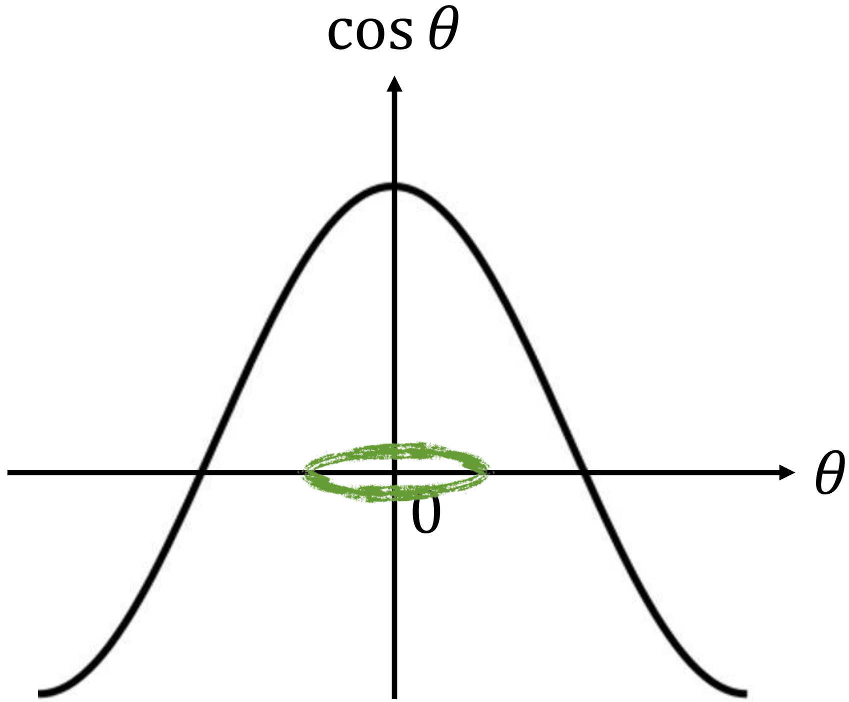

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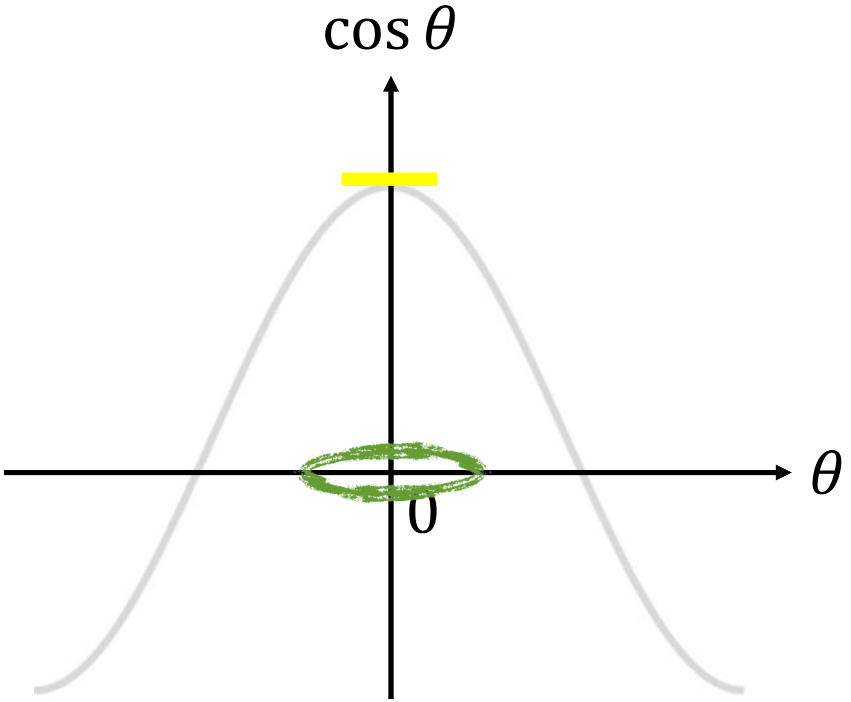

$$\cos(0+\Delta\theta) = 1 - \frac{\Delta\theta^2}{2!} + \frac{\Delta\theta^4}{4!} - \frac{\Delta\theta^6}{6!} + \cdots$$
$$\approx 1$$

$$\sin(0 + \Delta\theta)$$

$$\sin(0 + \Delta\theta) = \Delta\theta - \frac{\Delta\theta^3}{3!} + \frac{\Delta\theta^5}{5!} - \frac{\Delta\theta^7}{7!} + \cdots$$

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$$
\begin{aligned}
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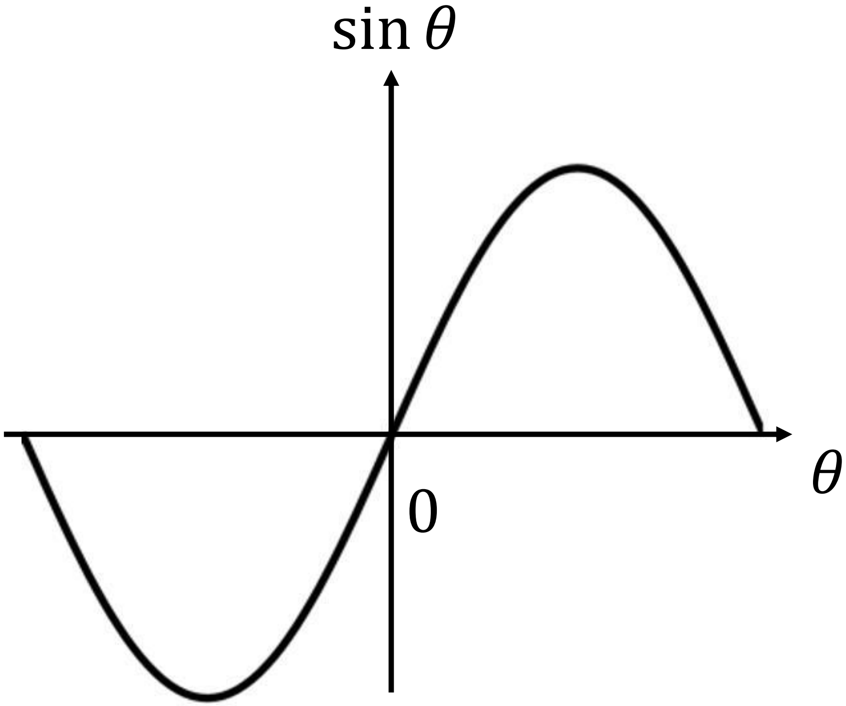
&\approx \Delta\theta
\end{aligned}
$$



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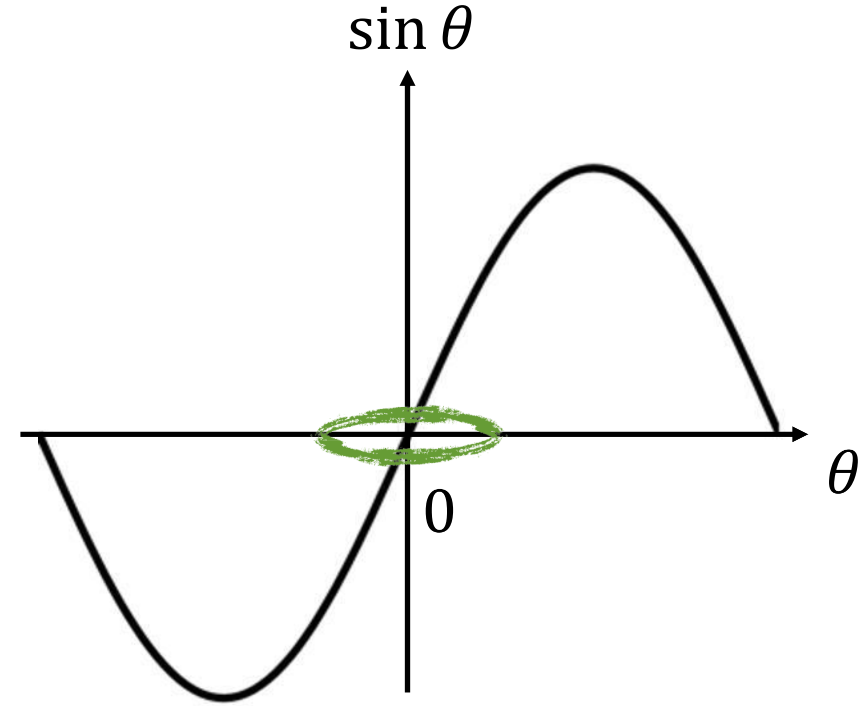

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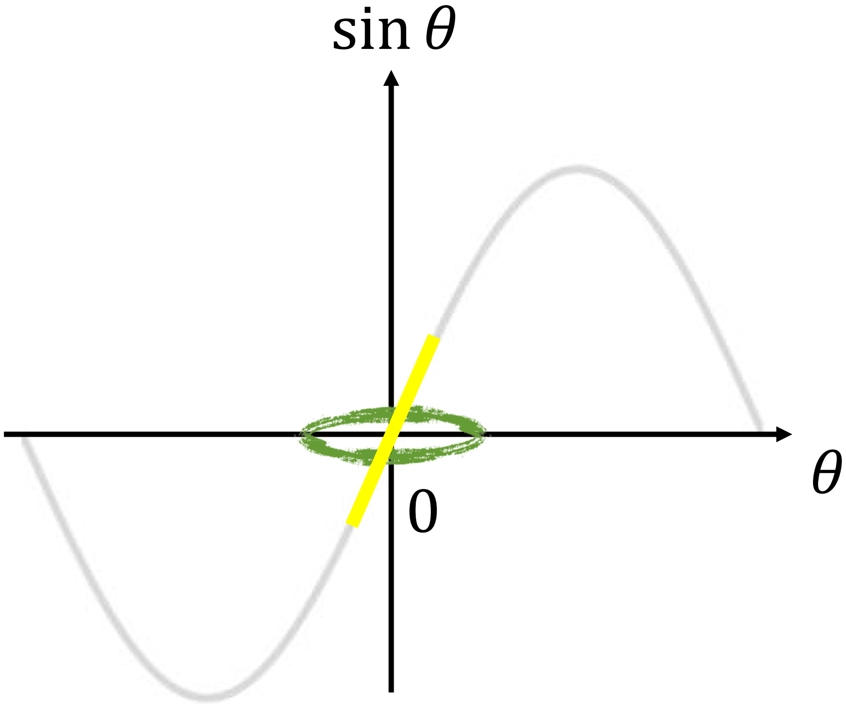

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$$\approx \Delta\theta$$

插曲

已结束

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\boldsymbol{\Omega})\mathbf{P} + \mathbf{T}$$

任意旋转可以表示为

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_x & -\sin\omega_x \\ 0 & \sin\omega_x & \cos\omega_x \end{pmatrix}\begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix}\begin{pmatrix} \cos\omega_z & -\sin\omega_z & 0 \\ \sin\omega_z & \cos\omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

因为我们要处理的是速度，所以可以用无穷小近似

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\boldsymbol{\Omega})\mathbf{P} + \mathbf{T}$$

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$$\cos\theta \approx 1$$

$$\sin\theta \approx \theta$$

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\mathbf{\Omega})\mathbf{P} + \mathbf{T}$$

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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_x & -\sin\omega_x \\ 0 & \sin\omega_x & \cos\omega_x \end{pmatrix}\begin{pmatrix} \cos\omega_y & 0 & \sin\omega_y \\ 0 & 1 & 0 \\ -\sin\omega_y & 0 & \cos\omega_y \end{pmatrix}\begin{pmatrix} \cos\omega_z & -\sin\omega_z & 0 \\ \sin\omega_z & \cos\omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cos\theta \approx 1$$

$$\sin\theta \approx \theta$$

$$\mathbf{R}(\mathbf{\Omega}) \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\omega_x \\ 0 & \omega_x & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & \omega_y \\ 0 & 1 & 0 \\ -\omega_y & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -\omega_z & 0 \\ \omega_z & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\boldsymbol{\Omega})\mathbf{P} + \mathbf{T}$$

近似

任意旋转可以表示为

$$\mathbf{R}(\boldsymbol{\Omega}) \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\omega_x \\ 0 & \omega_x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \omega_y \\ 0 & 1 & 0 \\ -\omega_y & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\omega_z & 0 \\ \omega_z & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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近似

任意旋转可以表示为

$$\mathbf{R}(\mathbf{\Omega}) \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\omega_x \\ 0 & \omega_x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \omega_y \\ 0 & 1 & 0 \\ -\omega_y & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\omega_z & 0 \\ \omega_z & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_x\omega_y + \omega_z & 1 - \omega_x\omega_y\omega_z & -\omega_x \\ \omega_x\omega_z - \omega_y & \omega_y\omega_z + \omega_x & 1 \end{pmatrix}$$

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\boldsymbol{\Omega})\mathbf{P} + \mathbf{T}$$

近似

任意旋转可以表示为

$$\mathbf{R}(\boldsymbol{\Omega}) \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\omega_x \\ 0 & \omega_x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \omega_y \\ 0 & 1 & 0 \\ -\omega_y & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\omega_z & 0 \\ \omega_z & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_x\omega_y + \omega_z & 1 - \omega_x\omega_y\omega_z & -\omega_x \\ \omega_x\omega_z - \omega_y & \omega_y\omega_z + \omega_x & 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix}$$

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\boldsymbol{\Omega})\mathbf{P} + \mathbf{T}$$

近似

任意旋转可以表示为

$$\mathbf{R}(\boldsymbol{\Omega}) \approx \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix}$$

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新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{R}(\mathbf{\Omega})\mathbf{P} + \mathbf{T}$$

近似

任意旋转可以表示为

$$\mathbf{R}(\mathbf{\Omega}) \approx \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix}$$

新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{R}\mathbf{P} + \mathbf{T}$$

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近似

任意旋转可以表示为

$$\mathbf{R}(\boldsymbol{\Omega}) \approx \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix}$$

新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{RP} + \mathbf{T} = \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{RP} + \mathbf{T} = \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

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位移由下式给出

新的3D位置可以表示为

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位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P}$$

新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{RP} + \mathbf{T} = \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P} = \begin{pmatrix} X - \omega_z Y + \omega_y Z + t_x \\ \omega_z X + Y - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + Z + t_z \end{pmatrix} - \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{RP} + \mathbf{T} = \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

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化简

新的3D位置可以表示为

$$\mathbf{P}_{\Delta t} = \mathbf{RP} + \mathbf{T} = \begin{pmatrix} 1 & -\omega_z & \omega_y \\ \omega_z & 1 & -\omega_x \\ -\omega_y & \omega_x & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P} = \begin{pmatrix} \cancel{X} - \omega_z Y + \omega_y Z + t_x \\ \omega_z X + Y - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + Z + t_z \end{pmatrix} - \begin{pmatrix} \cancel{X} \\ Y \\ Z \end{pmatrix}$$

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位移由下式给出

$$\mathbf{P}_{\Delta t}-\mathbf{P}=\begin{pmatrix}\cancel{X}-\omega_z Y+\omega_y Z+t_x\\ \omega_z X+\cancel{Y}-\omega_x Z+t_y\\ -\omega_y X+\omega_x Y+Z+t_z\end{pmatrix}-\begin{pmatrix}\cancel{X}\\ \cancel{Y}\\ Z\end{pmatrix}$$

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位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P} = \begin{pmatrix} \cancel{X} - \omega_z Y + \omega_y Z + t_x \\ \omega_z X + \cancel{Y} - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + \cancel{Z} + t_z \end{pmatrix} - \begin{pmatrix} \cancel{X} \\ \cancel{Y} \\ \cancel{Z} \end{pmatrix}$$

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位移由下式给出

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化简

$$= \begin{pmatrix} -\omega_z Y + \omega_y Z + t_x \\ \omega_z X - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + t_z \end{pmatrix}$$

位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P} = \begin{pmatrix} -\omega_z Y + \omega_y Z + t_x \\ \omega_z X - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + t_z \end{pmatrix}$$

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当$\Delta t \to 0$，位移除以时间得到速度

位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P} = \begin{pmatrix} -\omega_z Y + \omega_y Z + t_x \\ \omega_z X - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + t_z \end{pmatrix}$$

当$\Delta t \to 0$，位移除以时间得到速度

$$\mathbf{V} = \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}$$

位移由下式给出

$$\mathbf{P}_{\Delta t} - \mathbf{P} = \begin{pmatrix} -\omega_z Y + \omega_y Z + t_x \\ \omega_z X - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + t_z \end{pmatrix}$$

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位移由下式给出

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3D速度

$$\mathbf{V} = \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} -\omega_z Y + \omega_y Z + t_x \\ \omega_z X - \omega_x Z + t_y \\ -\omega_y X + \omega_x Y + t_z \end{pmatrix}$$

3D速度

如果相机在移动，而世界是静止的

$$\mathbf{V}=\begin{pmatrix}\dot{X}\\\dot{Y}\\\dot{Z}\end{pmatrix}=\begin{pmatrix}-\omega_z Y+\omega_y Z+t_x\\\omega_z X-\omega_x Z+t_y\\-\omega_y X+\omega_x Y+t_z\end{pmatrix}$$

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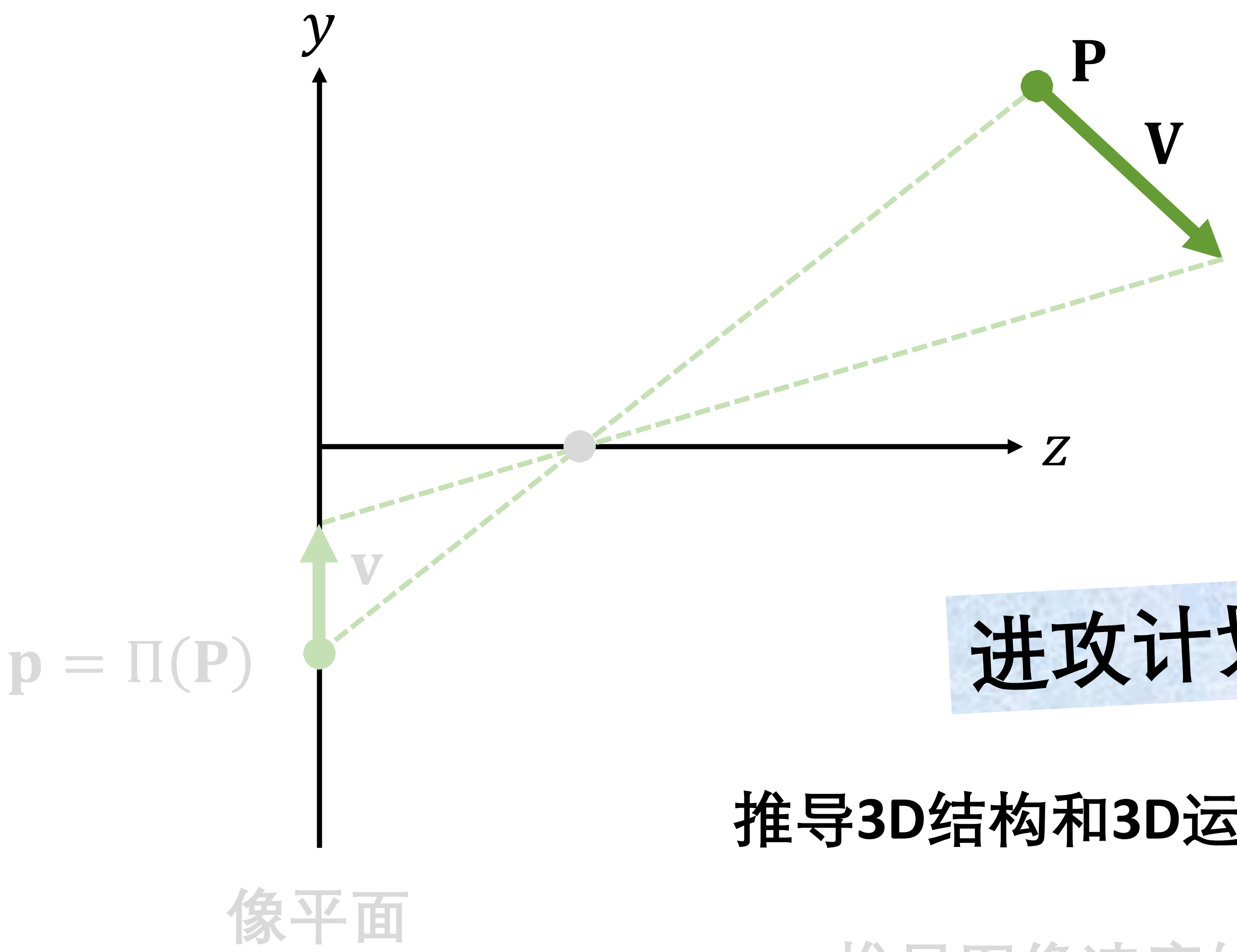
3D相对速度

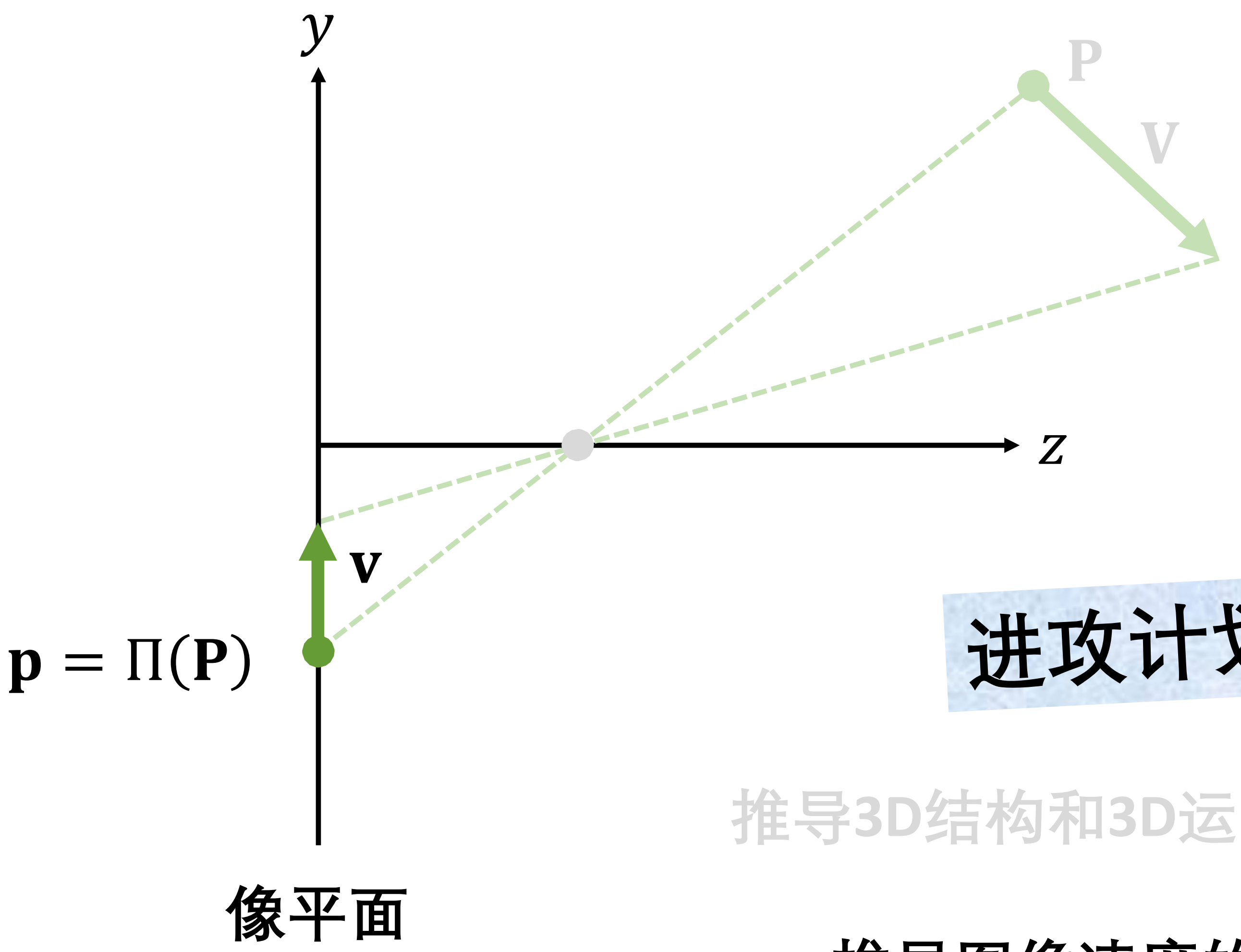


进攻计划

推导3D结构和3D运动的表达式

推导图像速度的表达式

关联3D和2D参数



进攻计划

推导3D结构和3D运动的表达式

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我们寻求图像速度的表达式

$$\frac{d\mathbf{p}}{dt} = \begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

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已知（为了不失一般性，假设$f = 1$）

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$$x = \frac{X}{Z} \qquad\qquad y = \frac{Y}{Z}$$

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取导数

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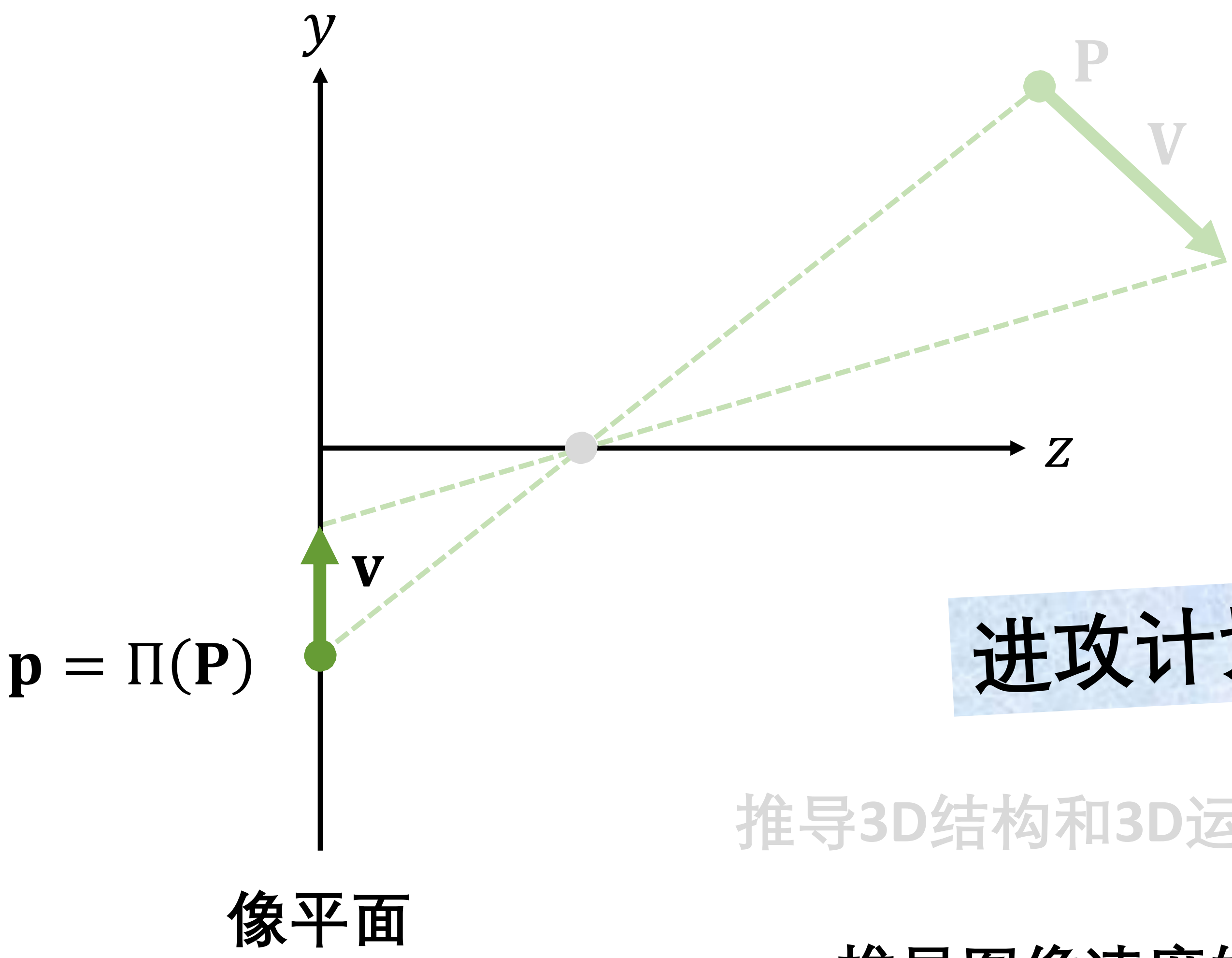
已知（为了不失一般性，假设$f = 1$**）**

$$x = \frac{X}{Z} \qquad y = \frac{Y}{Z}$$

取导数

$$\dot{x} = u = \frac{\dot{X}}{Z} - \frac{X\dot{Z}}{Z^2} \qquad \dot{y} = v = \frac{\dot{Y}}{Z} - \frac{Y\dot{Z}}{Z^2}$$



进攻计划

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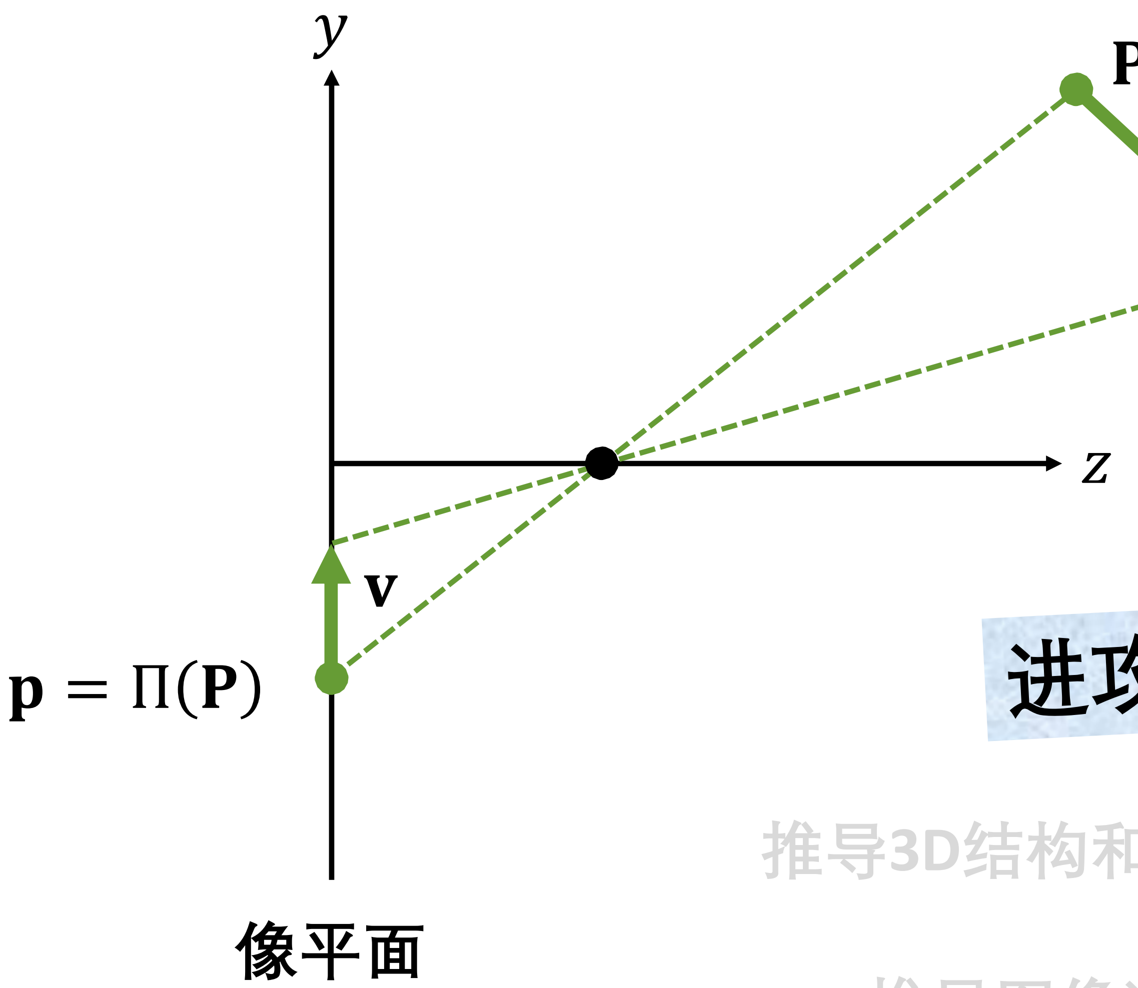

进攻计划

推导3D结构和3D运动的表达式

推导图像速度的表达式

关联3D和2D参数

我们有

$$\mathbf{V} = \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} \omega_z Y - \omega_y Z - t_x \\ -\omega_z X + \omega_x Z - t_y \\ \omega_y X - \omega_x Y - t_z \end{pmatrix}$$

我们有

$$\mathbf{V} = \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} \omega_z Y - \omega_y Z - t_x \\ -\omega_z X + \omega_x Z - t_y \\ \omega_y X - \omega_x Y - t_z \end{pmatrix}$$

和

$$\dot{x} = u = \frac{\dot{X}}{Z} - \frac{X\dot{Z}}{Z^2} \qquad \dot{y} = v = \frac{\dot{Y}}{Z} - \frac{Y\dot{Z}}{Z^2}$$

我们有

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代入 (以u 为例， v 同理)

我们有

$$\mathbf{V}=\begin{pmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{pmatrix}=\begin{pmatrix}\omega_z Y-\omega_y Z-t_x\\ -\omega_z X+\omega_x Z-t_y\\ \omega_y X-\omega_x Y-t_z\end{pmatrix}$$

和

$$\dot{x}=u=\frac{\dot{X}}{Z}-\frac{X\dot{Z}}{Z^2}\qquad\qquad \dot{y}=v=\frac{\dot{Y}}{Z}-\frac{Y\dot{Z}}{Z^2}$$

代入 (以u 为例， v 同理)

$$u=\frac{\dot{X}}{Z}-\frac{X\dot{Z}}{Z^2}=\frac{1}{Z}\left(\omega_z Y-\omega_y Z-t_x\right)-\frac{X}{Z^2}\left(\omega_y X-\omega_x Y-t_z\right)$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \frac{Z}{Z} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \cancel{\frac{Z}{Z}} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

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改写

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看起来眼熟吗？

$$u=\frac{1}{Z}(\omega_z Y-\omega_y Z-t_x)-\frac{X}{Z^2}(\omega_y X-\omega_x Y-t_z)$$

改写

$$=\left(\omega_z\frac{Y}{Z}-\omega_y\cancel{\frac{Z}{Z}}-\frac{t_x}{Z}\right)-\frac{X}{Z}\left(\omega_y\frac{X}{Z}-\omega_x\frac{Y}{Z}-\frac{t_z}{Z}\right)$$

看起来眼熟吗？

$$x=\frac{X}{Z}$$

$$u=\frac{1}{Z}\left(\omega_z Y-\omega_y Z-t_x\right)-\frac{X}{Z^2}\left(\omega_y X-\omega_x Y-t_z\right)$$

改写

$$=\left(\omega_z\frac{Y}{Z}-\omega_y\cancel{\frac{Z}{Z}}-\frac{t_x}{Z}\right)-\frac{X}{Z}\left(\omega_y\frac{X}{Z}-\omega_x\frac{Y}{Z}-\frac{t_z}{Z}\right)$$

代入

看起来眼熟吗？

$$x=\frac{X}{Z}$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \cancel{\frac{Z}{Z}} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

$$= \left(\omega_z \frac{Y}{Z} - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \cancel{\frac{Z}{Z}} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

$$= \left(\omega_z \frac{Y}{Z} - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

看起来眼熟吗?

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \cancel{\frac{Z}{Z}} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

$$= \left(\omega_z \frac{Y}{Z} - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

看起来眼熟吗?

$$y = \frac{Y}{Z}$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \cancel{\frac{Z}{Z}} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

$$= \left(\omega_z \frac{Y}{Z} - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

看起来眼熟吗？

$$y = \frac{Y}{Z}$$

$$u = \frac{1}{Z}\left(\omega_z Y - \omega_y Z - t_x\right) - \frac{X}{Z^2}\left(\omega_y X - \omega_x Y - t_z\right)$$

改写

$$= \left(\omega_z \frac{Y}{Z} - \omega_y \cancel{\frac{Z}{Z}} - \frac{t_x}{Z}\right) - \frac{X}{Z}\left(\omega_y \frac{X}{Z} - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

$$= \left(\omega_z \frac{Y}{Z} - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x \frac{Y}{Z} - \frac{t_z}{Z}\right)$$

代入

$$= \left(\omega_z y - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x y - \frac{t_z}{Z}\right)$$

$$u = \left(\omega_z y - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x y - \frac{t_z}{Z}\right)$$

$$u = \left(\omega_z y - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x y - \frac{t_z}{Z}\right)$$

整理

$$u = \left(\omega_z y - \omega_y - \frac{t_x}{Z}\right) - x\left(\omega_y x - \omega_x y - \frac{t_z}{Z}\right)$$

整理

$$= \frac{1}{Z}(x t_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$u=\left(\omega_z y-\omega_y-\frac{t_x}{Z}\right)-x\left(\omega_y x-\omega_x y-\frac{t_z}{Z}\right)$$

整理

$$=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

运动场方程

$$u=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

$$v=\frac{1}{Z}(yt_z-t_y)+\omega_x(y^2+1)-\omega_y(xy)-\omega_z(x)$$

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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观察

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

观察

旋转和平移分量不直接相互作用

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

旋转

观察

旋转和平移分量不直接相互作用

运动场方程

$$u = \frac{1}{Z}\left(xt_z - t_x\right) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

平移

观察

旋转和平移分量不直接相互作用

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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平移

观察

旋转和平移分量不直接相互作用

结构分量只与平移分量相互作用

运动场方程

$$u=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

$$v=\frac{1}{Z}(yt_z-t_y)+\omega_x(y^2+1)-\omega_y(xy)-\omega_z(x)$$

平移

观察

旋转和平移分量不直接相互作用

结构分量只与平移分量相互作用

平移和结构只能恢复到相差一个未知的尺度因子

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

平移

观察

旋转和平移分量不直接相互作用

结构分量只与平移分量相互作用

旋转引起的图像速度与结构无关

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

平移

观察

- 旋转和平移分量不直接相互作用
- 结构分量只与平移分量相互作用
- 旋转引起的图像速度与结构无关
  - 旋转分量不携带深度信息

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

旋转

观察

- 旋转和平移分量不直接相互作用
- 结构分量只与平移分量相互作用
- 旋转引起的图像速度与结构无关
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平移

运动场方程

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表面平行于相机移动

特殊情况

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面平行于相机移动

$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}}$ 且 $t_z = 0$

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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表面平行于相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z = 0$$

代入

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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表面平行于相机移动

$$\mathbf{\Omega} = (\omega_x, \omega_y, \omega_z)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z = 0$$

代入

$$u = -\frac{t_x}{Z} \qquad v = -\frac{t_y}{Z}$$

表面平行于相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z = 0$$

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运动场是什么样的?

表面平行于相机移动

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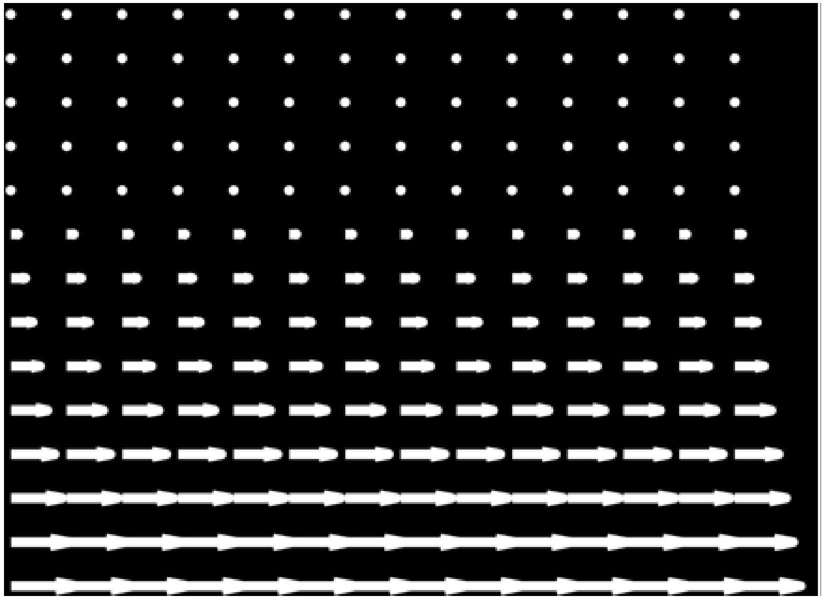

表面平行于相机移动

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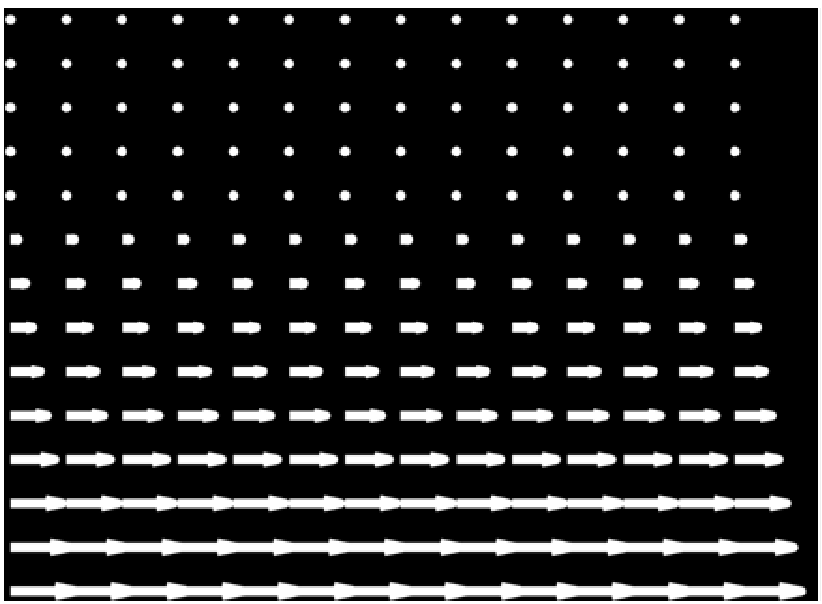

彼此相互平行

表面平行于相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z = 0$$

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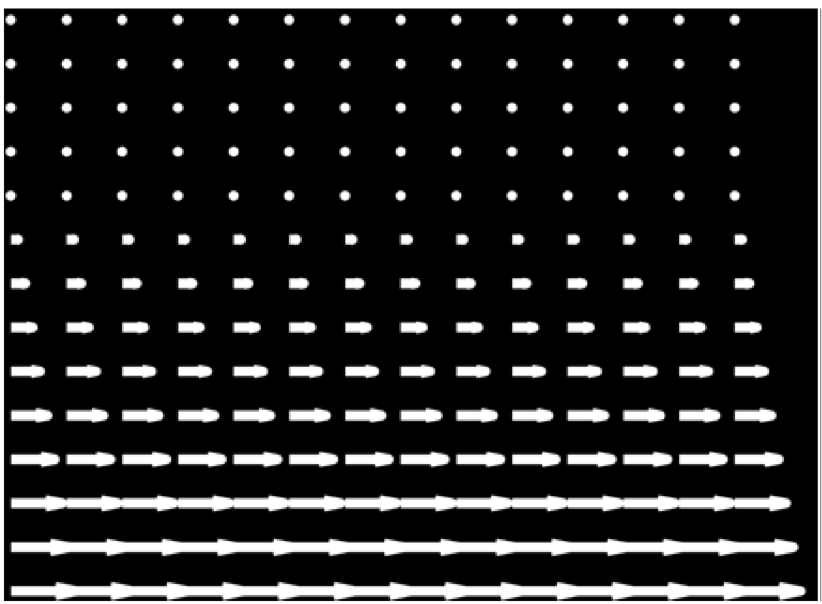

彼此相互平行

长度与深度成反比

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2+1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2+1) - \omega_y(xy) - \omega_z(x)$$

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2+1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2+1) - \omega_y(xy) - \omega_z(x)$$

平移

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面朝相机移动

特殊情况

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

代入

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

代入

$$u = \frac{t_z}{Z}\left(x - \frac{t_x}{t_z}\right) \qquad v = \frac{t_z}{Z}\left(y - \frac{t_y}{t_z}\right)$$

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

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$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

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运动场是什么样的?

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

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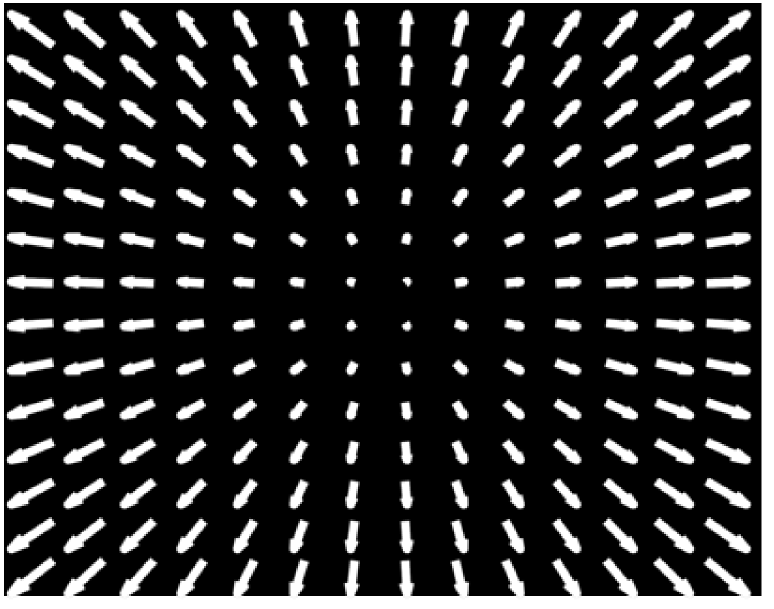

表面朝相机移动

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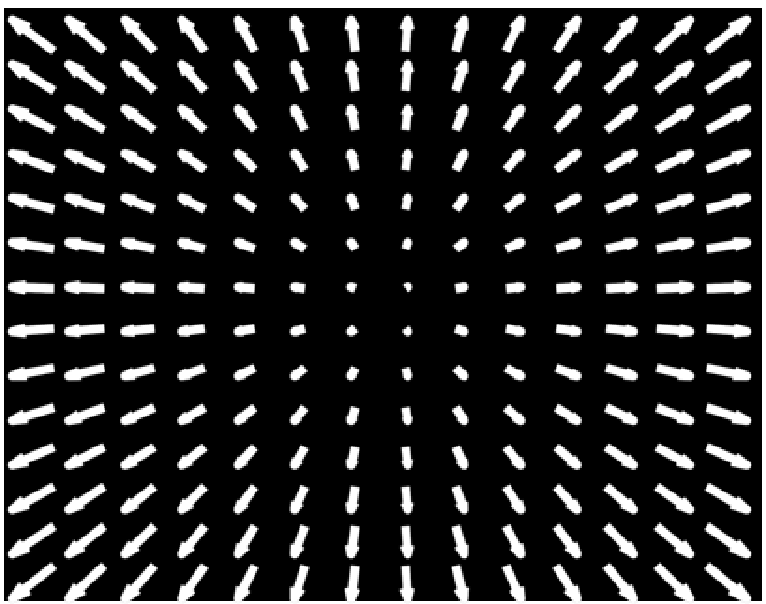

向外辐射状

表面朝相机移动

$$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

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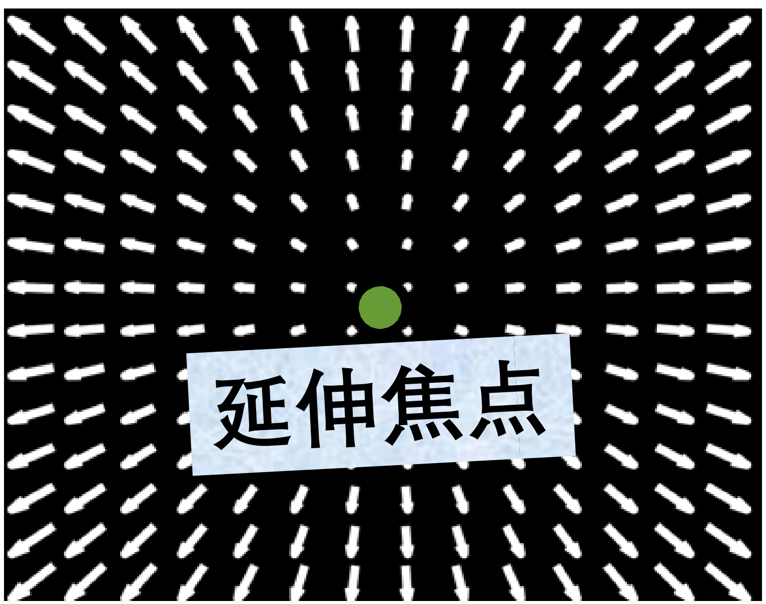
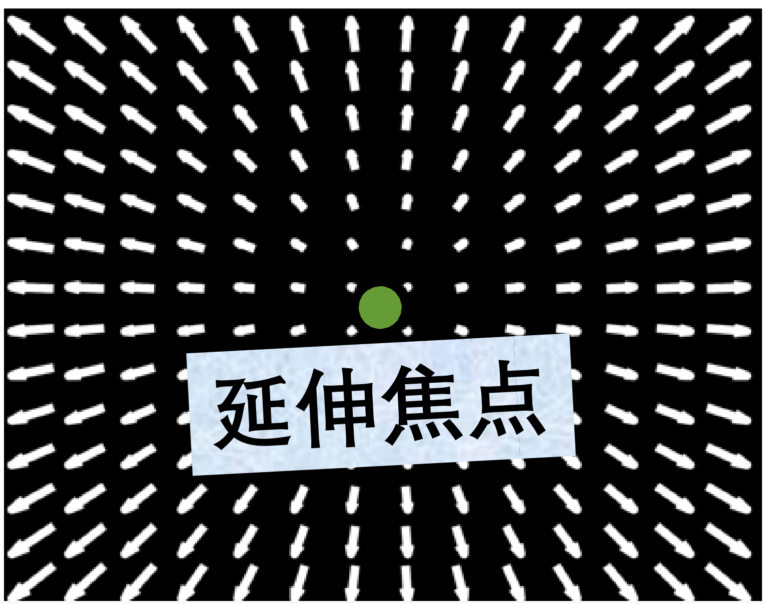


向外辐射状

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

$$u = \frac{t_z}{Z}\left(x - \frac{t_x}{t_z}\right) \qquad v = \frac{t_z}{Z}\left(y - \frac{t_y}{t_z}\right)$$



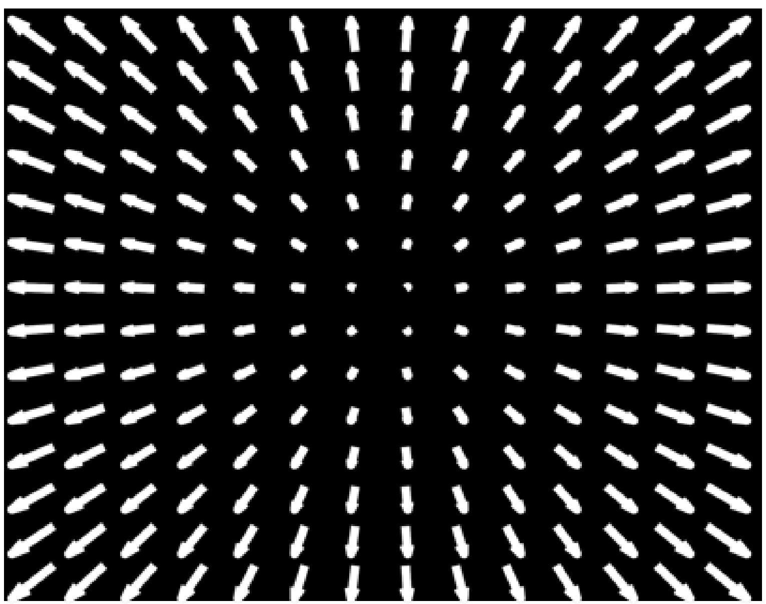
向外辐射状

延伸焦点 (Focus of Expansion, FOE): $\left(\frac{t_x}{t_z}f, \frac{t_y}{t_z}f\right)$

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z > 0$$

$$u = \frac{t_z}{Z}\left(x - \frac{t_x}{t_z}\right) \qquad v = \frac{t_z}{Z}\left(y - \frac{t_y}{t_z}\right)$$



向外辐射状

长度与深度成反比

运动场方程

$$u=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

$$v=\frac{1}{Z}(yt_z-t_y)+\omega_x(y^2+1)-\omega_y(xy)-\omega_z(x)$$

表面远离相机移动

特殊情况

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面远离相机移动

$$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面远离相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

代入

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

表面远离相机移动

$$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

代入

$$u = \frac{t_z}{Z}\left(x - \frac{t_x}{t_z}\right) \qquad v = \frac{t_z}{Z}\left(y - \frac{t_y}{t_z}\right)$$

表面朝相机移动

$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}}$ 且 $t_z < 0$

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运动场是什么样的?

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

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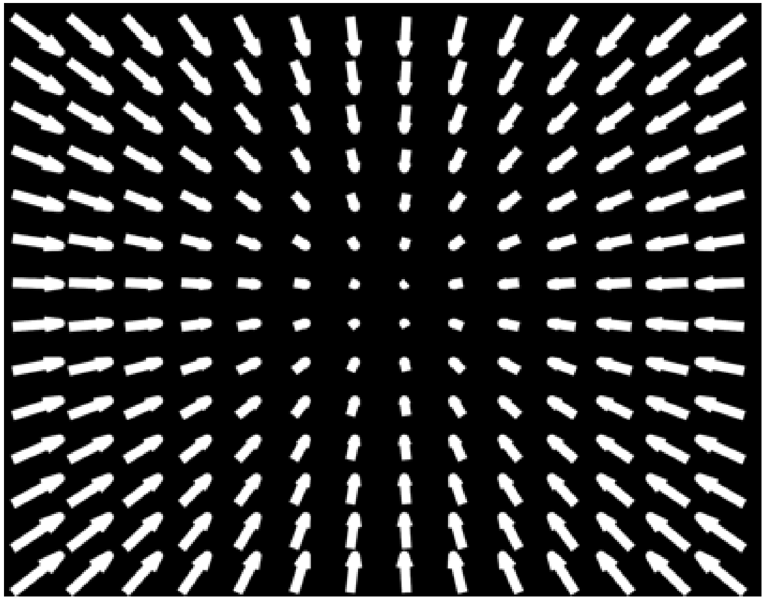

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

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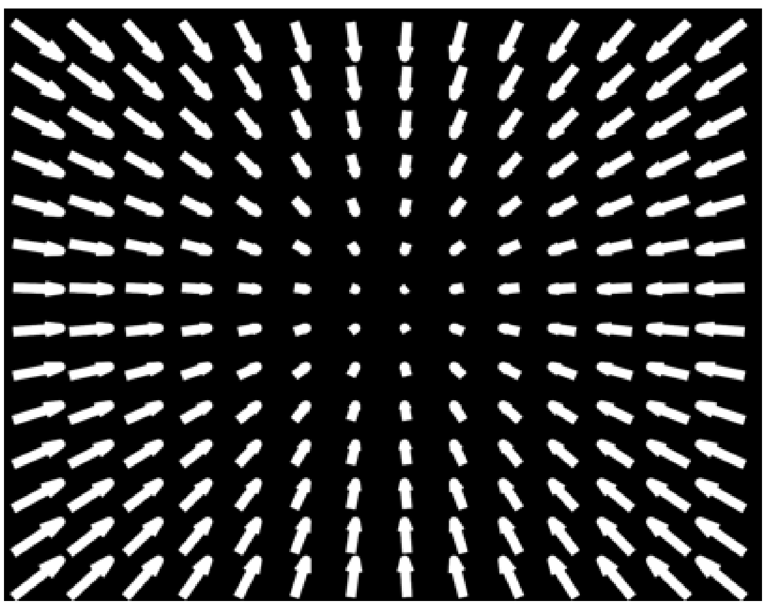

向内辐射状

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

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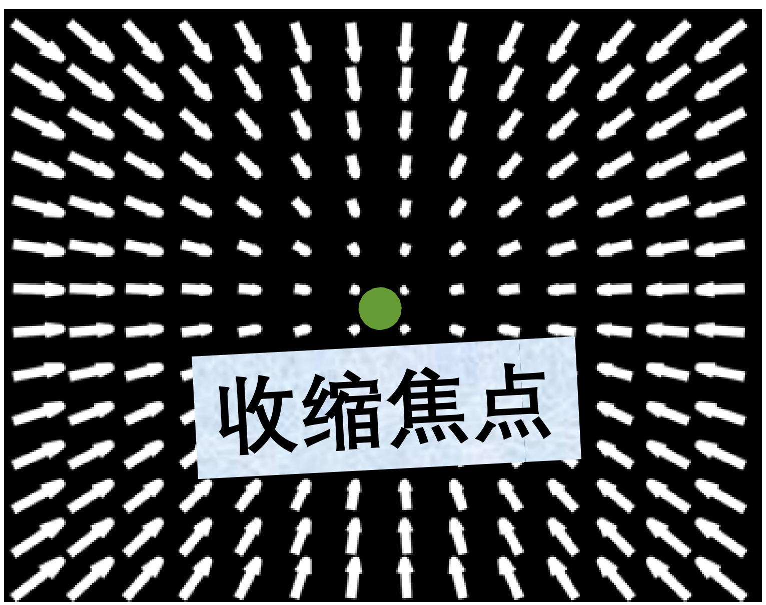

向内辐射状

表面朝相机移动

$$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

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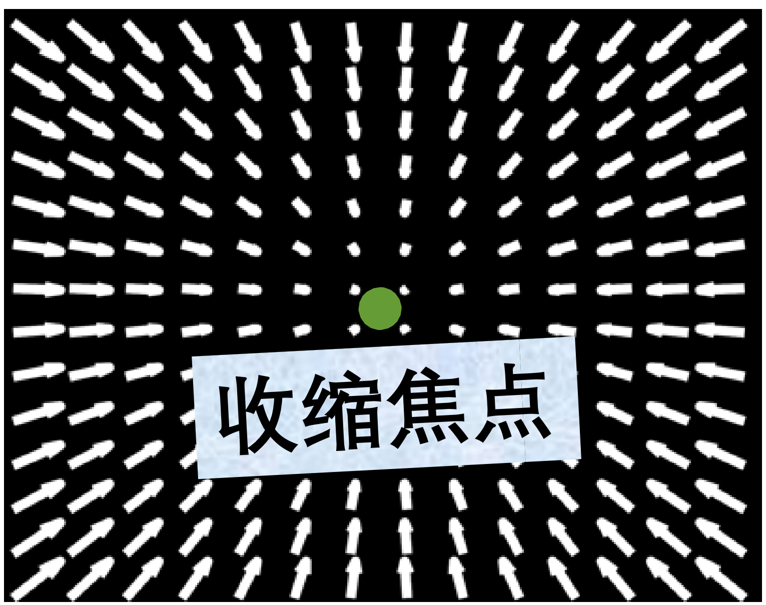


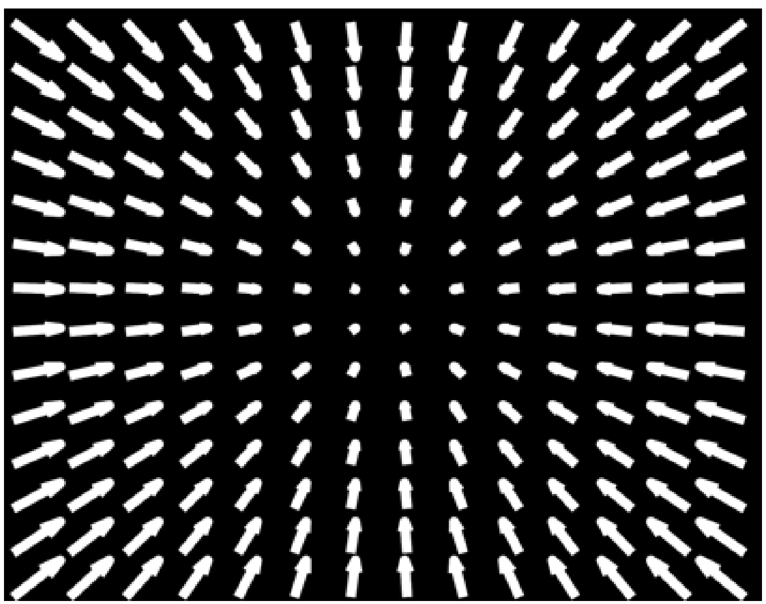
向内辐射状

收缩焦点 (Focus of Contraction, FOC): $\left(\frac{t_x}{t_z}f, \frac{t_y}{t_z}f\right)$

表面朝相机移动

$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} = (0,0,0)^{\mathrm{T}} \text{ 且 } t_z < 0$$

$$u = \frac{t_z}{Z}\left(x - \frac{t_x}{t_z}\right) \qquad v = \frac{t_z}{Z}\left(y - \frac{t_y}{t_z}\right)$$



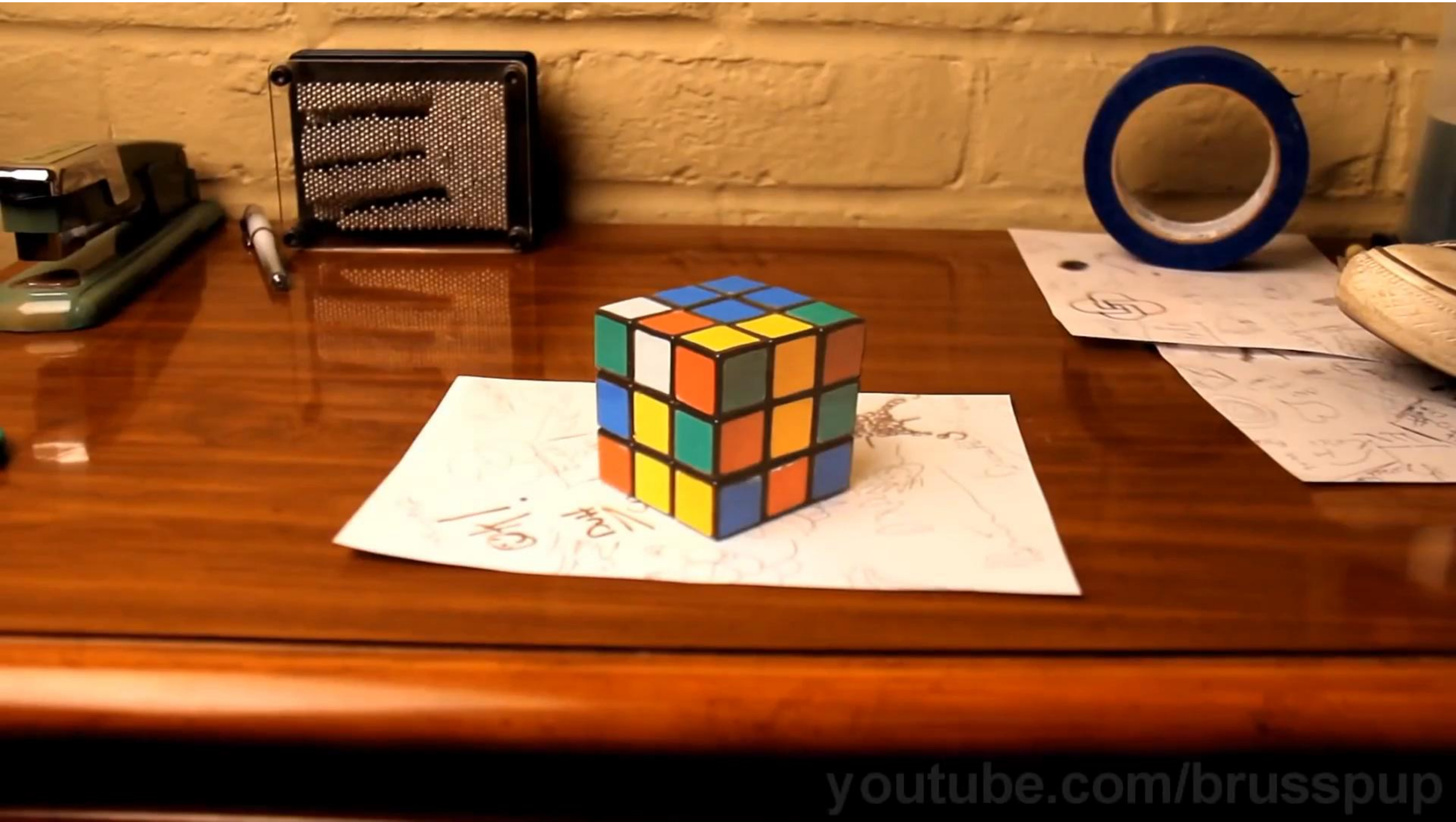
向内辐射状

长度与深度成反比

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$



运动场方程

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“纯”旋转

特殊情况

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$$\mathbf{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}} \text{ 且 } \mathbf{T} = \mathbf{0}$$

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代入

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图像坐标的二次函数

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运动场是什么样的?

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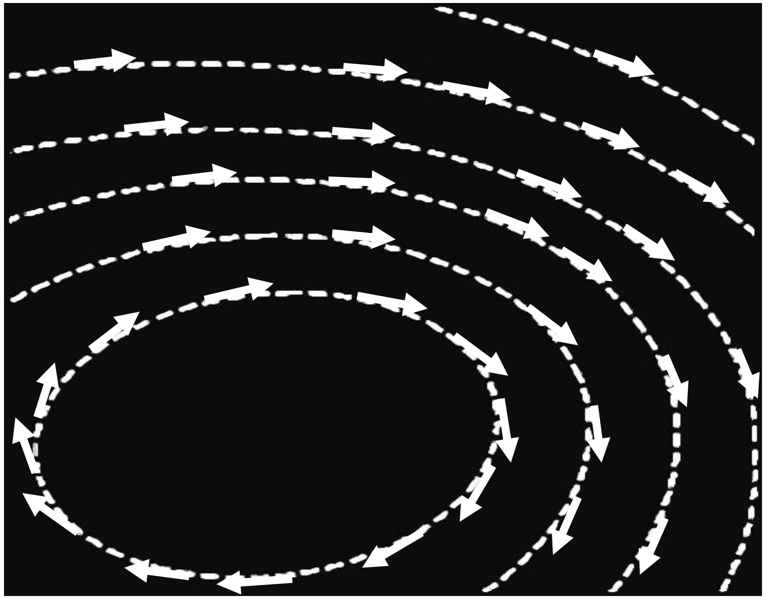

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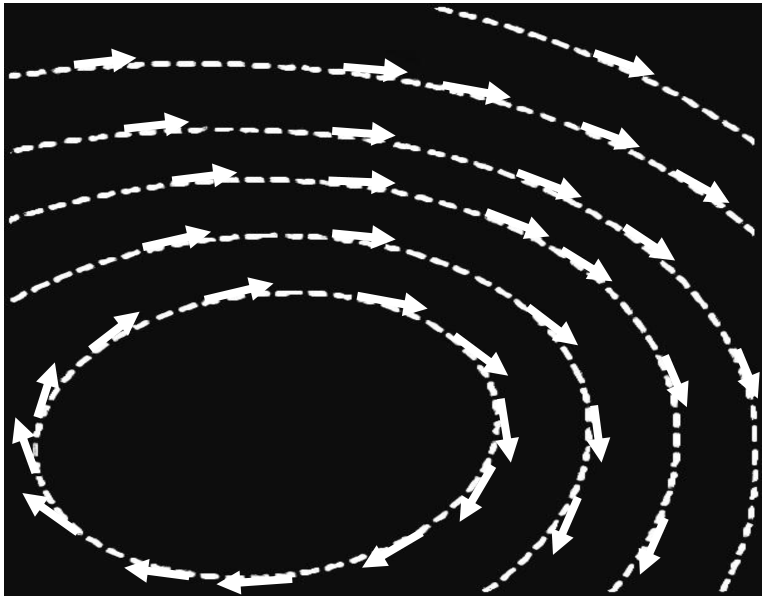

旋转状

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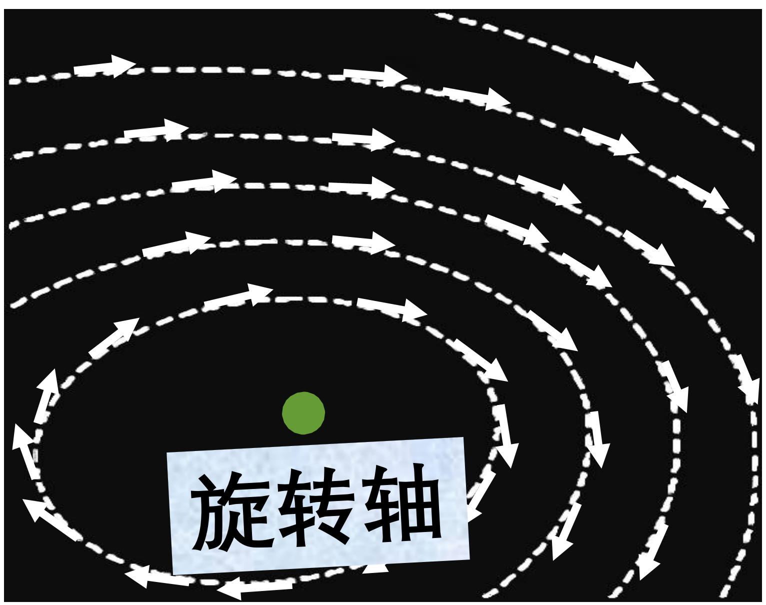

旋转状

旋转轴 (Axis of Rotation, AOR)

穿过图像平面： $\left(\dfrac{\omega_x}{\omega_z} f, \dfrac{\omega_y}{\omega_z} f\right)$

“纯”旋转

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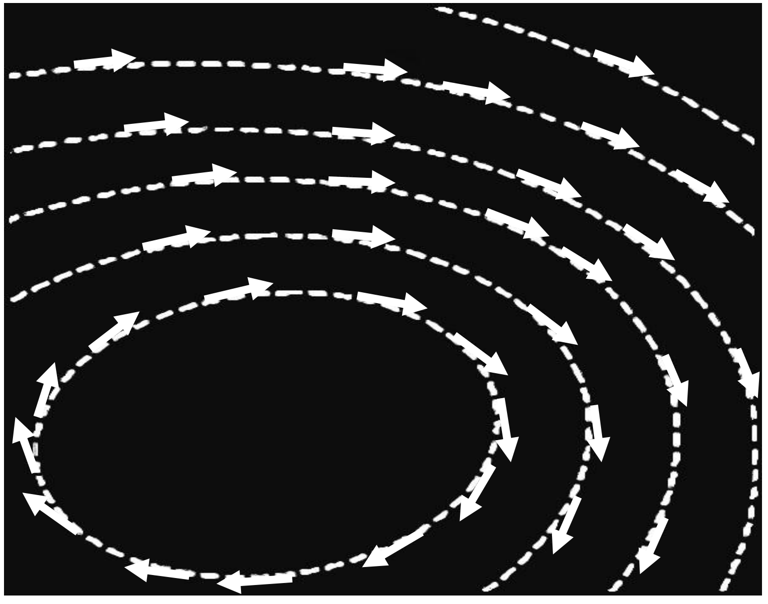
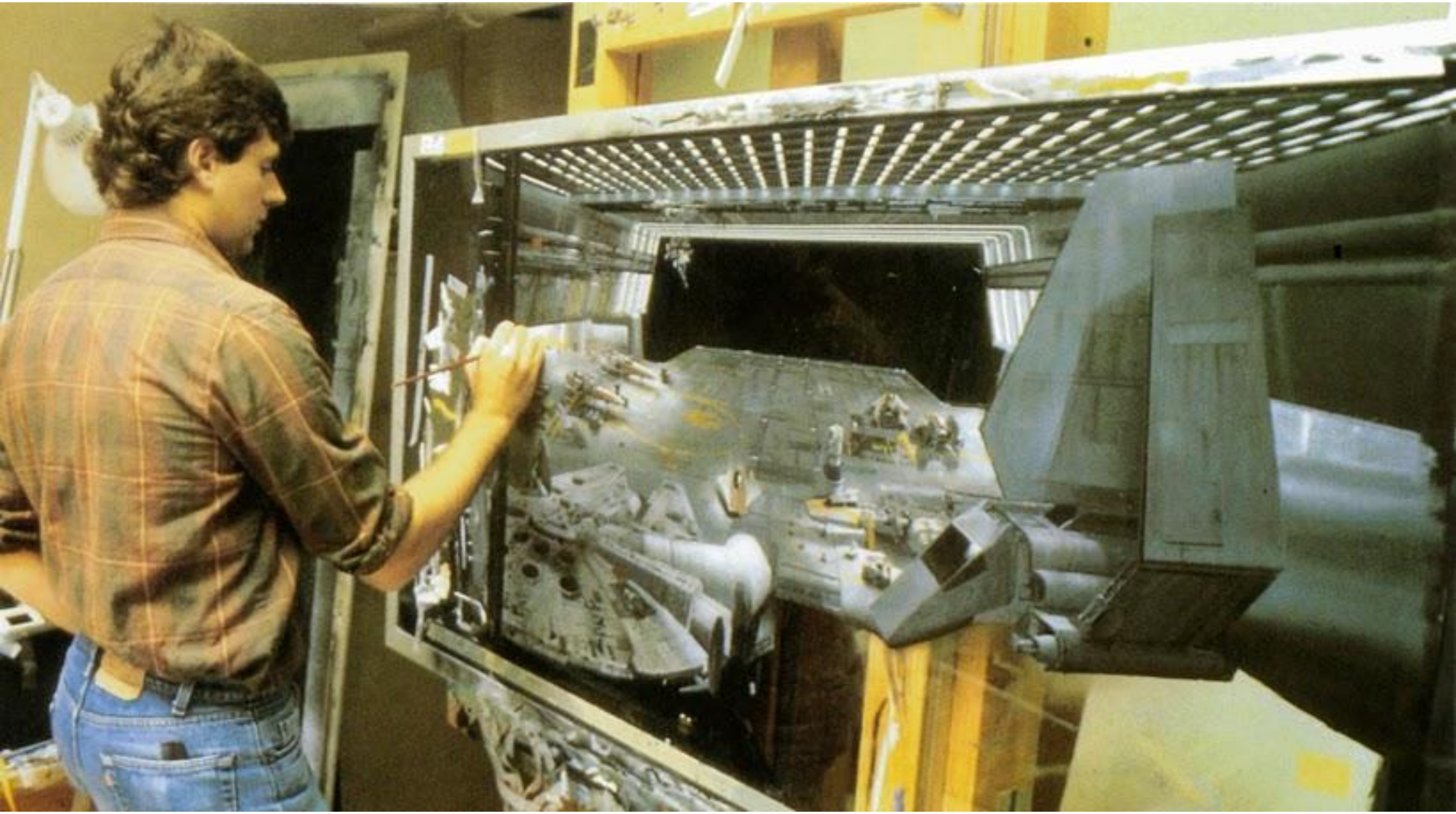


旋转状

长度与深度无关

星球大战VI：绝地归来



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旋转


Я
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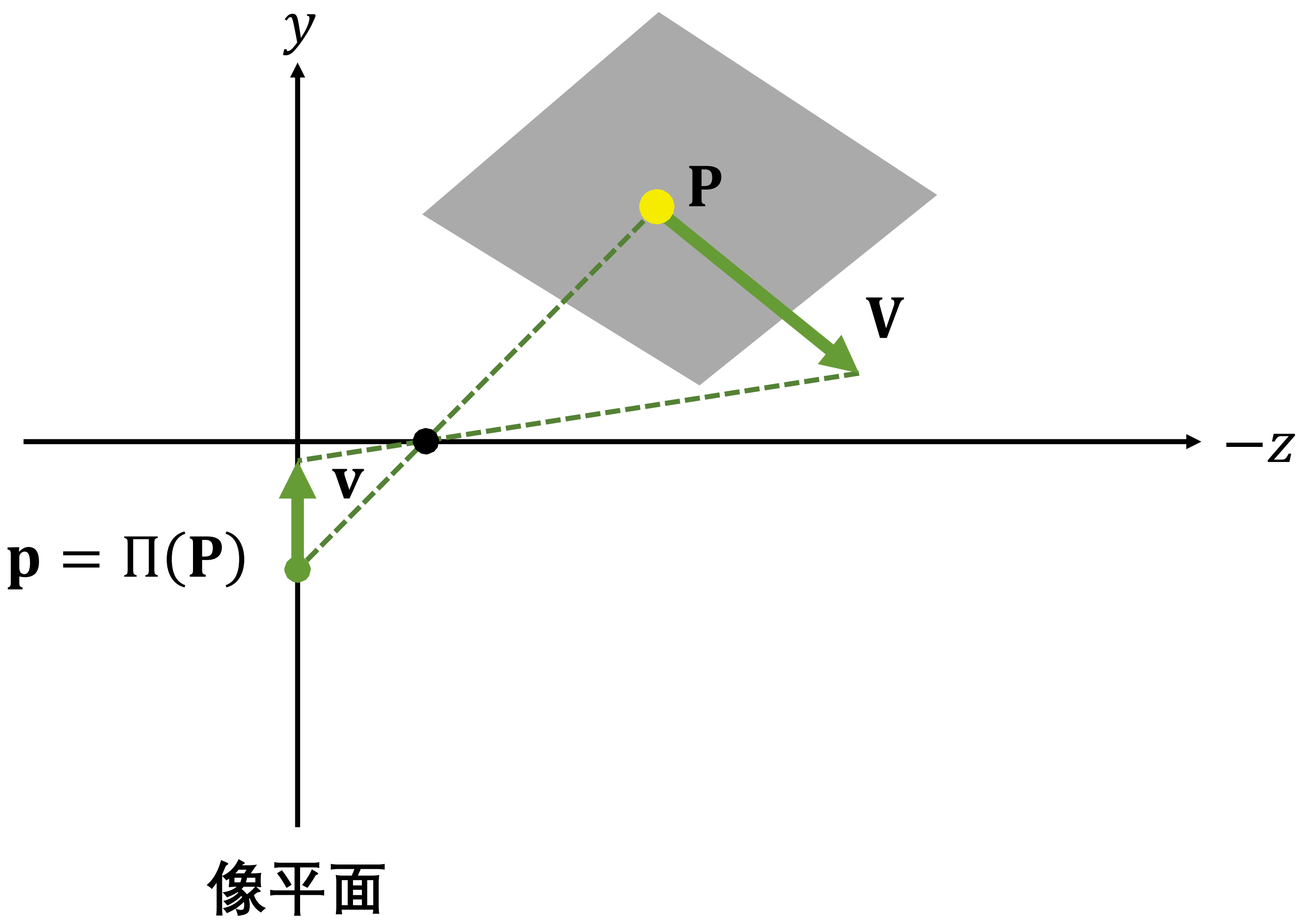
表面是一个平面



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表面是一个平面： $\alpha X + \beta Y + \gamma Z = \delta$

特殊情况

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已知（为了不失一般性，假设$f = 1$）

$$x = \frac{X}{Z} \qquad y = \frac{Y}{Z}$$

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整理

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a_i是未知平面和相机运动参数的函数

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平面表面的运动场由二次多项式

精确地、全局地给出

参数化流

$$I(x, y, t) = I(x + \Delta x(\mathbf{x}; \mathbf{p}), y + \Delta y(\mathbf{x}; \mathbf{p}), t + \Delta t)$$

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参数模型描述的运动

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光滑表面的流可以用一个低阶多项式来近似

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光滑表面的流可以用一个低阶多项式来近似

仿射模型

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给定3D运动参数和光流，估计结构

特殊情况

运动场方程

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给定3D运动参数和光流，估计结构

分离Z

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分离Z

$$u - \omega_x(xy) + \omega_y(x^2 + 1) - \omega_z(y) = \frac{1}{Z}(xt_z - t_x)$$

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“非旋转”光流

给定3D运动参数和光流，估计结构

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一个未知数的两个方程

给定3D运动参数和光流，估计结构

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Z可以从其中任一方程中恢复

给定3D运动参数和光流，估计结构

$$u-\omega_x(xy)+\omega_y(x^2+1)-\omega_z(y)=\frac{1}{Z}(xt_z-t_x)$$

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Z可以从其中任一方程中恢复

$$Z(x,y)=\frac{xt_z-t_x}{u-\omega_x(xy)+\omega_y(x^2+1)-\omega_z(y)}$$

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假设没有平移并给定光流，恢复3D旋转

特殊情况

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代入

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给定3个测量值可以恢复旋转速度

假设没有平移并给定光流，恢复3D旋转

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如何求解旋转速度?

假设没有平移并给定光流，恢复3D旋转

$\boldsymbol{\Omega} = \left(\omega_x, \omega_y, \omega_z\right)^{\mathrm{T}}$ 且 $\mathbf{T} = \mathbf{0}$

$$u = \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

使用最小二乘法求解

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使用最小二乘法求解

$$\underset{\omega_x, \omega_y, \omega_z}{\arg\min} \sum_x \sum_y \Big[\big(u - [\omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)]\big)^2 + \big(v - [\omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)]\big)^2\Big]$$

假设没有平移并给定光流，恢复3D旋转

使用最小二乘法求解

$$\arg\min_{\omega_x,\omega_y,\omega_z} \sum_x \sum_y \Big[\big(u - [\omega_x(xy) - \omega_y(x^2+1) + \omega_z(y)]\big)^2 + \big(v - [\omega_x(y^2+1) - \omega_y(xy) - \omega_z(x)]\big)^2\Big]$$

如何求解该目标函数？

假设没有平移并给定光流，恢复3D旋转

使用最小二乘法求解

$$\underset{\omega_x,\omega_y,\omega_z}{\arg\min} \sum_x \sum_y \Big[\big(u - [\omega_x(xy) - \omega_y(x^2+1) + \omega_z(y)]\big)^2 + \big(v - [\omega_x(y^2+1) - \omega_y(xy) - \omega_z(x)]\big)^2 \Big]$$

如何求解该目标函数?

求导，令其等于零并求解

假设没有平移并给定光流，恢复3D旋转

使用最小二乘法求解

$$\arg\min_{\omega_x,\omega_y,\omega_z} \sum_x \sum_y \left[\left(u - \left[\omega_x(xy) - \omega_y(x^2+1) + \omega_z(y)\right]\right)^2 + \left(v - \left[\omega_x(y^2+1) - \omega_y(xy) - \omega_z(x)\right]\right)^2\right]$$

如何求解该目标函数?

使用伪逆求解

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

特殊情况

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

代入

运动场方程

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

代入

$$u = \frac{\alpha}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1)$$

$$v = \frac{\alpha}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy)$$

运动场方程

$$u=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

$$v=\frac{1}{Z}(yt_z-t_y)+\omega_x(y^2+1)-\omega_y(xy)-\omega_z(x)$$

假设给定线速度方向且旋转角$\omega_z=0$，给定光流，恢复3D旋转

未知尺度因子

代入

$$u=\frac{\alpha}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)$$

$$v=\frac{\alpha}{Z}(yt_z-t_y)+\omega_x(y^2+1)-\omega_y(xy)$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$u = \frac{\alpha}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1)$$

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假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$u = \frac{\alpha}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1)$$

$$v = \frac{\alpha}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy)$$

计算流方程与下式的点积

$$\begin{pmatrix} t_y - yt_z \\ -t_x + xt_z \end{pmatrix}$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$u = \cancel{\frac{a}{Z}(xt_z - t_x)} + \omega_x(xy) - \omega_y(x^2 + 1)$$

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计算流方程与下式的点积

$$\begin{pmatrix} t_y - yt_z \\ -t_x + xt_z \end{pmatrix}$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$u = \cancel{\frac{d}{Z}(xt_z - t_x)} + \omega_x(xy) - \omega_y(x^2 + 1)$$

$$v = \cancel{\frac{d}{Z}(yt_z - t_y)} + \omega_x(y^2 + 1) - \omega_y(xy)$$

计算流方程与下式的点积

$$\begin{pmatrix} t_y - yt_z \\ -t_x + xt_z \end{pmatrix}$$

$$\begin{aligned} u(t_y - yt_z) + v(-t_x + xt_z) = \\ &[\omega_x(xy) - \omega_y(x^2 + 1)](t_y - yt_z) + \\ &[\omega_x(y^2 + 1) - \omega_y(xy)](-t_x + xt_z) \end{aligned}$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$
\begin{aligned}
u\left(t_y - yt_z\right) + v(-t_x + xt_z) = \\
&\left[\omega_x(xy) - \omega_y(x^2+1)\right]\left(t_y - yt_z\right) + \\
&\left[\omega_x(y^2+1) - \omega_y(xy)\right](-t_x + xt_z)
\end{aligned}
$$

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$
\begin{aligned}
u(t_y - y t_z) + v(-t_x + x t_z) = \\
&[\omega_x(xy) - \omega_y(x^2+1)](t_y - y t_z) + \\
&[\omega_x(y^2+1) - \omega_y(xy)](-t_x + x t_z)
\end{aligned}
$$

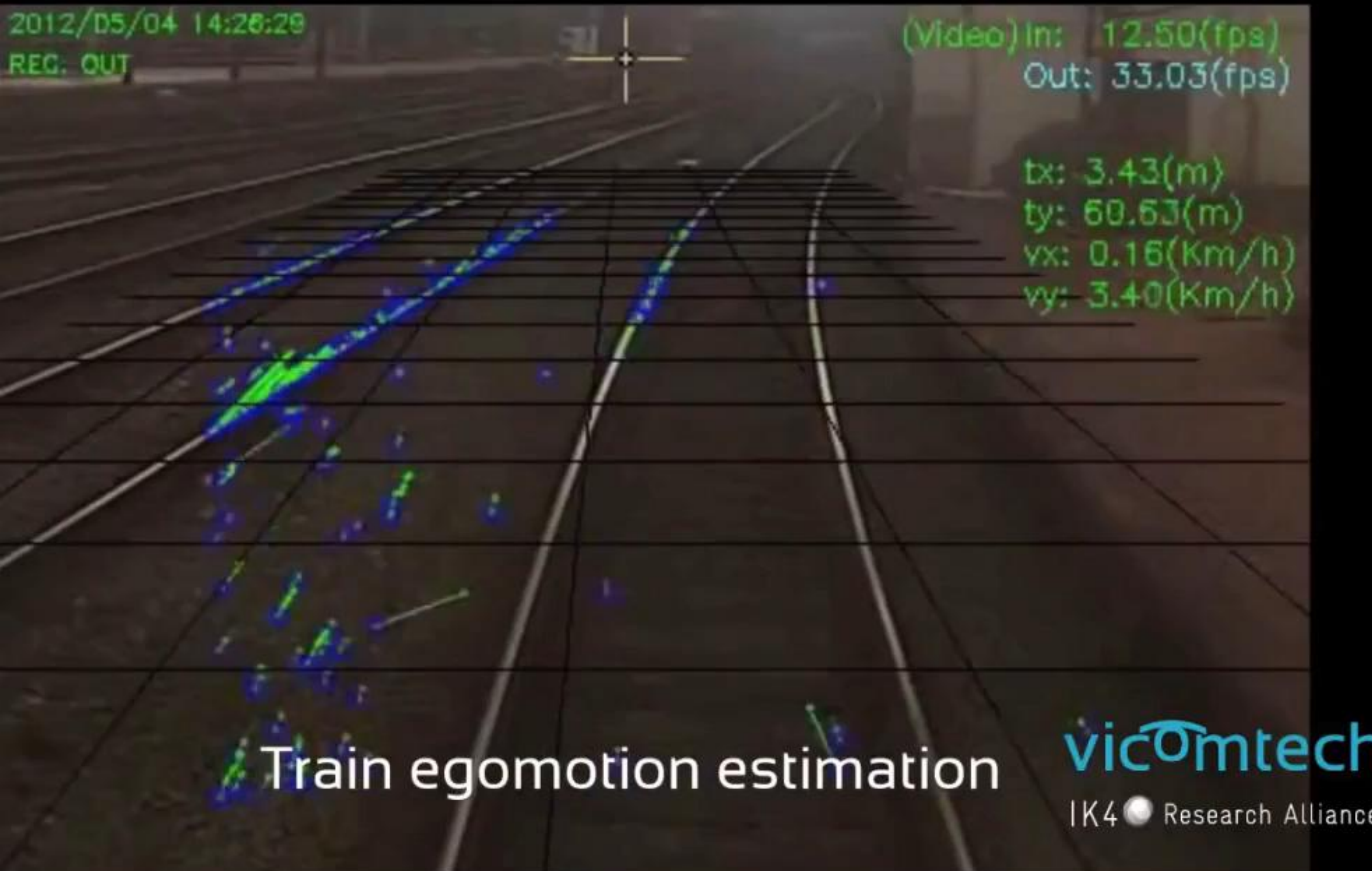
得到一个两个未知数的线性非齐次方程

假设给定线速度方向且旋转角$\omega_z = 0$，给定光流，恢复3D旋转

$$u(t_y - y t_z) + v(-t_x + x t_z) =$$
$$\left[\omega_x (xy) - \omega_y (x^2 + 1)\right](t_y - y t_z) +$$
$$\left[\omega_x (y^2 + 1) - \omega_y (xy)\right](-t_x + x t_z)$$

得到一个两个未知数的线性非齐次方程

可以使用最小二乘法求解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Out: 33.03(fps)
tx: 3.43(m)
ty: 60.63(m)
vx: 0.16(Km/h)
vy: 3.40(Km/h)
Train egomotion estimation
vicomtech
IK4 Research Alliance

碰撞时间 (time-to-contact)

如何计算碰撞时间?

如何计算碰撞时间？

相机原点到达被观察表面前的持续时间

碰撞时间

碰撞时间

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

碰撞时间

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计算光流场的散度

碰撞时间

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

微积分和几何学

碰撞时间

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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计算光流场的散度

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碰撞时间

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计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x + xt_z) + 2\rho t_z + \frac{\partial \rho}{\partial y}\left(-t_y + yt_z\right)$$
$$+y\omega_x - 2x\omega_y + 2y\omega_x - x\omega_y$$

碰撞时间

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

计算光流场的散度

深度倒数

$$\mathrm{div}(u, v) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x + xt_z) + 2\rho t_z + \frac{\partial \rho}{\partial y}(-t_y + yt_z)$$
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$$u=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

$$v=\frac{1}{Z}\left(yt_z-t_y\right)+\omega_x(y^2+1)-\omega_y(xy)-\omega_z(x)$$

计算光流场的散度

$$\mathrm{div}(u,v)=\frac{\partial\rho}{\partial x}(-t_x+xt_z)+2\rho t_z+\frac{\partial\rho}{\partial y}\left(-t_y+yt_z\right)$$
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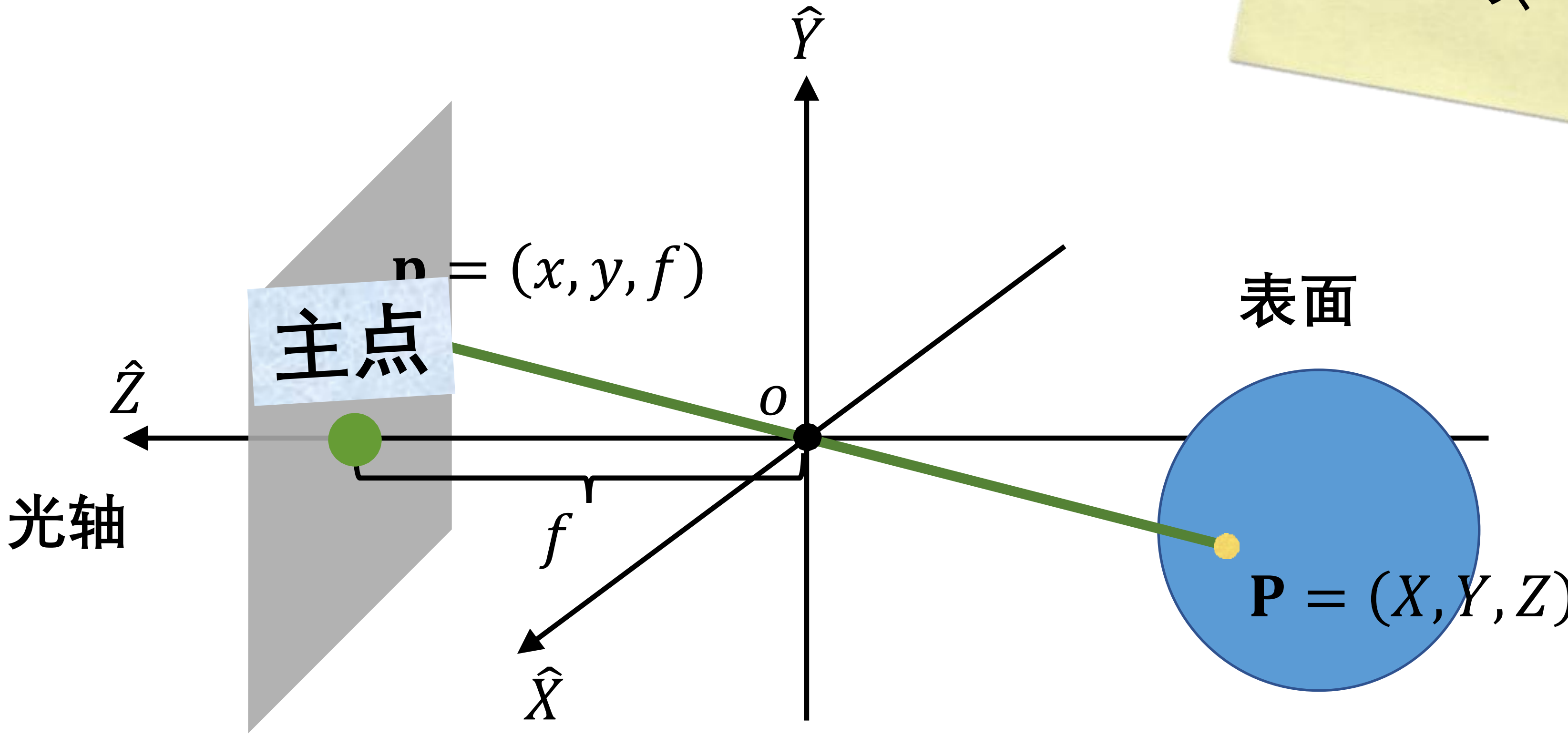
计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial\rho}{\partial x}(-t_x + xt_z) + 2\rho t_z + \frac{\partial\rho}{\partial y}\left(-t_y + yt_z\right)$$
$$+y\omega_x - 2x\omega_y + 2y\omega_x - x\omega_y$$

计算主点的散度

回顾



$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x + xt_z) + 2\rho t_z + \frac{\partial \rho}{\partial y}\left(-t_y + yt_z\right)$$
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计算主点的散度

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计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x + \cancel{xt_z}) + 2\rho t_z + \frac{\partial \rho}{\partial y}\left(-t_y + \cancel{yt_z}\right)$$
$$+\cancel{y\omega_x} - \cancel{2x\omega_y} + \cancel{2y\omega_x} - \cancel{x\omega_y}$$

计算主点的散度

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计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x + \cancel{xt_z}) + 2\rho t_z + \frac{\partial \rho}{\partial y}(-t_y + \cancel{yt_z})$$
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计算主点的散度

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x) + 2\rho t_z + \frac{\partial \rho}{\partial y}(-t_y)$$

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x) + 2\rho t_z + \frac{\partial \rho}{\partial y}\left(-t_y\right)$$

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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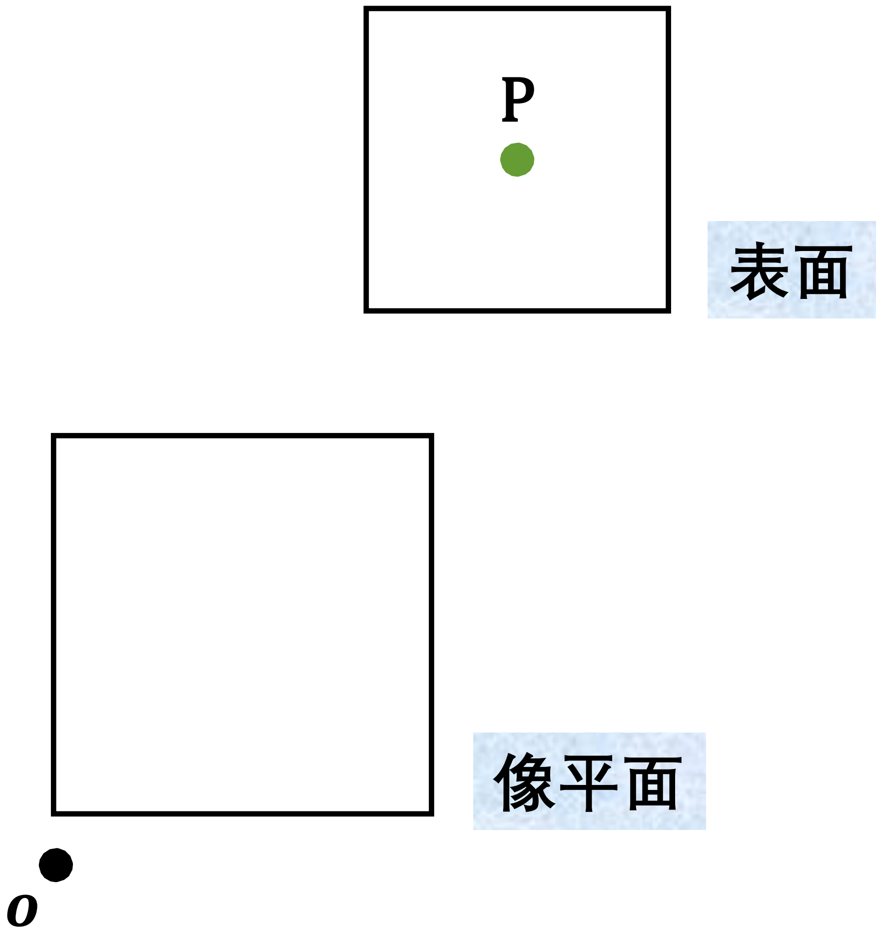
计算光流场的散度

$$\mathrm{div}(u, v) = \frac{\partial \rho}{\partial x}(-t_x) + 2\rho t_z + \frac{\partial \rho}{\partial y}\left(-t_y\right)$$

假设成像的表面是一个正面平行平面


P
表面
像平面
o

$$u=\frac{1}{Z}(xt_z-t_x)+\omega_x(xy)-\omega_y(x^2+1)+\omega_z(y)$$

$$v=\frac{1}{Z}\left(yt_z-t_y\right)+\omega_x(y^2+1)-\omega_y(xy)-\omega_z(x)$$

计算光流场的散度

$$\operatorname{div}(u,v)=\frac{\partial\rho}{\partial x}(-t_x)+2\rho t_z+\frac{\partial\rho}{\partial y}\left(-t_y\right)$$

假设成像表面是一个正面平行平面

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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计算光流场的散度

$$\mathrm{div}(u, v) = \cancel{\frac{\partial \rho}{\partial x}(-t_x)} + 2\rho t_z + \frac{\partial \rho}{\partial y}(-t_y)$$

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计算光流场的散度

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计算光流场的散度

$$\mathrm{div}(u, v) = \cancel{\frac{\partial\rho}{\partial x}(-t_x)} + 2\rho t_z + \cancel{\frac{\partial\rho}{\partial y}(-t_y)}$$

假设成像表面是一个正面平行平面

$$\mathrm{div}(u, v) = 2\rho t_z$$

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}\left(yt_z - t_y\right) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

计算光流场的散度

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计算光流场的散度

$$\mathrm{div}(u, v) = 2\rho t_z$$

碰撞时间可直接从散度估计

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

计算光流场的散度

$$\mathrm{div}(u, v) = 2\rho t_z$$

碰撞时间可直接从散度估计

$$T_{\mathrm{time-to-contact}} = \frac{Z}{t_z}$$

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

计算光流场的散度

$$\mathrm{div}(u, v) = 2\rho t_z$$

碰撞时间可直接从散度估计

$$T_{\text{time-to-contact}} = \frac{Z}{t_z} = \frac{2}{\mathrm{div}(u, v)}$$

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

$$v = \frac{1}{Z}(yt_z - t_y) + \omega_x(y^2 + 1) - \omega_y(xy) - \omega_z(x)$$

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碰撞时间可直接从散度估计

$$T_{\text{time-to-contact}} = \frac{Z}{t_z} = \frac{2}{\operatorname{div}(u, v)}$$

图像测量

$$u = \frac{1}{Z}(xt_z - t_x) + \omega_x(xy) - \omega_y(x^2 + 1) + \omega_z(y)$$

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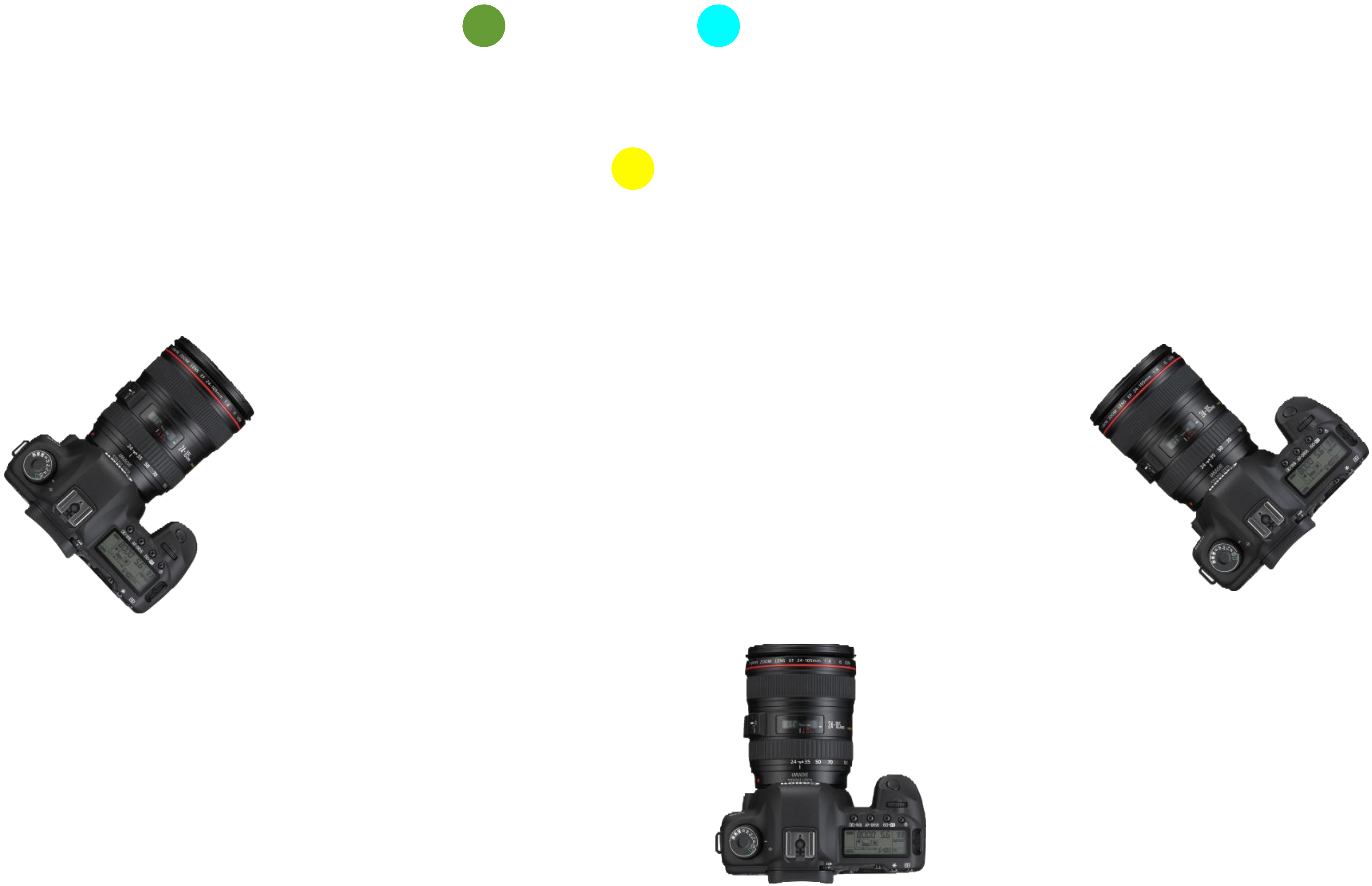
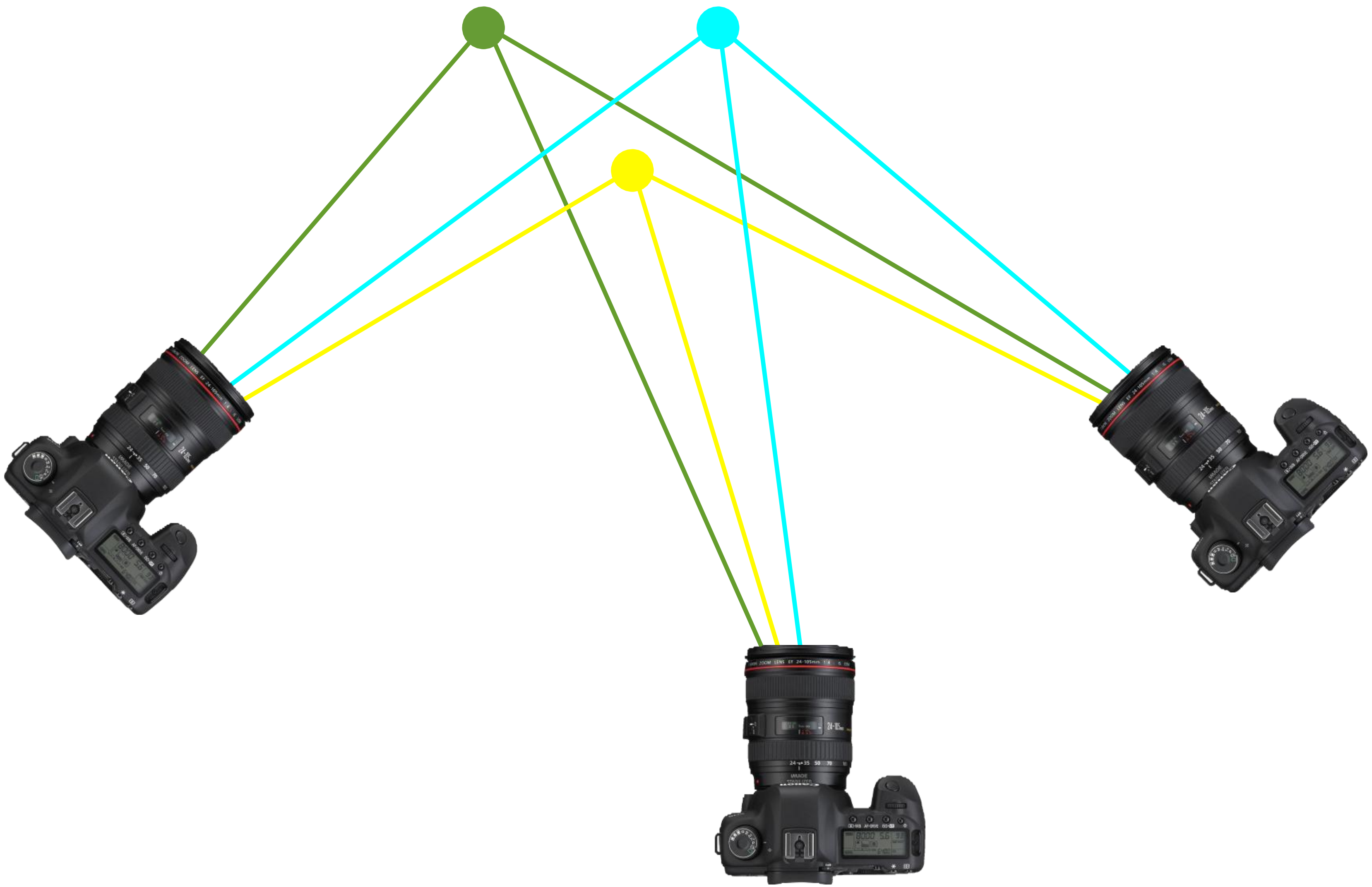
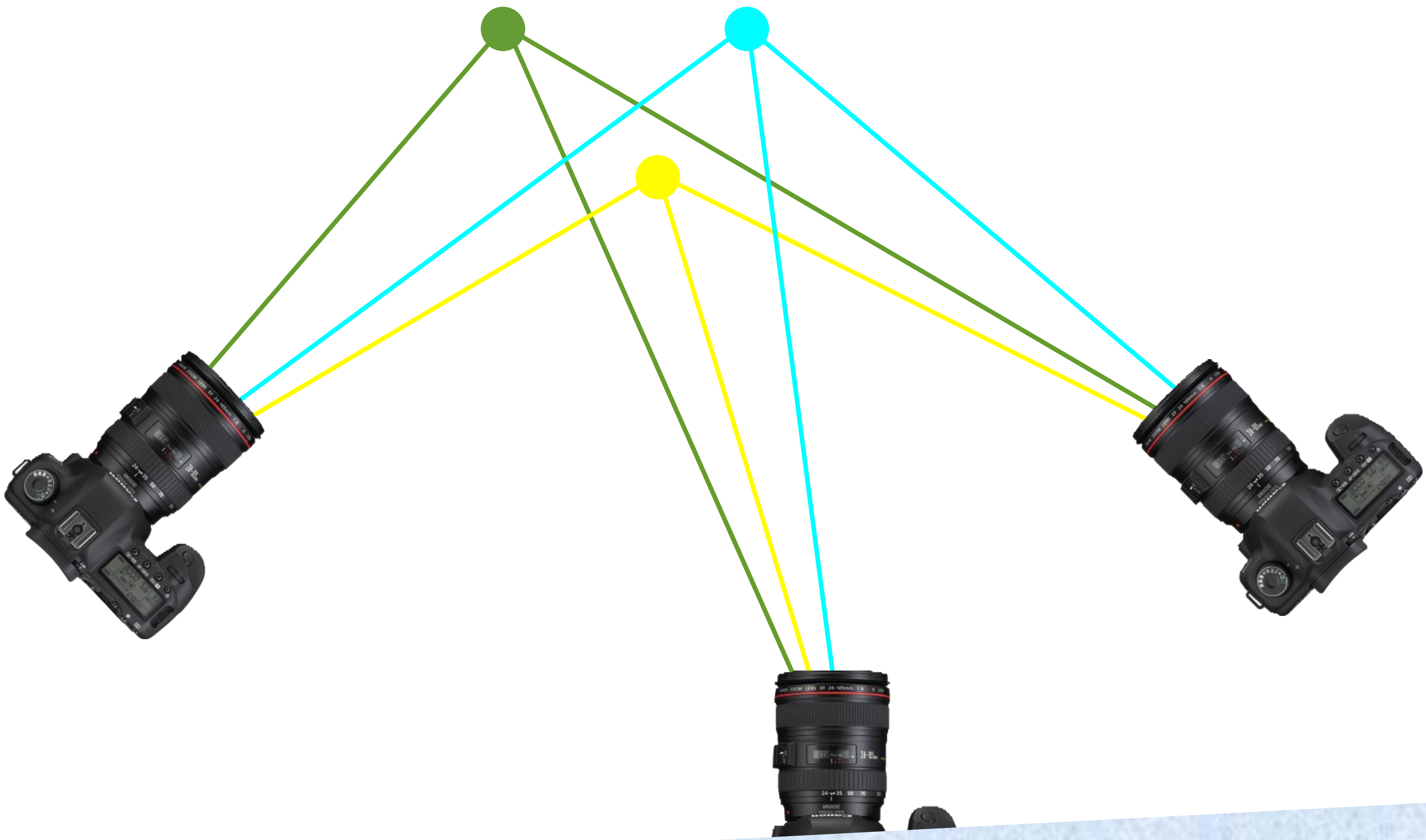
碰撞时间可直接从散度估计

$$T_{\text{time-to-contact}} = \frac{Z}{t_z} = \frac{2}{\operatorname{div}(u, v)}$$

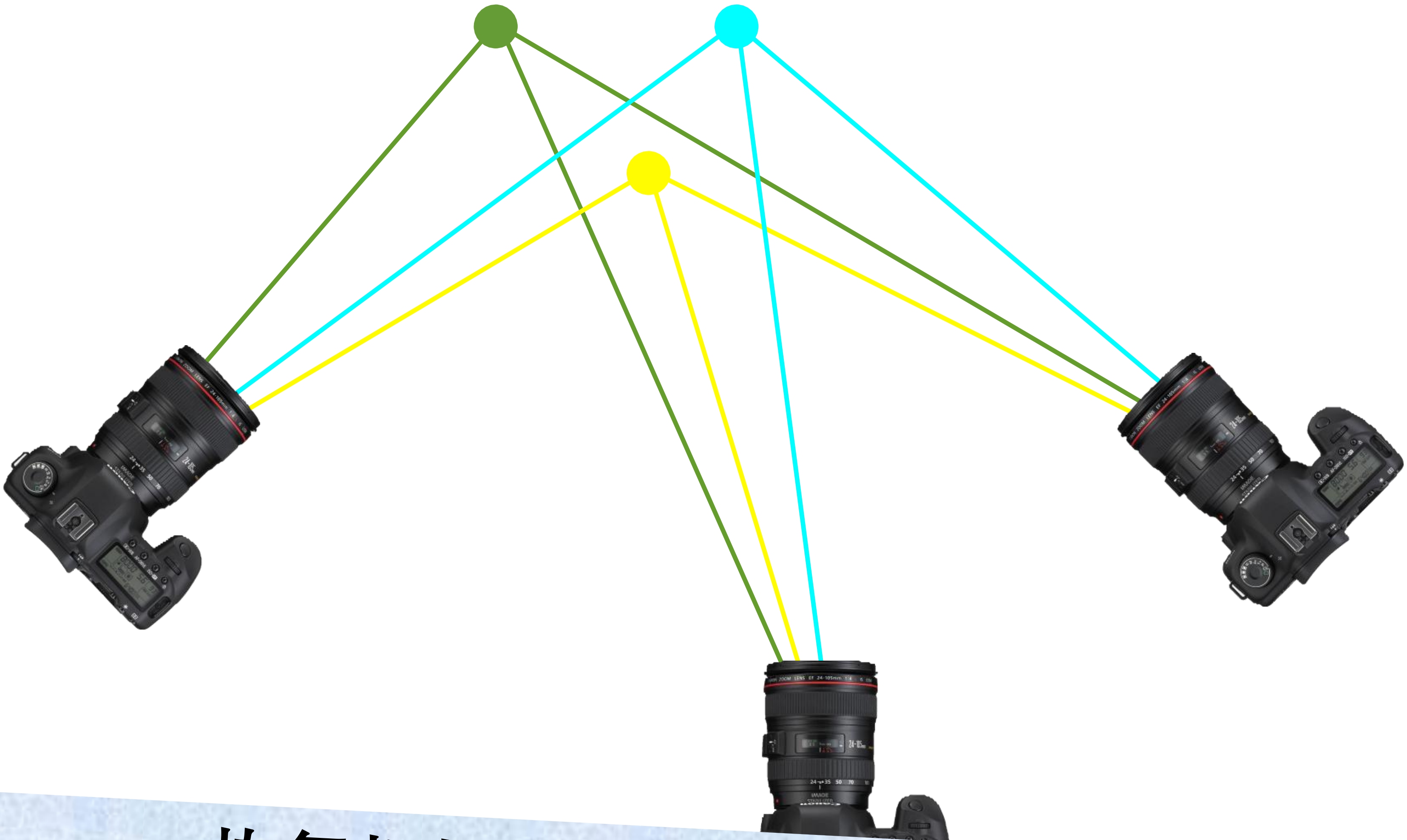
由运动恢复结构
(Structure from Motion)
有限位移法





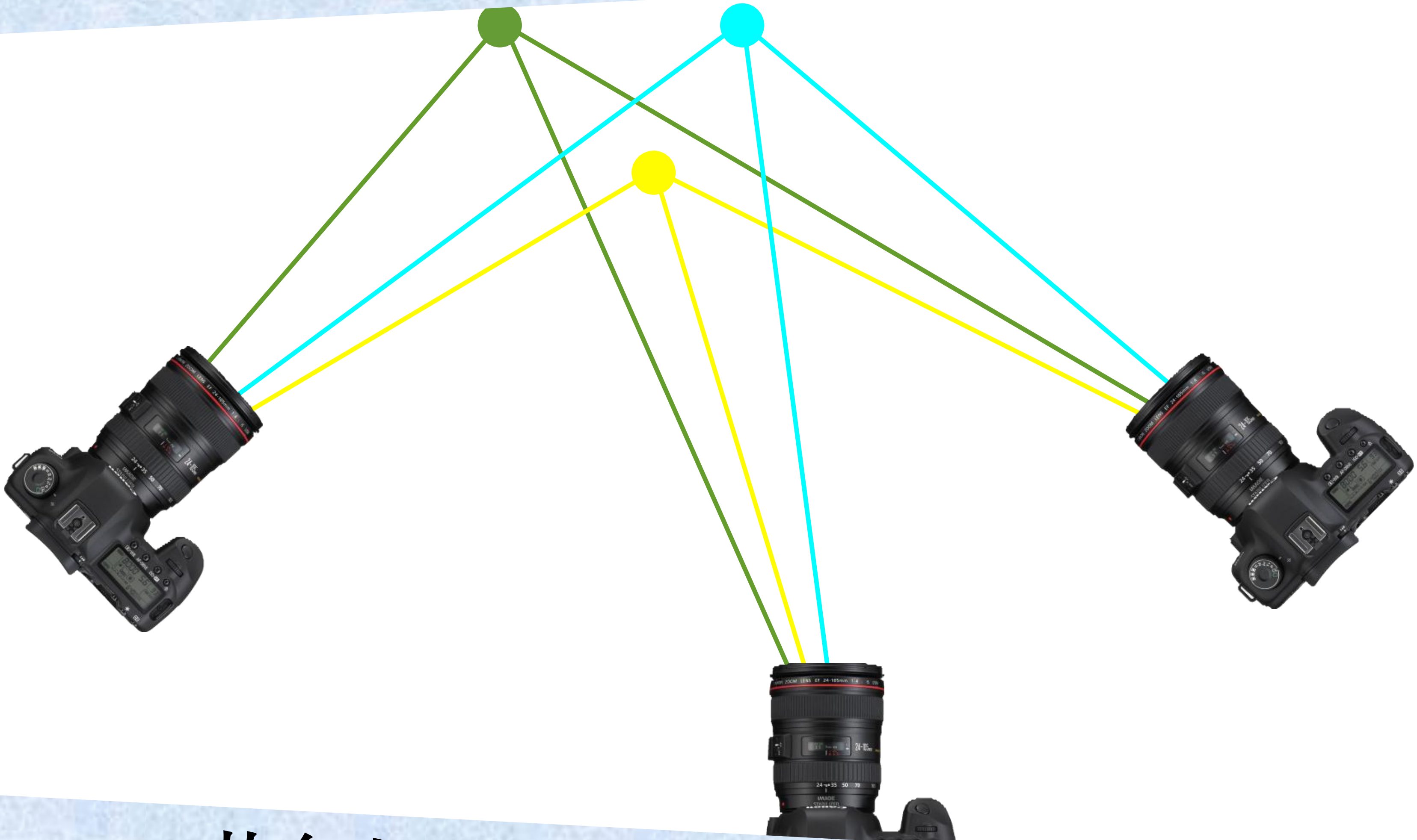


给定跨多个视图的点对应关系



恢复相机的相对3D旋转和平移

恢复3D场景结构



恢复相机的相对3D旋转和平移

Shape and Motion from Image Streams under Orthography: a Factorization Method

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Abstract
Inferring scene geometry and camera motion from a stream of images is possible in principle, but is an ill-conditioned problem when the objects are distant with respect to their size. We have developed a *factorization method* that can overcome this difficulty by recovering shape and motion under orthography without computing depth as an intermediate step

International Journal of Computer Vision, 1992

Tomasi and Kanade, 1992

tinyurl.com/SFM-KGD